A Reduced Rank STAP and Staggered PRF for Multiple Target Situations

Samira Dib\textsuperscript{1}, Mourad Barkat\textsuperscript{2}, \textit{IEEE Fellow}, Jean-Marie Nicolas\textsuperscript{3}, Morad Grimes\textsuperscript{1}

\textsuperscript{1} University of Jijel, Algeria
\textsuperscript{2} King Saud University, Saudi Arabia
\textsuperscript{3} Telecom Paris, France
samiradib@yahoo.fr

ABSTRACT
In this paper, we first present the principles of STAP and discuss the properties of optimum detector, as well as problems associated with estimating the adaptive weights such as ambiguities and the high computational cost. The performances are evaluated highlighting the influence of radar parameters on the detection of slow targets. To resolve problem of high computational cost of optimal STAP, reduced-rank methods are used. And to resolve Doppler ambiguities, staggering of PRF is used. The simulation results are presented and the performances of STAP are discussed. In addition, the effect of an interfering target is analyzed. The performances of detection are discussed using two parameters: power and direction of the interfering target. Numerical evaluation is based on two models of staggering PRF: quadratic and pseudo-random, with two methods for reducing rank: Principal Components and SINR metric.

It was shown that the primary target is completely masked if the interfering target is powerful and this for any direction of the second target. It was also proven as when this one is located according to particular directions of the jammers, detection is largely degraded. To cure these problems, one considered an analysis of the staggered PRF of the STAP with the two methods which showed their effectiveness.

KEYWORDS
Signal processing, Radar, STAP, Eigenanalysis, Staggered PRF, Reduced Rank STAP.

1 INTRODUCTION
For ground radar, all clutter echoes are received with a Doppler frequency zero, while airborne radar, the total of all arrivals produces a Doppler clutter broadband. The goal of multifunction airborne radar is to detect, locate and represent all potential targets. The detection of fast targets is done by using MTI; however, the detection of slow target is much more difficult because of the spectral width of ground clutter. The space-time processing, STAP, can provide a rejection of clutter to detect slow targets. STAP is a technique of two-dimensional filtering which allows simultaneous suppression of the clutter and white noise. Typically, STAP means the simultaneous processing of the spatial signals received by multiple elements of an array antenna, and the temporal signals provided by the echoes from coherent pulse interval (CPI). For airborne radars, a target in a noisy scenario is shown in Figure 1. In
azimuth projections, the lobe principal of the clutter covers the target; while in Doppler projections, it is covered by the whole spectrum of the white noise and the secondary lobes of the clutter. However, it is clear that the target is separable from the clutter and noise in the two-dimensional field angle-Doppler [1,2]. Space-time processing exploits the fact that the clutter spectrum is basically a narrow ridge. A space-time clutter filter therefore has a narrow clutter notch, so that even slow targets fall into the pass band. Brennan and Reed [1] first introduced STAP to the radar community in 1973. With the recent advancement of high speed, high performance digital signal processors, STAP is becoming an integral part of airborne or space-borne radars for MTI functions. However, the main disadvantage of STAP is its high computational cost, since it utilizes complex matrix operations and often in an iterative fashion. For this reason, some reduced-rank STAP algorithms have been developed. In [1-14], it was shown that STAP has a good ability to unmask or extract targets from Doppler-spread clutter. It has the unique property of compensating for the Doppler spread induced by the platform motion and thus, making the detection of slow targets possible.

In this paper, we first discuss the effect of radar parameters on the optimal STAP, and the reduced rank STAP (RR STAP) on the airborne radar as well as the staggered PRF on the RR STAP performance in the two cases of known and unknown covariance matrix. Then, we examine the performances of the STAP and the staggered PRF in the case where an interfering target is present in the radar environment.

In what follows, we will analyze the influence of the parameters of radar on the target detection using the improvement factor as a study tool. Then, we illustrate the importance of the separation of subspace interference-noise which will reduce the rank of the covariance matrix. In Section 2, we present the formulation problem and the mathematical data describing the environment in which the radar operates and the model of the environment with the secondary target. Reduced rank STAP processing and staggering PRF are presented in Section 3. The discussion of the results is presented in section 4 including a comparative study with and without interfering target, while the conclusion illustrating the main results is presented in Section 5.

Figure 1. Diagram of two-dimensional filtering

2 PROBLEM FORMULATIONS AND MATHEMATICAL MODEL

The basic principles of space-time adaptive processing (STAP) is defined as a linear combination which adds the spatial samples of the elements of an antenna array and temporal samples of a multiple pulse coherent waveform. To calculate the weight vector adaptive STAP, the statistics of interference environment are determined by forming a covariance matrix. Typically, this
matrix is not known a priori and must be estimated from the sample space-time secondary radar as shown in Figure 2.

![Figure 2. Conventional chain of STAP](image)

### 2.1 Problem Formulation

Consider the case where an interfering target is present in the field of monitoring of the radar. We will discuss its effect on the capacity of detection of the primary target. The secondary target is characterized by the following two parameters:

(i) $\beta = |\beta| e^{j\Phi}$: Complex amplitude with random phase $\Phi$ uniformly distributed between 0 and $2\pi$,

(ii) the direction $\theta$ or the angle of arrival of the signal coming from the interfering target.

We note that, in this paper, we consider the case of two uncorrelated targets. The analysis of the case of correlated targets constitutes a future work.

### 2.2 Mathematical Model of Data:

We consider a linear space-time array with $N$ sensors uniformly spaced and $M$ delay elements for each sensor. The radar transmits a coherent range of $M$ pulses at a constant pulse repetition frequency, $PRF$. For each $PRI$ ($PRI = 1/PRF$), the range cells are collected to cover the range constituting a two-dimensional model, called "data cube" of STAP. The data are processed at one range of interest, which corresponds to a slice of the CPI data cube as shown in Figure 3. This 2-D snapshot is a space-time data structure which consists of element space information and PRI space-Doppler information. The snapshot is stocked to form a $MN \times 1$ vector $X$.

![Figure 3. Data Cube of STAP](image)

A space time snapshot at range $k$ in the presence of a target is given by [1]

$$X = \alpha S + X_i$$  \hspace{1cm} (1)$$

where, $X_i$ is the vector of interferences (noise, jamming and clutter), $\alpha$ is the target amplitude and $S$ is the space-time steering vector given by $S = S_t \otimes S_s$,

$S_t = [1; e^{-j2\pi F_t}; e^{-j2\pi 2F_t}; \ldots; e^{-j2\pi (M-1)F_t}]$ and

$S_s = [1; e^{-j2\pi F_s}; e^{-j2\pi 2F_s}; \ldots; e^{-j2\pi (N-1)F_s}]$;

where $F_t = F_d / PRF$ and $F_s = d \cdot \sin \theta / \lambda$ are, respectively, the normalized Doppler and spatial frequency. $d$ is the distance between the antennas and $\theta$ is the azimuth angle. The optimum weight of the STAP, which maximizes the signal to interference noise ratio SINR, is obtained to be [1]
\[ W_{opt} = \alpha R^{-1} S \]  

(2)

\( R \) is the covariance matrix of the interferences, which is supposed to be known and its structure is given by [3]

\[ R = E[nn^H] = R_c + R_j + R_n \]  

(3a)

where \( R_c, R_j, R_n \) are the covariance matrices of clutter, jammers and thermal noise, respectively, as follows

\[ R_c = \sum_{k=1}^{Nc} \zeta_k (S_{sk} S_{sk}^H) \otimes (S_{sk} S_{sk}^H) \]  

(3b)

\[ R_j = \sum_{i=1}^{N_j} \sum_{j=1}^{N_j} \alpha_i \alpha_j^H \otimes S_{si} S_{si}^H = \Lambda \mathbf{E} \mathbf{A}^H \]  

(3c)

and

\[ R_n = E[X_n X_n^H] = \sigma^2 \mathbf{I}_j \otimes I_k = \sigma^2 I_{kj} \]  

(3d)

where \( A = [S_{s1}, S_{s2}, \ldots, S_{sNj}] \) and

\[ E = \text{diag}(\sigma^2 \zeta_1, \sigma^2 \zeta_2, \ldots, \sigma^2 \zeta_{Nj}). \]

\( \zeta_k \) is Interference to Noise Ratio, INR; \( S_{si} \) and \( S_{si} \) are the vectors of direction temporal and space, respectively, of the interfering target.

The secondary target behaves like a target

The vector of the interfering signal is defined as follows

\[ X_i = \beta S_i \]  

(6)

Finally, the matrix of space-time covariance of the second target is

\[ R_i = E[X_i X_i^H] \]  

(7)

The new structure of the matrix of covariance is the sum of the matrices of jammers, clutter, white noise and the secondary target

\[ R = R_c + R_j + R_n + R_i \]  

(8)

In practice, \( R \) is not known and must be estimated from the snapshots. The well-known SMI gives an estimate of the matrix by averaging over the secondary range cells, such that

\[ \hat{R} = \frac{1}{L} \sum_{l=1}^{N} X_l X_l^H \]  

(9)

where \( k \) is the test range cell, and \( L \) is the number of secondary range cells. The
training data should not contain the signal, because it would be considered as interference and suppressed. For this reason, both the range cell under test and any adjacent range cells which might contain significant signal power due to range sidelobes (the so called “guard cells”) are usually excluded from the training data. This is illustrated in Fig. 2.

The SMI weight vector is then

$$W_{SMI} = \alpha \hat{R}^{-1} S$$  \hspace{1cm} (10)

The performance of the processor can be discussed in terms of the Improvement Factor (IF). IF is defined as the ratio of the SINR of the output to that of the input of the Direct Form Processor (DFP) and given by [1]

$$IF_{opt} = \frac{W^H S S^H \text{tr}(R)}{W^H RW S^H S}$$  \hspace{1cm} (11)

$W$ is the optimum weights of the interference plus noise rejection filter.

Note that a notch, which is a reversed peak of the clutter, appears at the frequency in the direction of sight of the radar, while the width of this notch gives a measurement of the detection of slow moving targets.

3 REDUCED RANK STAP AND STAGGERED PRF

The objective of the partially adaptive STAP is to reduce the complexity of the problem of adaptation, while maintaining almost the same optimal performance. The partially adaptive algorithms of the STAP consists in transforming the data with a matrix $V \in \mathbb{C}^{MN \times r}$, where $r << MN$. There are several methods for the covariance matrix rank reduction [3-14], which may differ in the shape of the processor as well as in the selection of the columns of the matrix.

3.1 Principal Components Method

The principal component, PC (also known as the eigencanceler method [5]) is based on the eigenvectors conservation of the matrix of covariance of interferences corresponding to the dominant eigenvalues [7]. The resulting PC-based DFP (Direct Form Processor) weight vector $W = W_{PC-DFP}$ has the form

$$W_{PC-DFP} = S - \sum_{i=1}^{r} \frac{\lambda_i - \lambda_{min}}{\lambda_i} (v_i^H S) v_i$$  \hspace{1cm} (12)

Where $\{\lambda_i\}_{i=1}^{MN}$ are the eigenvalues of $R$ and $\{v_i\}_{i=1}^{MN}$ are the associated eigenvectors and $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{MN}$.

3.2 SINR metric Method

The objective is to choose the $r$ columns of $V$ such that the loss in the performances of the SINR will be minimized. Berger and Welsh [8] chose the columns of $V$ as being the eigenvectors of $R$, which minimized the reduction in the performance of the SINR method. Clearly, the partial sum in (7) is maximized by selecting the $r$ columns of $V$ to be the eigenvectors which maximize the quantity [8]

$$\left| v_i^H S \right|^2 / \lambda_i$$  \hspace{1cm} (13)

This is referred to as the SINR metric.

The improvement factor of the reduced rank can then be written as

$$IF_{RR} = S^H V (V^H R V)^{-1} V^H S \text{tr}(V^H R V) / S^H V V^H S$$  \hspace{1cm} (14)
3.3 Analysis of STAP with eigenvalues

The analysis with eigenvalues (eigenanalysis) was first proposed by Klemm [2]. This analysis is based on the development of spatial-temporal covariance matrix to reduce the rank without losing the information captured by the antenna. It reduces the degree of freedom of filter and the computation time for real-time applications. The covariance matrix has a small number of high eigenvalues and a large number of low eigenvalues. The number of high eigenvalues, predicted by the theorem of Landau-Pollak [2], is limited by

\[ r \leq M + N - 1 \] (15)

Another method for the evaluation of eigenvalues was presented by Brennan and Staudahar in [4], where the number of high eigenvalues is given by

\[ r = \text{int}(N + (M - 1) \gamma) \] (16)

where
- \( \gamma = (2V_R \cdot \text{PRI})/d; \)
- \text{int} indicates integer part;
- \( V_R \) is the speed of the radar.

The eigenanalysis suggests that most of the power is concentrated in the \( r \) values.

3.4 STAP with Staggered PRF

It is known that if the PRF has a low value, Doppler ambiguities occur and are caused by the overlapping of the edge lines with the true spectrum. This overlapping decreases gradually with each time the PRF is increased because the edge lines move away one from the other by leaving the true spectrum without a shift. Therefore, the idea of using the change of PRF appeared to solve the problem of Doppler ambiguities.

The quadratic change of PRF consists of increasing (or decreasing) the PRI in certain stages. Therefore, the PRI in the temporal frequency of the direction vectors of the target and clutter is multiplied by the term \( (1+\varepsilon_m/M) \) for each impulse \( m \) [12].

In the pseudo-random staggering, as its name indicates it, consists in varying the interval of repetition of impulses PRI in a pseudo-random way by multiplying it by the term \( (1+\varepsilon_u(m)) \) where the random part \( u(m) \) is uniformly distributed on the interval \([-1, 1]\).

4 Results and Discussion

In this Section, we discuss the influence of some radar parameters and some algorithms, based on the reduction of the rank of the covariance matrix and staggered PRF, on the detection of a target with a low power (SNR=0dB) and with a slow speed.

The simulated environment is a linear side looking network of \( N \) antennas spaced out by \( d \) and \( M \) impulses in the CPI. The elevation angle is fixed to 20°. The speed of the airborne radar is \( V_R=100\text{m/s} \), and the frequency of transmission is 0.3GHz. The environment of the interferences is composed of:

(i) Five jammers whose angles of azimuth are: 0°, 180°, 60°, 90°, and 72°, with jammer reports/ratios on noise (JNRs) of 13dB, 12dB, 11dB, 10dB, and 9dB respectively.

(ii) Clutter of ground covering the band [-30°, 30°], and of clutter to noise ratio (CNR) equal to 8 dB.

In addition to the above assumptions, the environment of the interferences is composed of an interfering target whose power and direction are the parameters
of analysis. The performance of detection will be discussed in the case of a scenario without ambiguities, \( \text{PRF} = \frac{8V_R}{\lambda} \), for better seeing the effect of the interfering target. This does not prevent us from mentioning the case of a scenario with ambiguities.

All the simulations were carried out over 20 Monte Carlo runs.

### 4.1 Sample size of the network

In Figure 4, we plot the improvement factor (IF) versus the normalized Doppler frequency (Ft). It is noted that the clutter notch becomes thinner when the number of antennas increases (a). This confirms the results obtained in [2]. Similar results are obtained when increasing the number of pulses (b). It can be concluded that improving the detection can be accomplished by increasing a single parameter (number of antennas or pulses). To achieve good performances, SMI method needs a large number of snapshots which increases the computational cost and time for implementation in real time (c) and (d).

**Figure 4.** Improvement Factor of the optimal processor, DFP with \( \text{PRF} = 4V_R/\lambda \) and \( d/\lambda = 0.5 \):
(a) known \( R \)  (b) unknown \( R \)

### 4.2 Space Subsampling

Figure 5(a) shows the IF versus Ft of a scenario where the antennas are spaced at a distance \( d = \lambda \) that is to mean, double of the spatial frequency sampling or Nyquist frequency \( (F_{\text{Nyquist}}) \). It is clear
that ambiguous notches appear in the sidebands, which is not the case in Figure 5(b) where the distance between the antennas is \(d = \lambda/2\). In this Figure, we can see that there is disappearance of some ambiguous notches and thus the detection is improved. Similar results are noticed for SMI method but for large number of data. Therefore, we can say that the subsampling in space \(d \geq \lambda/2\) leads to a spatial ambiguity or range ambiguity.

![Figure 5](image)

Figure 5. Improvement Factor for known \(R\): (a) \(d = \lambda\), (b) \(d = \lambda/2\)

4.3 Temporal Subsampling

From Figure 6, we note that the notch is thin and leads to the detection of slow targets. In the case where PRF is equal to \(2V_{R}/\lambda\), we see the appearance of ambiguous notches with speeds associated are called blind velocities and the detection becomes relatively difficult. However, for the case of PRF equal to \((4V_{R}/\lambda)\), we notice that the ambiguous notches are eliminated and thus the detection is improved. We can conclude that the temporal subsampling leads to Doppler ambiguity. This is due to the overlap of the sidebands that becomes more important with the decline in PRF.

The explanation will be more convincing if we consider the problem of sampling and the well known Shannon theorem. Indeed, if one compares the two results in (a) and (b) where it was considered a PRF equal to \(F_{Nyquist}\), and a PRF equal to \((F_{Nyquist})/2\), respectively, although we note the aliasing phenomenon, expressed here by the appearance of sidelobe echoes of clutter.

![Figure 6](image)

Figure 6. Improvement Factor for the DFP with constant PRF, \(N = 8, M = 10\). \(d / \lambda = 0.5\): (a) \(PRF = 4V_{R}/\lambda\), (b) \(PRF = 2V_{R}/\lambda\)
This analysis can be addressed by using the spectral analysis and varying the PRF. Figure 7 shows the minimum variance spectra of the received signal in the presence of two jammers for different values of PRF. There is the presence of aliasing of the clutter for high values of PRF. This translates into an increase of range of the covariance matrix as is illustrated in Figure 8. We can therefore say that the choice of the PRF is essential to ensure good detection.

We can conclude that constant PRF leads to Doppler ambiguities and the clutter spectrum becomes ambiguous so that blind velocities occur.

Figure 7. Angle/Doppler spectrum for known $R$ in the presence of two jammers at $-40^\circ$ et $60^\circ$.
number of jammers. The high-input eigenvalue is about \( \lambda_{\text{max}} = \text{CNR} + 10 \log(\text{NM})(\text{dB}) \).

This causes a slow convergence which is inherent to adaptive algorithms [15]. These values can actually be read directly from Figure 9.

On the other hand, we consider three cases: a small number of data \((MN = 20)\), average number of data \((MN = 40)\), large amounts of data \((MN = 80)\). It can be seen clearly from Figure 10, that an increase in the number of data causes an increase in those of the eigenvalues and the power of the latter itself has increased.

To see the effect of the number of data on the detection of slow target, we consider an unambiguous scenario, \( \text{PRF} = 4V_R / \lambda \) for \( MN = 20 \) and \( MN = 80 \). From Figure 11, we note that the two notches are relatively thin, which means that the slow moving targets are detected. In addition, the increase in \( MN \) leads to an increase in the detection capability, and thus a narrowing of the notch, while the cost of computations becomes increasingly high because it is linked to the number \( MN \times MN \). However, undulations appear due to the increasing effects of clutter and noise.
4.5 Reduction of the rank using PC and SINR metric methods

It is important to consider the performances of the SINR for each method partially adaptive according to the rank. We notice from Figure 12 that there is a strong degradation in the performances for a small rank. It is obvious that the method of the PC cannot attain the optimal SINR of exit until the rank is equal to the dimension of the eigenstructure of the noise subspace. The SINR metric is best when the rank is reduced below full dimension.

In Figure 13, we show IF versus Ft for the DFP-PC, parameterized by rank $r$. We consider the unambiguous case, $PRF = 4V_R / \lambda$. The same detection results are obtained when $r = 20$ and $r = 40$, and thus to save calculation time, we choose $r=20$ as a good reduction of the rank. We observe that if the rank is very small, the slow moving targets will be removed with the clutter, and thus will not be detected. Similar findings were made for an ambiguous scenario. We can then say that we should not reduce up to the rank to very low values.

The same observations are made in the case of the SINR metric processor.

Figure 12. SINR performance versus the rank

Figure 13. Improvement factor for the PC-DFP with different values of $r$ and with $PRF = 4V_R / \lambda$
We notice from Fig. 14(b) that there is a strong degradation in the performances for PC and SINR metric methods. Thus, to alleviate this problem of ambiguous notches, we consider the use of staggered PRF technique. by leaving undulations in the bandwidth of the STAP. Same results are obtained when using pseudo random change of PRF (Figure 17). The use of the change of PRF does not have any effect on the detection the slow targets. In addition that SMI method needs only MN snapshots to estimate the covariance matrix, a low number comparatively to the case of constant PRF.

**Figure 14.** Improvement Factor for the DFP with constant PRF, $N = 8$, $M = 10$, $d/\lambda = 0.5$, $r=20$: (a) $PRF = 4V_\rho/\lambda$, (b) $PRF = 2V_\rho/\lambda$

### 4.6 Reduction of the rank with staggered PRF

Figures 15 and 16 show IF versus Ft with the quadratic change of PRF applied to the PC and SINR metric methods for the two cases of known and SMI method in an ambiguous scenario ($PRF = 2V_\rho/\lambda$). We note that the application of the change of PRF removes the ambiguous notches clutter.

**Figure 15.** Improvement Factor with $PRF=2V_\rho/\lambda$, for: (a) DFP-PC (b) DFP-PC with quadratic change of $PRF$ (c) $MN$ SMI-PC with quadratic change of $PRF$

**Figure 16.** IF versus Ft with $PRF=2V_\rho/\lambda$, for: (a) DFP- SINR metric (b) DFP- SINR metric with quadratic change of $PRF$ (c) $MN$ SMI- SINR metric with quadratic change of $PRF$
4.7 STAP in the Presence of Interfering Target

**Optimum processor with constant PRF**

Figure 18 represents IF according to $F_t$ when the interfering target is present in the direction of the primary target ($\theta=\pi/12$). It is observed that as the strength of the target increases powerful, the detection deteriorates. In Figure 19, the interfering target is located in a direction of one jammer equalizes to 180°. Being in opposite position to the primary target, the interference contributes by a harmful excess of power, which will cause the degradation of detection. If $\beta=10$, the notch tends to disappear and thus the primary target will be poorly detected; this was expected because of the strong power of the second target. A small contracting of the notch is noted when $\beta=0.1$, and thus detection is relatively good. We can conclude that detection is degraded more especially as the PRF decreases and the power of the secondary target increases.
Figure 19. Improvement factor versus Ft for the DFP with constant PRF=$8V_R/\lambda$, in the presence of an interfering target ($\theta=\pi$):
(a) $\beta=0.1$    (b) $\beta=10$

In Figure 20, the interfering target is present in the direction of Jammers. We note that as the secondary target approaches the primary one while varying its direction according to that of the jammers, its effect is more remarkable. In other words, undulations and ambiguous notches appear when the angle of the direction of the target decreases or converges towards that of the primary target. However, the latter is completely masked if $\beta=10$ regardless the direction. For $\beta=0.1$, only a shift also occurred (the notch is not centred at the zero) and detection is relatively good.

Figure 20. Improvement factor versus Ft for the DFP with constant PRF=$8V_R/\lambda$, in the presence of an interfering target ($\theta=\pi/3$):
(a) $\beta=0.1$    (b) $\beta=10$

**Quadratic staggering of PRF**

We propose to study the extreme case of which the effect of the second target is most unfavourable for better seeing the utility of the change of PRF for the elimination of the ambiguous notches. It means that we start with a PRF less than Nyquist frequency, so that we will have ambiguities due to the sub-sampling in addition to the presence of strange secondary target close to the primary one. Therefore, we consider that the second target is close to the primary target, $\theta=\pi/12$, and of strong power $\beta=10$. The scenarios with and without ambiguities are examined in Figure 21, by varying $\varepsilon$. We notice the disappearance of some ambiguous notches in the edge lines for $\varepsilon=0.001$, in comparison where $\varepsilon=0$. Whereas for $\varepsilon=0.1$ or $\varepsilon=1$, one clearly sees an almost total disappearance of the ambiguous notches with the persistence of some undulations. Detection thus becomes much better than before the application of change of PRF. What leads us to conclude that the quadratic change of PRF is very effective to improve the capacity of detection and to eliminate...
ambiguities even in the presence of an interfering target.

**Reduction of the rank using the SINR metric method and the pseudo random change of PRF**

The results are similar to those of the two other methods of reduction of the rank and change of PRF. The same curves are found. The change of PRF is primarily used to solve the problem of Doppler ambiguities which appears because of the low value of PRF (lower than the bandwidth of the clutter). We can thus say that the two methods of change of PRF are effective to solve the problem of the presence of an interfering target. Indeed, the ambiguous notches are eliminated. In addition, the reduction of the rank does not have an effect on detection of the slow targets (the width of the notch does not vary) only to ranks very reduced.

**Comparative study**

In Figures 22 and 23, we consider two cases: absence and presence of an interfering target characterized by the direction $\theta=\pi/12$ and the power $\beta=10$ with constant PRF and quadratic change of PRF, respectively. By comparing the two Figures 22(a) and 22(b), we note the appearance of dense undulations and the notches in the edge lines. This is due to the effect of the second target which is powerful and is located at the same direction as the primary one. These undulations degrade the detection. We can see clearly, from Figure 23, that the harmful effect of the interfering target on the detection of the primary target is relatively raised by the reduction of the rank.

---

**Figure 21.** Effect of the quadratic change of the PRF on the detection in the presence of a secondary target, $\beta=10$, $\theta=\pi/12$: (1) $\varepsilon=0.001$  
(2) $\varepsilon=0.1$  
(3) $\varepsilon=1$  
(4) $\varepsilon=0$
5 Conclusion

In this paper, we presented the fundamental concepts of space-time adaptive processing, STAP, while considering a direct form processor. The simulation results are discussed highlighting the influence of various parameters on the performance of radar detection of slow targets. It was shown that to have a good detection, it is recommended to select a PRF greater than or equal to the Nyquist frequency to avoid Doppler ambiguities and take \((d = \lambda /2)\) to avoid the range ambiguities. Unfortunately, if radar uses a constant PRF, the clutter suppression will also lead to the suppression of moving target echoes with certain radial velocities, so-called blind velocities. Thus, increasing the PRF is useful for mitigating Doppler ambiguities, and reducing the PRF is useful for achieving unambiguous range. Choosing the PRF of a radar mode therefore involves a compromise between the unambiguous target velocity and the unambiguous range.

In addition, the results concerning the separation of subspace interference-noise...
can be exploited in algorithms for reducing the rank of the covariance matrix. The results of simulations showed that the reduction of rank has no effect on the detection of slow targets (the width of the notch does not vary) unless the rank is very small. We did the analysis using two methods: PC and SINR metric. The results showed that these methods allow a reduction in the computation time and a reduction of the rank at low values without affecting the detection of slow moving targets. However, some ambiguous notches persist in the bandwidth. To alleviate this problem, we applied the quadratic change of PRF. We showed that this method solves well the problem of ambiguities. We can summarize that Doppler ambiguities can be removed by PRF staggering. While, range ambiguities can be avoided by employing an array antenna with a vertical adaptivity. Also, it is shown that all the algorithms outperform SMI method when the covariance matrix is estimated from a data set with limited support.

In addition, we studied the effect of the presence of an interfering target in the radar environment. We derived the expression of the vector of interference and the new structure of the matrix of covariance. An analysis with the eigenvalues showed that when the interfering target is of a great power, the detection of the primary target was not possible. It was completely masked by dense undulations and ambiguous notches, and that was independently to its direction. Also, we proved that when the second target is located according to some directions of the jammers, the capacity of detection decreased seriously. Whatever its effect is negligible according to others directions. Then, we proposed the use of the reduction of the rank and the change of PRF to overcome the problem of the masking of the primary target. To conclude the study, we presented a comparative study, with and without interference, in which the effectiveness of the methods used were highlighted.

6 REFERENCES