Modeling and Simulation for Orientation Control of an Under-actuated Drill Machine

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ABSTRACT

Modelling for Orientation control of a Drill Machine belonging to a class of under actuated system is presented. The orientation control along x-axis and y-axis of the drill machine is achieved through a single pair of electromagnetic poles. Based on the concept of pulse width modulation a controlling signal for the actuators which caters for all practical limitations of the system is generated. The derived discrete time equivalent model overcomes the practical limitation of control techniques used for such class of systems. It also opens a new dimension especially for under-actuated and non-linear systems which are otherwise difficult rather impossible through orthodox control schemes. A closed form based on discrete time equivalent model supported by simulation results will be helpful in designing overall feedback control for the orientation of the system.

KEYWORDS

Under Actuated System, Control, Pulse Equivalent Area, Pulse width Modulation

1. INTRODUCTION

Multi-axis machining is almost two to three decades old but with innovative ideas, challenging machining requirements and invention of new type of materials keeps this technological field still fertile and green for scientist and engineers for further research and experimentation. The ideas are not limited only for exploring new & innovative and versatile means of machining but field is open and rather more importantly to carry out further improvements including addressing of practical problems, improve efficiency, decrease wastage in terms of materials, time and maintenance cost etc, to carryout performance and error analysis in existing and already developed techniques [1,2,3,4]. The extensive use of multi-axis drilling in the industry [5,6] especially in the field of precision machinery for the drilling of soft materials, PCBs or even drilling on the curved surfaces where the drill bit should be kept perpendicular to the surface is foremost important.

A similar application was discussed in [7], where a two axis drill machine was used. The drill bit is free to move along X & Y axis as shown in figure 1, while it spins or rotates about its z-axis. The movement in X & Y axis are executed or implemented by electromagnetic windings on the stator and permanent magnets mounted on the rotor which in this case is drill bit itself. The magnetic field produced by the

Figure 1. Under-actuated Drill Machine [7]
poles of stator winding interacts with that of permanent magnets mounted on the drill bit produces a torque which in turns is used for changing the drill orientation. The stator poles are excited at a particular phase in every revolution. Due to the mounting and weight limitations a single pair of electromagnets was used and therefore the desired orientation is achieved only by using single actuation thereby reducing the dual axis actuation to single for a two degree of freedom control. Hence the overall system reduces to under-actuated system as the degrees of freedom are greater than the number of control inputs.

Although the control of actuator as a rotor from the excitation of the stator windings is a well-established technique [8,9,10,11]. But all these techniques are based on fully actuated principles. The control of under-actuated systems is challenging and that is the reason it is one of the favorite topics in field of controls and dynamics [12,13,14,15,16,17,18]. However mostly these systems are time invariant but here the system under discussion is time variant.

The approach of [7] utilizes discrete-time controller to determine the magnitude and the phase of actuation pulse for the desired orientation of the drill bit on the basis of sampled state feedback. A discrete-time equivalent model was developed by considering one complete revolution of drill bit as a discrete step. The model was derived in discrete system which makes it easier for implementation and overcoming under actuation limitations. Although the solution provided by [7] is very precise and effective in controlling the orientation, but has a major practical limitation that the duration of the pulse is fixed and the level or intensity of control force is translated into the magnitude of the pulse. In physical system such unlimited amplitude is not available as it leads to saturation. On the other hand if a constraint is put on the maximum amplitude of control signal then this will pose serious limitation on the control input which restricts the overall movement of the drill and leads to stability issues too. The proposed remedy to this limitation is the use of a Pulse Width modulation technique [19, 25], in which a pulse of varying width having fixed amplitude is used. This resulted in overcoming the control effort limitation as well as provides a smooth control for precise movement of the drill bit.

The remainder of the paper is organized as follows: section II covers the structure and construction of the drill machine followed by system modeling in section III. In section IV, the already established techniques which were and can be applied. Section V covers the discrete time equivalent model, and the conclusions are drawn in Section VI followed by references.

2. STRUCTURE & CONSTRUCTION

The plant is a small drill machine and its construction is shown in the figure 1[7]. It spins or rotates about its z-axis while the up & down and right & left movements are about x-axis and y-axis respectively. Due to the mounting limitations mainly because of its miniature size only a single pair of magnets is used. As the drill bit is rotating therefore both axis can be excited but only one axis can be excited at one time, thus converting the fully actuated drill machine into an under actuated system.

Detailed description of drill machine was explained in detail [7]. The stator is a cylindrical body comprising of single phase winding while the rotor is a permanent magnet mounted on a spherical joint which is allowing its motion about the three axes. The two frames of reference one that is attached with the rotor (X’, Y’, Z’) and the other with the stator (X, Y, Z)

![Figure 2. Frames of References](image-url)
are shown in the Figure 2. When the stator coil is energized at specific roll instants a torque about the $X'$ axis of the rotor is generated. This results the rotation of drill bit about its $Y'$ axis. As the rotor is spinning at $\omega$ radians per second therefore due to precession phenomenon the applied torque in one axis causes a motion in the axis which is perpendicular to itself and to the spin axis as well.

3. MATHEMATICAL MODELING

A spinning drill bit with a high angular rate about $z$-axis resembles two degrees of freedom gyroscope, therefore following model of gyroscope [20] is used :-

$$
\begin{align*}
J_\dot{\theta}_x + b\dot{\theta}_x + H\dot{\theta}_y &= \tau_x \\
J_\dot{\theta}_y + b\dot{\theta}_y - H\dot{\theta}_x &= \tau_y
\end{align*}
$$

(1)

$\dot{\theta}_x$ & $\dot{\theta}_y$ are the angular position of the bit about $X$ & $Y$ axes respectively while $\tau_x$ & $\tau_y$ are the applied torques in stator frame of reference. Due to single pair of pole is mounted on the rotor, the applied torque available in stator frame is given by (2)

$$
\begin{align*}
\tau_x &= k_i \tau \cos\omega t \\
\tau_y &= k_i \tau \sin\omega t
\end{align*}
$$

(2)

Substituting (2) in (1) we have (3)

$$
\begin{align*}
J_\dot{\theta}_x + b\dot{\theta}_x + H\dot{\theta}_y &= k_i \tau \cos\omega t \\
J_\dot{\theta}_y + b\dot{\theta}_y - H\dot{\theta}_x &= k_i \tau \sin\omega t
\end{align*}
$$

(3)

The combined moment of inertia of the drill bit and permanent magnet along $X$ & $Y$ axes is assumed to be same because of axis symmetry and is denoted by $J$ and $J_z$ about $Z$ axis respectively. Whereas $b$ & $K_i$ represents the coefficient of friction and torque constant. $H$ being the angular momentum is given by the following relationship:-

$$
H = (J_z - J)\omega
$$

(4)

To represent (3) in state space form, $\dot{\theta}_x$ & $\dot{\theta}_y$ are represented as $x_3$ & $x_4$ and $\dot{x}_3$ & $\dot{x}_4$ as $x_1$ & $x_2$. The applied torques $\tau_x$ & $\tau_y$ are represented as $u_1$ & $u_2$. Finally the state space representation in stator frame of reference is given in (5).

$$
\begin{align*}
\dot{x}_1 &= \frac{-b}{j}x_1 - \frac{H}{j}x_2 + \frac{k_i}{j}u_1 \\
\dot{x}_2 &= \frac{H}{j}x_1 - \frac{b}{j}x_2 + \frac{k_i}{j}u_2 \\
\dot{x}_3 &= x_1 \\
\dot{x}_4 &= x_2
\end{align*}
$$

(5)

The (5) can be written in matrix form (6), the output vector $y(t)$ is formed by $x_3$ & $x_4$ states and the input $u(t)$ is given by (7).

$$
\dot{x}(t) = Ax(t) + Bu(t)
$$

(6)

$$
y(t) = Cx(t)
$$

(7)

$$
A = \begin{bmatrix}
-\frac{b}{j} & -\frac{H}{j} & 0 & 0 \\
\frac{H}{j} & -\frac{b}{j} & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix},
B = \begin{bmatrix}
k_i \sin\omega t \\
0 \\
0 \\
k_i \cos\omega t
\end{bmatrix},
C = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(8)

Using the transformation (9) the stator frame of reference is transformed into rotor frame of reference (11) by changing the state variables for time variant systems [22] through a rotation matrix (10). The output vector $\tilde{y}(t)$ comprises of angular positions $\dot{\theta}_x$ & $\dot{\theta}_y$ in stator frame of reference.

$$
z(t) = T^{-1}(t)x(t)
$$

(9)

Where

$$
T^{-1}(t) = \begin{bmatrix}
\cos\omega t & \sin\omega t & 0 & 0 \\
-\sin\omega t & \cos\omega t & 0 & 0 \\
0 & 0 & \cos\omega t & \sin\omega t \\
0 & 0 & -\sin\omega t & \cos\omega t
\end{bmatrix}
$$

(10)
\[ \dot{z}(t) = \bar{A} z(t) + \bar{B} u(t) \]
\[ \bar{y}(t) = \bar{C} \bar{z}(t) \]
(11)

where
\[
\bar{A} = \begin{bmatrix}
-\frac{b}{J} & -\frac{H}{J} & -\omega & 0 & 0 \\
\frac{H}{J} + \omega & -\frac{b}{J} & 0 & 0 \\
1 & 0 & 0 & -\omega \\
0 & 1 & \omega & 0 
\end{bmatrix}
\]
\[
\bar{B} = \begin{bmatrix}
\frac{K}{J} \\
0 \\
0
\end{bmatrix}, \quad \bar{C} = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 
\end{bmatrix}
\]
(12)

The output vector \( \bar{y} \) comprises of angular positions \( \theta_x \) & \( \theta_y \) in rotor frame of reference.

4. PULSE EQUIVALENT AREA

The approach used in [7] for orientation control of the drill bit was based on state space approach of a discrete time equivalent model. The practical limitations of provision of unlimited magnitude, saturation and stability issues are serious in nature. Another probable approach for such a system is to use the Principle of Equivalent Areas (PEA) for Pulse width modulation (PWM) scheme. The PEA implies that even when two input signals have different waveforms, they can still results in the similar outputs if they have the same areas [21]. When this concept is applied to a PWM signal whose pulse width is to be modified to achieve a PEA equivalent of a given control signal the relationship is governed by following [21]:-

\[ \int_{kT}^{(k+1)T} u(t) \, dt = U\sigma_k \]
(13)

Where \( U \) is pulse amplitude, \( T \) is the PEA internal, \( u(t) \) is the given control signal to be converted into pulse signal and \( \sigma_k \) is the pulse-width to be determined.

Although PEA concept is practically implementable but is based on approximation. Another limitation is of high moduation frequency as the averaging response becomes closer to the given control signal at high frequencies. The higher model formulas are more difficult to convert using PEA concept [23, 24]. Even the minimum pulse width which cannot be ignored is a major limitation for developing exact PEA signal. At last some rules of thumb need to be established for setting design parameters to obtain desired and more accurate PEA signal.

In the light of above two techniques which have specific limitations a novel technique based on PWM principle is developed which is explained in the next section.
5. EQUIVALENT MODEL IN DISCRETE TIME

Figure 4 depicts one actuation cycle of period $T$ applied for one complete revolution of drill bit. Where $\Delta$ defines the shift of rectangular pulse center from the start of revolution. It is actually the representation of phase of actuation. Whereas $\alpha$ is the fixed amplitude of the pulse. $P_w$ defines the width of rectangular pulse and varies according to the requirement of the control signal magnitude on the lines of Pulse width modulation concept. This marks the difference in approach from the [7] where a pulse with fixed width having varying amplitude was used for the actuation.

Using the input signal $u(t)$ characteristics of figure (4) a closed form is built by considering one complete revolution of the drill bit from time $KT$ to $(K+1)T$. This model is developed by dividing the signal into three separate intervals i.e., interval I, II and III. It must be noted that during interval I and III the system is un-actuated.

Interval I : $KT \rightarrow (KT + \Delta - \frac{P_w}{2})$ the states at $(KT + \Delta - \frac{P_w}{2})$ using [22] are given by (14)

$$x(KT + \Delta - \frac{P_w}{2}) = e^{A(KT+\Delta-\frac{P_w}{2}-KT)}x(KT)$$  \hspace{1cm} (14)

Interval II : $(KT + \Delta - \frac{P_w}{2}) \rightarrow (KT + \Delta + \frac{P_w}{2})$ using result of (14) we have (15)

$$x(KT + \Delta + \frac{P_w}{2}) = e^{AP_w}e^{A(\Delta-\frac{P_w}{2})}x(KT) + \frac{P_w}{2}e^{AP_w}e^{A(\Delta-\frac{P_w}{2})}d\Phi BU$$  \hspace{1cm} (15)

Interval III : $(KT + \Delta + \frac{P_w}{2}) \rightarrow x((K+1)T)$ we have (16)

$$x((K+1)T) = e^{A(T-\Delta+\frac{P_w}{2})}x(KT + \Delta + \frac{P_w}{2})$$  \hspace{1cm} (16)

Substituting values from (15) into (16) we get:-

$$x((K+1)T) = e^{AT}x(KT) + e^{A(T-\Delta)}\frac{P_w}{2}e^{-A\Phi}d\Phi BU$$  \hspace{1cm} (17)

After further simplifying (17) we get:-

$$x((K+1)T) \equiv e^{AT}x(KT) + e^{A(T-\Delta)}\left(P_w + \frac{A^2Pw^3}{24} + \frac{A^4Pw^5}{1920}\right)BU$$  \hspace{1cm} (18)
Thus (18) describe the states at time (T+1) for a specific $P_w$ and $\Delta$. This was also verified from a simulation results as shown in figure 5, when same input signal is given to the drill bit model (8) and its derived solution (18).

6. CONCLUSION

A closed form in discrete time domain is presented for orientation control of an under-actuated drill machine. PWM approach was used to overcome the practical limitation of controlling signal. Technique of averaging the control force over one revolution e.g., Principle of Equivalent Areas (PEA) is avoided to achieve precise control. The derived model is also verified by comparing it with actual model through simulations. The derived model is useful for designing the feedback control system for orientation control. An pulse equivalent area based novel technique [26] has already been developed for orientation control of similar model under discussion since the typical control techniques [27,28,29,30,31] are not applicable for this class of model. This derived model will be useful in developing fast optimization schemes for this technique.

REFERENCES


[7] M. B. Malik, Fahad M. Malik, and Khalid Munawar, "Orientation control of a 3-d under-


