ACCELERATED STABLE STOCHASTIC MOBILE MULTI-AGENTS USING BOUNDARY EFFECTS

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ABSTRACT

This paper describes how to coordinate stable autonomous multi-agents which move over finite resources consisting of cells using boundary effects. Each of the agents stochastically moves in accordance with transition probabilities over the resources. We suppose there are interactions or coordination among agents such that (1) each agent can not move to a destination cell occupied by agents more than the agents of a current cell and (2) their agents have time-lag. Then, the multi-agent behavior becomes more stable by giving an appropriate average moving speed for every agents. This paper presents more accelerated stable stochastic mobile multi-agent behavior by varying the average moving speed depending on the agent locations of resources.

KEYWORDS

Autonomous Multi-Agent Systems, Coordination, Stable Mobile Agents, and Boundary Effect.

1 INTRODUCTION

This paper describes autonomous stochastic mobile multi-agents with time-lag and average moving speed. Each of the agents moves on the cells along finite resources arranged over a straight line according to a transition probabilities in synchronization. The moving of each agent is restricted so that it depends on the number of other agents on destination cells within a specific ranged window. So there is coordination among agents with time-lag. The paper[19] showed that the multi-agent behavior becomes more stable by giving an appropriate average moving speed for every agents. The average moving speed of all the agents is the same, i.e. it is independent from resource locations. This paper provides more accelerated stable stochastic mobile multi-agent behavior by varying the average moving speed depending on the agent locations. Our analysis and experiments show that the variations with respect to the number of agents on cells become lower, that is more stable, when all the agents have an appropriate average moving speed according to the agent locations, i.e. we can design more stable multi-agent configurations if we vary their average moving speed depending on the agent locations. According to the principle of minimum entropy in the law of nature, the stochastic average moving speed for their agents is accelerated autonomously.

In our real world, there are a lot of unusual beings with unexpected phenomena that are beyond human understanding. In fact, a multi-agent behavior is one of them, while it is quite difficult to analyze the behavior of multi-agents in general theoretical frameworks, because there are interactions or coordination among agents.

The studies of complex systems[1] have been expected to explore new unexpected phenomena which are created by artificial systems. The most attractive one is that the behavior of entire systems does not obvious from a simple combination of each agent behavior. Our stochastic mobile multi-agents or multi-objects exactly behave more stable by giving an appropriate average moving speed for their agents.

We are in need of a simple model with no fat in mobile multi-agents for analyzing complex systems. Fortunately, Sen et al.[17] proposed a simple basic model of mobile multi-agents, and Rustogi et al.[16] presented the fundamental results of the former model. Ishiduka et al.[8] also introduced a time lag and showed the relationship between time lag and stability in mobile multi-agents. The above models are intended to clarify how fast the mobile multi-agents fall into a complete stable state, i.e. a hole state in absorbing Markov chain[5], thus the goal is to design a coordinative system which falls into a stable hole in shorter passage time as soon as possible.

On the other hand, in physics, Toyabe et al. [20] experimentally demonstrated that information-to-energy conversion is possible in an autonomous single stochastic mobile agent. In other words, the paper presented a solution of Maxwell's devil. The idea is that if an agent goes up the spiral stairs during stochastic movements, it sets the stopper on the stairs so that the agent does not to come down. This approach needs an explicit control that the agent does not come down the spiral stairs. It is single agent against multi-agents, and our ultimate goal is to get the energy from stochastic mobile multi-agents. In multi-agent models, Hiyama[7] presented the precise theoretical calculation providing the interactions among different types of objects in nucleus.

Our model, Multi-Agent behavior with Time lag and Moving Speed: MATMS, is based on Sen et al.[17] and the developed model with time-lag proposed by Rustogi and Singh[16]. We note that our purpose is different from the papers [17, 16, 8] which try to clarify the relationships between time-lag and stability in multi-agent systems. In other words, the papers try to find the multi-agent configurations satisfying autonomous uniform resource allocation in a shortest passage time. On the other hand, our multi-agents initially start from a most stable state, each agent on resource stochastically moves over cells, and it just likes atoms in a liquid. The agents are always moving on resources stochastically, and they never stop. In addition, we extend their models to have moving average speed. Our model satisfies Markov condition and irreducible so that the configurations do not depend on the initial configurations in the limit, and our problem is to find more stable multi-agent configurations accompanying agent movements. It just likes as a molecule has an energy so that it is always moving while the agents are alive, and it depends on the manner of substances. The paper[19] showed that a stochastic mobile multi-agent system, whose the agents move slowly as a whole on average, is more stable than other ones not having average moving speed as a whole theoretically and experimentally. This paper demonstrates more stable agent behavior by considering the agent locations on resources.

This paper is organized as the following. First, we define our model in Section 2. Section 3 shows that there exists a new distinct behavior based on theoretical analysis for small size 3×3, and Section 4 presents our experimental results to confirm the theoretical analysis. In the following section, we discuss the related works. Finally, we conclude this paper in Section 6.

2 MATMS: A MULTI-AGENT MODEL WITH TIME-LAG AND MOVING SPEED

We define a multi-agent consisting of k agents, $k \geq 2$. All the agents are arranged over a finite resource consisting of cells $S(i)$, $i = 1, .., n$ on a straight line $[1, n]$ instead of a circle[17, 16], and move to synchronize over the resource according to the following transition probability $p_{i,j}$ in stochastic manner. In the following, sometimes we are simply expressed as i a resource $S(i)$.

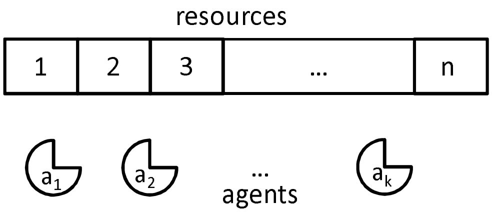

Figure 1: The resources of multi-agents.

First, we define a weight function $f_{i,j}$, $i, j = 1, .., n$ as

$$f_{i,j} = \begin{cases} 1, & i = j \\ 0, & i \neq j, r_i < r_j \\ 1 - \frac{1}{1+\gamma \exp(\frac{move(r_i - r_j, i, j) - \alpha}{\beta})}, & \text{otherwise}, \end{cases} \quad (1)$$

where r_i are the number of agents on i-th cell, and α, β and γ are constants. α is called an "inertia" which is the tendency of an agent to stay in its resource[16], and *move* is an accelerated function to give average moving speed on either left or right defined by (2) and (3) in later. Rustogi et al. model[16] does not satisfy the condition irreducible, while our model satisfies Markov property under the condition not to restrict the moving directions of agents, and the model becomes irreducible.

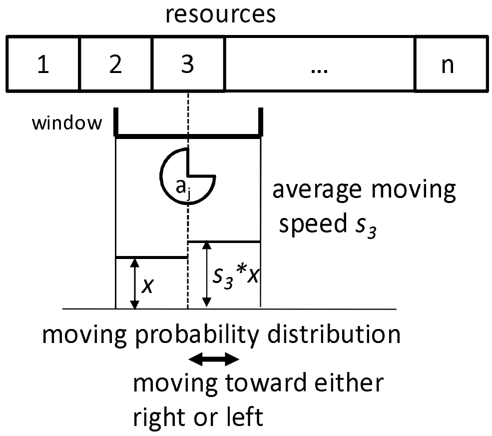

Figure 2: The model MATMS.

Our model also has an average moving *speed* such as every agents move with an average moving speed s_i ($s_i \geq 1$) either of left or right directions on the cells. Their agents move along the resources arranged over the straight line according to the probability $p_{i,j}$ in stochastic manner, where i and j indicate i-th and j-th cells, respectively. In the case of right average moving speed s_i, the function $move(x, i, j)$, which describes the ratio of imbalance from a cell i to a destination cell j with the difference $x(= r_i - r_j)$ in the numbers of agents on i and j, is defined by

$$move(x, i, j) = \begin{cases} s_i \times x, & i < j, \\ x, & \text{otherwise}, \end{cases} \quad (2)$$

wheres s_i is an average moving speed at i-th cell. On the other hand, for the left average moving, *move* is defined by

$$move(x, i, j) = \begin{cases} s_i \times x, & i > j, \\ x, & \text{otherwise.} \end{cases} \quad (3)$$

The function $move(x, i, j)$ is not symmetric with respect to x, and s_i represents the ratio of imbalance at i-th cell in the function *move*. As a special case, $move(x, i, j)$ becomes symmetric with respect to x and the agents do not move on average if $s_i = 1$. The average moving directions are inverted with the same average moving speed if each agent arrives at the leftmost or rightmost cells, i.e. the cells on the boundaries. The average moving speed depends on the agent locations, and each agent moves towards either left or right directions on average independently. Thus, all the agents are randomly choosing the moving directions which are apart from the effect on the left and right boundaries.

A moving transition probability $p_{i,j}$ from a cell $S(i)$ to a destination cell $S(j)$ is defined by the normalization of $f_{i,j}$ with probability 1 as

$$p_{i,j} = \frac{f_{i,j}}{\sum_k f_{i,k}}, \quad i, j = 1, .., n, \quad (4)$$

based on $f_{i,j}$. Rustogi et al. [16] introduced a window $win(i)$ with a fixed size for analyzing the behavior of multi-agent systems with time-lag. Then, a moving transition probability $p_{i,j}$ from a current cell $S(i)$ to a destination cell $S(j)$ is defined by

$$p_{i,j} = \begin{cases} \dfrac{f_{i,j}}{\sum_{k \in win(i)} f_{i,k}}, & i = 1, .., n, j \in win(i), \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

where w is a window size, and $win(i)$ is the set $[i - w, i + w]$. A time delay which is local properties is proportional to the window size w (see [16, 8]).

There are no constraints on the moving of agents such that each cell has a fixed upper limit capacity to occupy agents, while there is another constraint in the model, i.e. the moving transition probability $p_{i,j}$ is 0 if the number of agents on a cell $S(i)$ is less than the number of agents on a destination cell $S(j)$.

Our proposed model, Multi-Agent behavior with Time delay and Moving Speed: MATMS, is similar to the models [16, 8]. But, there are some differences between MATMS and [16, 8], and we describe them in the following.

The resources in MATMS are arranged over a straight line $[1, n]$ as in [8], and the wind function $win(i)$ is the set $[i - w, i + w] \cap [1, n]$ if w is a window size.

We note that there are two choices on the moving average directions which are either left or right. Suppose an agent moves towards left on average at the previous step. Which is the moving direction at the next step? If we exclude the cases that the agents stay on boundaries, there are two exclusive cases(or a model protocol) for each agent independently: (1) we inherit the directions at the previous steps, i.e. left on average in above, or (2) we randomly select it at each step according to even probability either left or right, i.e. half to half rule for the direction. The second case (2) is suite to Markov property. The first case (1) does not satisfy Markov property so that the systems depend on the initial configurations.

3 THEORETICAL STABLE ANALYSIS OF 3×3 MODEL

In this section, we discuss a concrete mobile multi-agent such that a multi-agent having an appropriate average speed is more stable than a multi-agent not having moving average speed, i.e. staying the current cell on average.

Suppose the multi-agent of which the number of cells and agents are 3 together. This is a minimal model to examine a coordination among agents. We first use the parameter values $\beta = 2$ and $\gamma = 1$, and fix the window size w to 1.

Suppose 1, 2 and 3 are their cell names(Figure 3). We do not distinguish the names among the agents for the simplicity, and represent it just a. Suppose that the same average moving speed s_1 and s_3 at 1st and 2nd cells are both s_b ($s_b \geq 1$), respectively, since the resource is symmetric. Also suppose that the average moving speed s_2 at 2nd cell is s_c ($s_c \geq 1$). The moving directions of the agents are randomly selected either left l or right r in half and half at every steps. The multi-agent state is a set of three agent states. $[(a, 1)$, $(a, 2)$, $(a, 2)]$ is an example of the multi-agent states, where a are agents and 1, 2 and 3 are the resources.

An example of the agent moving configuration is represented by $(a, r, 1)$ if the agent a on the cell 1 moves towards right with average moving speed s_b. The multi-agent moving configuration consists of three agent configurations in this minimal model. For an example, $[(a, r, 1)$, $(a, r, 1)$, $(a, r, 1)]$ is a multi-agent moving configuration.

In our minimal model, the directions of the stochastic mobile agents are stochastically chosen at every steps so that the multi-agent becomes Markov chain and irreducible. In this setting, there are 10 multi-agent states shown in Figure 3, and we must consider 136 probabilistic transition rules shown in Appendix A. That is, the number of the states(Figure 3), the state transition rules (Appendix A) and the multi-agent moving configurations (The top items of Appendix A) are 10, 136 and 20, respectively. The illustration of the transition rules (b-2) in Appendix A is shown in Figure 4. The more details of (g-2) consisting 27 rules in Appendix A are illustrated in Figure 5.

This simple model satisfies Markov condition and it is irreducible, so we easily compute the eigenvectors of the state transition matrix

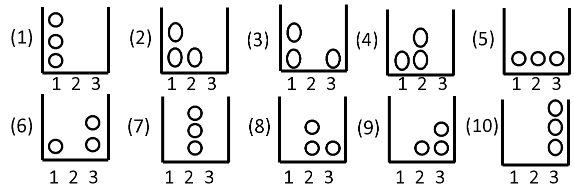

Figure 3: The states of the multi-agent: $cells = 3$, $agents = 3$ and $w = 1$.

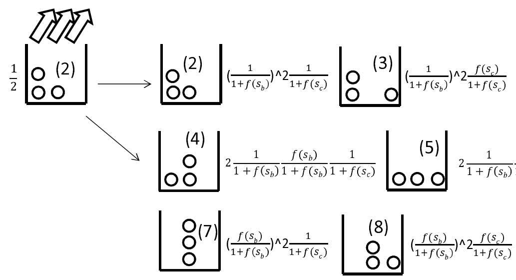

Figure 4: The transition rules (b-2) in Appendix A.

with the size 10×10 using Appendix A, and compute the transition probabilities among every states in the limit by changing the moving speed s. The theoretical computational results of the existence probabilities for every states are shown in Table 1.

The expected averages and variances with respect to the number of the agents on the cell 1 are m_1 and v_1, respectively, given by the following:

$$\begin{aligned} m_1 &= 3p_1 + 2(p_2 + p_3) + p_4 + p_5 + p_6, \quad (6) \\ v_1 &= (3 - m_1)^2 p_1 + (2 - m_1)^2 (p_2 + p_3) \\ &\quad + (1 - m_1)^2 (p_4 + p_5 + p_6) \\ &\quad + (0 - m_1)^2 (p_7 + p_8 + p_9 + p_{10}), \quad (7) \end{aligned}$$

where p_i are the probabilities of the correspondence states i shown in Table 1.

By similar way, we can compute the expected averages (m_1 and m_2) and variances (v_2 and v_3) with respect to the numbers of agents

Table 1: The probabilities staying the states in the case $cells = 3$, $agents = 3$ and $w = 1$ based on theoretical analysis.

state	$\alpha = 4$ $s_b = 2$, $s_c = 3$	$\alpha = 10$ $s_b = s_c = 6.5$
(1)	0.004028236	0.0001453373
(2)	0.070348532	0.0090297468
(3)	0.094054467	0.0112430473
(4)	0.108065877	0.0213383576
(5)	0.431800494	0.9154153079
(6)	0.094054467	0.0112430473
(7)	0.015205284	0.0010717141
(8)	0.108065877	0.0213383576
(9)	0.070348532	0.0090297468
(10)	0.004028236	0.0001453373

Table 2: The means and variances with respect to the numbers of agents on each cell based on theoretical analysis: $cells = 3$, $agents = 3$, $w = 1$.

	$\alpha = 4$ $s_b = 2$, $s_c = 3$	$\alpha = 10$ $s_b = s_c = 6.5$
m_1	0.9748115	0.9889783
v_1	0.3775294	0.05231782
m_2	1.050377	1.022043
v_2	0.2541528	0.04546244
m_3	0.9748115	0.9889783
v_3	0.3775294	0.05231782
m_a	1	1
v_a	0.4098153	0.05820631

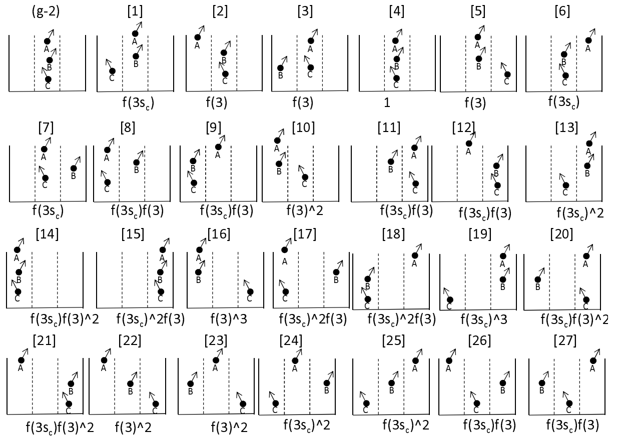

Figure 5: The details of the transition rules (g-2) in Appendix A. The denominators $(1 + f(3s_c) + f(3))^3$ are abbreviated.

on the cells 2 and 3:

$$\begin{aligned}
m_2 &= 3p_7 + 2(p_4 + p_8) + p_2 + p_5 + p_9, \quad (8)\\
v_2 &= (3 - m_2)^2 p_7 + (2 - m_2)^2 (p_4 + p_8)\\
&\quad + (1 - m_2)^2 (p_2 + p_5 + p_9)\\
&\quad + (0 - m_2)^2 (p_1 + p_3 + p_6 + p_{10}), \quad (9)\\
m_3 &= 3p_{10} + 2(p_6 + p_9) + p_3 + p_5 + p_8, \quad (10)\\
v_3 &= (3 - m_3)^2 p_{10} + (2 - m_3)^2 (p_6 + p_9)\\
&\quad + (1 - m_3)^2 (p_3 + p_5 + p_8)\\
&\quad + (0 - m_3)^2 (p_1 + p_2 + p_4 + p_7). \quad (11)
\end{aligned}$$

The expected variance v_a with respect to the number of agents over the resource is computed as

$$\begin{aligned}
v_a &= [p_1(4 + 1 + 1) + p_2(1 + 1) + p_3(1 + 1)\\
&\quad + p_4(1 + 1) + p_6(1 + 1) + p_7(4 + 1 + 1)\\
&\quad + p_8(1 + 1) + p_9(1 + 1)\\
&\quad + p_{10}(4 + 1 + 1)]/3 \quad (12)
\end{aligned}$$

Table 1 shows the existence probability of each state. Then, the theoretical expected means and variances are shown in Table 2. We also show our analyses in Figure 6-10.

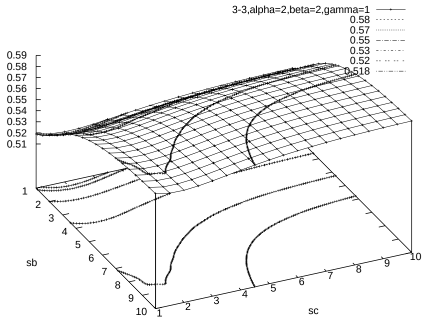

Figure 6: The theoretical analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 2$.

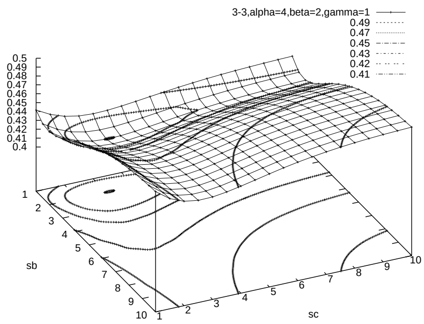

Figure 7: The theoretical analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 4$.

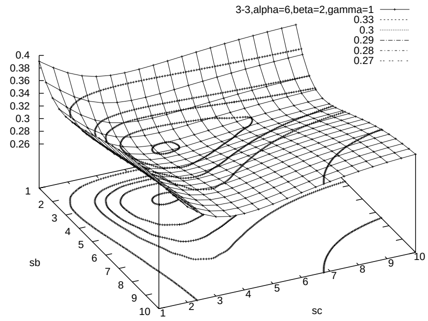

Figure 8: The theoretical analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 6$.

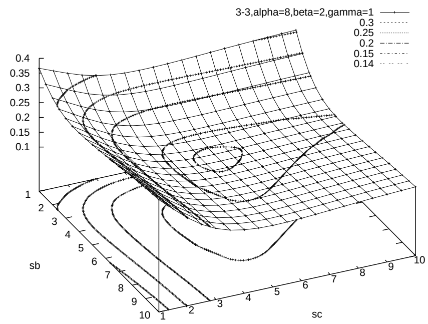

Figure 9: The theoretical analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 8$.

4 EXPERIMENTS

In this section, we present our experimental results and make the comparisons of the theoretical analysis and the experimental results.

We initially configure the multi-agent which is staying most stable state, i.e. the state (5) in Figure 3, and observed it during 50,000 steps. We computed the variance with respect to the number of agents between 10,001 and 50,000 steps. Then, we used Mersenne twister random number generator for a long period.

We show our experimental results in Figure 12-15 by changing α, s_b and s_c. Compare Figure 6-10 and Figure 11-15 to the order in which the have listed up. The experimental results and the theoretical analysis are almost the same. We also show the summary of the experiments shown in Table 3. Table 4 shows the optimum moving speed, and Figure 16 shows the optimum points of the v_b versus v_c. While the boundary effect is greater in small α, the effect is small in large α.

5 RELATED WORKS

Sen et al.[17] presented our basic model, and Rustogi et al.[16] also proposed the their extended model with time delay and presented

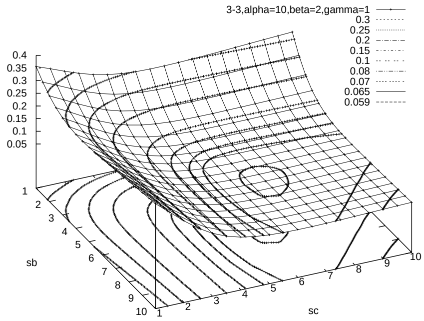

Figure 10: The theoretical analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 10$.

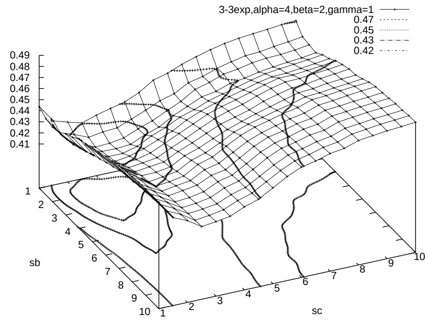

Figure 12: The experimental analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 4$.

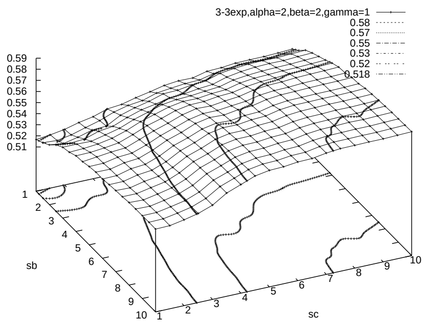

Figure 11: The experimental analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 2$.

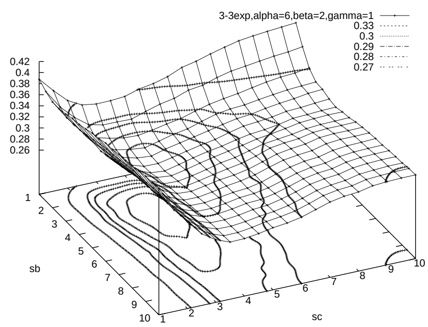

Figure 13: The experimental analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 6$.

the excellent results. Ishiduka et al.[8] introduced a time lag for the propagation speed explicitly in addition to a window, and showed the relationships between stability and time lags. We note that Sen and Rustogi models employ the resources on circles. On the other hand, the resources of Ishiduka model are on a straight line. A straight line of resources are more realistic and natural compares to a circle. How's the boundary effect? How's the circular effect?

There are a lot of discussions on the stability of multi-agents. Chlie et al. [3] tries to find time Markov chains to be stable when its state has converged to an equilibrium distribution. Bracciali et al. [2] presents an abstract declarative semantics for multi-agent systems based on the idea of stable set. Moreau [13] discusses the compelling model of network of agents interacting via time-dependent communication links. Finke and Passino [4] discusses a behavior of a group of agents and their interactions in a shared environment. Lee et al. [9] considers the kinematicd based dynamics-based flocking model on graphs, and the model of the behavior is unstable. They proposed a stable information framework for multi-agents. Mohanarajah and Hayakawa [12] discusses the for-

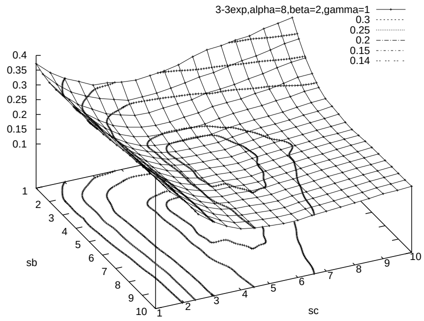

Figure 14: The experimental analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 8$.

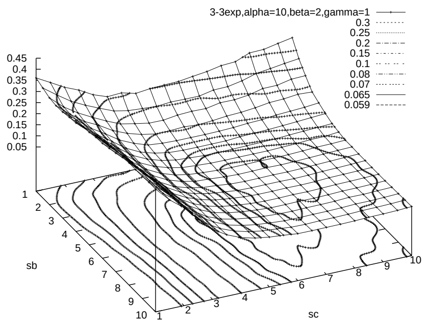

Figure 15: The experimental analysis of the variances v_a w.r.t. s_b and s_c. $\alpha = 10$.

Table 3: The means and variances with respect to the numbers of agents on each cell by experiments: $cells = 3$, $agents = 3$ and $w = 1$.

	$\alpha = 4$ $s_b = 2,\ s_c = 3$	$\alpha = 10$ $s_b = s_c = 6.5$
m_1	1.0066	0.98535
v_1	0.3760	0.0507
m_2	1.0833	1.0222
v_2	0.4783	0.0660
m_3	0.9101	0.9924
v_3	0.3682	0.0502
m_a	1	1
v_a	0.4125	0.0559

Table 4: The experimental results of the optimal moving speed.

α	s_b	s_c	v_a
2	1	2	0.515
4	2	3	0.410
6	3.5	4.3	0.270
8	5.0	5.4	0.138
10	6.5	6.5	0.058

mation control of multi-agent dynamical systems in the case of limitation on the number of communication channels. Hirayama et al. [6] introduced the distributed Lagrangian protocol for finding the solutions of distributed systems. These papers are intended to control the multi-agent systems in corporative stable states. However, our model is one of the natural models to achieve the coordination without controls and without communication among agents.

From the viewpoint of multi-agent coordination, consensus or agreement, Lessor et al. [11] proposed a domain-independent generic agent architecture for the next generation of multi-agent systems. Shehory et al.[18] addresses the implementation of distributed coalition formation algorithms within a real-world multi-agent system. Lee et al. [10] Jadbabaie et al. showed that all agents move in the same heading, provided that these agents are periodically linked together. Robertson et al.[15] proposed that a novel style of multi-agent system specification and deployment is described, in which familiar methods from computational logic are re-interpreted to a new context. Beard and Atkins [14] provides a survey of consensus problems in multi-agent cooperative control with the goal of promoting research in this

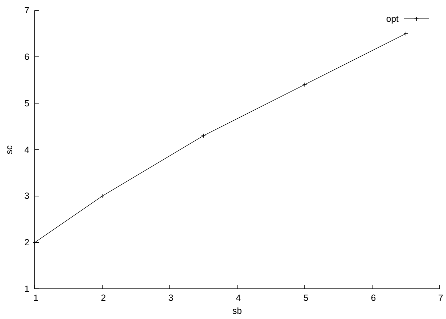

Figure 16: The v_b versus v_c in Table 4.

area.

6 CONCLUSIONS

In this paper, we considered a stochastic mobile multi-agent model, and presented that the model, Multi-Agent behavior with Time delay and Moving Speed: MATMS, having appropriate average moving speed is stable more than ones not having average moving speed. This shows that each agent needs the moving acceleration to stay more stable states. The acceleration is a *speed* in this paper.

We also discussed the resource utilization of the multi-agents, and presented that the model having appropriate average moving speed enables us higher resource utilization than ones not having average moving speed. This shows that each agent needs the moving acceleration to stay high usage of cells.

In our model, we showed that there is an appropriate speed to achieve the most stable or utilizable configuration for each inertia α.

Since individual objects in nature are governed by the lower entropy, all the objects which move randomly with interactions over resources cause naturally flow. Then, each object may independently take a different direction for moving rather than coordination, i.e. no need to control agents. We may extract the energy from the multi-agents which move randomly in a closed region, then we may think them just like atoms. This kind of work has done by Toyabe et al.[20] in single agent. The applications of this research are online algorithm and numerical analysis.

REFERENCES

[1] R. Axelrod. *The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration.* Princeton University Press, Princeton University, 1984.

[2] A. Bracciali, P. Mancarella, and K. Stathis. Stable multi-agent systems. *LNAI 3451*, pages 322–334, 2005.

[3] M. Chli, P.D. Wilde, J. Goossenaerts, V. Abramov, N. Szirbik, L. Correia, P. Mariano, and R. Ribeiro. Stability of multi-agent systems. *IEEE International Conference on Systems, Man and Cybernetics*, 1:551–556, 2003.

[4] J. Finke and K.M. Passino. Stable cooperative multiagent spatial distributions. *Decision and Control, 2005 and 2005 European Control Conference*, pages 3566–3571, 2005.

[5] C.M. Grinstead and J. L. Snell. Introduction to probability. *American Mathematical Society*, 1997.

[6] K. Hirayama, T. Matsui, and M. Yokoo. Adaptive price update in distributed lagrangian relaxation protocol. *AAMAS*, 2009.

[7] E. Hiyama and et al. Three- and four-body cluster models of hypernuclei using the g-matrix an interaction. *Progress of Theoretical Physics*, 97(6):881–899, 1997.

[8] Y. Ishiduka and K. Iwanuma. A relationship between time delay and the amount of knowlege in distributed cooperation of multi-agent systems. *IEICE D-I*, J86-I:117–120, 2003. Japanese.

[9] D. Lee and M.W. Spong. Stable flocking of multiple inertial agents on balanced graphs. *American Control Conference*, 2006.

[10] S. Lee and H. Wang. Multi-agent coordination using nearest-neighbor rules: Revisiting the vicsek model. *Cornell University*, 2004.

[11] V.R. Lessor. Reflections on the nature of multi-agent coordination and its implications for an agent architecture. *Autonomous Agents and Multi-Agent Systems*, 1:89–111, 1998.

[12] G. Mohanarajah and T. Hayakawa. Formation stability of multi-agent systems with limited information. *American Control Conference*, 2008.

[13] L. Moreau. Stability of multiagent systems with time-dependent communication links. *IEEE Transactions on Automatic Control*, 50, 2005.

[14] W. Ren, R.W. Beard, and E.M. Atkins. A survey of consensus problems in multi-agent coordination. *American Control Conference*, 2005.

[15] D. Robertson. Multi-agent coordination as distributed logic programming. *ICLP, LNCS 3132*, pages 416–430, 2004.

[16] S.K. Rustogi and M.P. Singh. Be patient and tolerate imprecision: How autonomous agents can coordinate effectively. *6th IJCAI*, 1999.

[17] S. Sen, S. Roychowdhury, and N. Arona. Effects of local information on group behavior. *Proceedings of the Second International Conference on Multiagent Systems, AAAI*, 1996.

[18] O.M. Shehory, K. Sycara, and S. Jha. Multi-agent coordination through coalition formation. *4th International Workshop, ATAL'97*, pages 143–154, 1998.

[19] I. Shioya and T. Miura. An autonomous accelerated coordination of stochastic moving multi-agents under variations and resource utilization. *International Journal of Digital Information and Wireless Communications (IJDIWC)*, 2:943–957, 2012.

[20] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano. Experimental demonstration of information-to-energy conversion and validation of the generalized jarzynski equality. *Nature Physics*, 6:988–992, 2010.

A THE STATE TRANSITION RULES AND THEIR PROBABILITIES OF 3×3 MODEL

First, let us define the auxiliary function $f(x)$ as

$$f(x) = 1 - \frac{1}{1 + \gamma \times \exp((x - \alpha)/\beta)},$$

where $cells = 3$, $agents = 3$ and $w = 1$.

(a) The multi-agent moving configuration $[(a, r, 1),\ (a, r, 1),\ (a, r, 1)]$ with the conditional probability $cp_a = 1$ moves to one of the following states in accordance with the transition probabilities.

(1) $[(a, 1),\ (a, 1),\ (a, 1)]$, $\frac{1}{(1+f(3s_b))^3}$

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(2) $[(a,1),\ (a,1),\ (a,2)],\ 3\frac{f(3s_b)}{(1+f(3s_b))^3}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ 3\frac{f(3s_b)^2}{(1+f(3s_b))^3}$

(7) $[(a,2),\ (a,2),\ (a,2)],\ \frac{f(3s_b)^3}{(1+f(3s_b))^3}$

(b-1) $[(a,r,1),\ (a,r,1),\ (a,l,2)],\ cp_{b1} = \frac{1}{2}$

(2) $[(a,1),\ (a,1),\ (a,2)],\ \frac{1}{(1+f(s_b))^2(1+f(1))}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ \frac{f(1)}{(1+f(s_b))^2(1+f(1))}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ 2\frac{f(s_b)}{(1+f(s_b))^2(1+f(1))}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ 2\frac{f(s_b)f(1)}{(1+f(s_b))^2(1+f(1))}$

(7) $[(a,2),\ (a,2),\ (a,2)],\ \frac{f(s_b)^2}{(1+f(s_b))^2(1+f(1))}$

(8) $[(a,2),\ (a,2),\ (a,3)],\ \frac{f(s_b)^2 f(1)}{(1+f(s_b))^2(1+f(1))}$

(b-2) $[(a,r,1),\ (a,r,1),\ (a,r,2)],\ cp_{b2} = \frac{1}{2}$

(2) $[(a,1),\ (a,1),\ (a,2)],\ \frac{1}{(1+f(s_b))^2(1+f(s_c))}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ \frac{f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ 2\frac{f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ 2\frac{f(s_b)f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(7) $[(a,2),\ (a,2),\ (a,2)],\ \frac{f(s_b)^2}{(1+f(s_b))^2(1+f(s_c))}$

(8) $[(a,2),\ (a,2),\ (a,3)],\ \frac{f(s_b)^2 f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(c) $[(a,r,1),\ (a,r,1),\ (a,l,3)],\ cp_c = 1$

(2) $[(a,1),\ (a,1),\ (a,2)],\ \frac{f(s_b)}{(1+f(2s_b))^2(1+f(s_b))}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ \frac{1}{(1+f(2s_b))^2(1+f(s_b))}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ 2\frac{f(2s_b)f(s_b)}{(1+f(2s_b))^2(1+f(s_b))}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ 2\frac{f(2s_b)}{(1+f(2s_b))^2(1+f(s_b))}$

(7) $[(a,2),\ (a,2),\ (a,2)],\ \frac{f(2s_b)^2 f(s_b)}{(1+f(2s_b))^2(1+f(s_b))}$

(8) $[(a,2),\ (a,2),\ (a,3)],\ \frac{f(2s_b)^2}{(1+f(2s_b))^2(1+f(s_b))}$

(d-1) $[(a,r,1),\ (a,r,2),\ (a,r,2)],\ cp_{d1} = \frac{1}{4}$

(1) $[(a,1),\ (a,1),\ (a,1)],\ \frac{f(1)^2}{(1+f(2s_c)+f(1))^2}$

(2) $[(a,1),\ (a,1),\ (a,2)],\ 2\frac{f(1)}{(1+f(2s_c)+f(1))^2}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ 2\frac{f(2s_c)f(1)}{(1+f(2s_c)+f(1))^2}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ \frac{1}{(1+f(2s_c)+f(1))^2}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ 2\frac{f(2s_c)}{(1+f(2s_c)+f(1))^2}$

(6) $[(a,1),\ (a,3),\ (a,3)],\ \frac{f(2s_c)^2}{(1+f(2s_c)+f(1))^2}$

(d-2) $[(a,r,1),\ (a,l,2),\ (a,l,2)],\ cp_{d2} = \frac{1}{4}$

(1) $[(a,1),\ (a,1),\ (a,1)],\ \frac{f(s_c)^2}{(1+f(s_c)+f(2))^2}$

(2) $[(a,1),\ (a,1),\ (a,2)],\ 2\frac{f(s_c)}{(1+f(s_c)+f(2))^2}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ 2\frac{f(s_c)f(2)}{(1+f(s_c)+f(2))^2}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ \frac{1}{(1+f(s_c)+f(2))^2}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ 2\frac{f(2)}{(1+f(s_c)+f(2))^2}$

(6) $[(a,1),\ (a,3),\ (a,3)],\ \frac{f(2)^2}{(1+f(s_c)+f(2))^2}$

(d-3) $[(a,r,1),\ (a,l,2),\ (a,r,2)],\ cp_{d3} = \frac{1}{2}$

(1) $[(a,1),\ (a,1),\ (a,1)],\ \frac{f(s_c)f(1)}{(1+f(s_c)+f(2))(1+f(1)+f(2s_c))}$

(2) $[(a,1),\ (a,1),\ (a,2)],\ \frac{f(s_c)+f(1)}{(1+f(s_c)+f(2))(1+f(1)+f(2s_c))}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ \frac{f(s_c)f(2s_c)+f(1)f(2)}{(1+f(s_c)+f(2))(1+f(1)+f(2s_c))}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ \frac{1}{(1+f(s_c)+f(2))(1+f(1)+f(2s_c))}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ \frac{f(2s_c)+f(2)}{(1+f(s_c)+f(2))(1+f(1)+f(2s_c))}$

(6) $[(a,1),\ (a,3),\ (a,3)],\ \frac{f(2s_c)f(2)}{(1+f(s_c)+f(2))(1+f(1)+f(2s_c))}$

(e-1) $[(a,r,1),\ (a,l,2),\ (a,l,3)],\ cp_{e1} = \frac{1}{2}$

(2) $[(a,1),\ (a,1),\ (a,2)],\ \frac{f(0)^2}{(1+f(0))^2(1+2f(0))}$

(3) $[(a,1),\ (a,1),\ (a,3)],\ \frac{f(0)}{(1+f(0))^2(1+2f(0))}$

(4) $[(a,1),\ (a,2),\ (a,2)],\ \frac{f(0)+f(0)^3}{(1+f(0))^2(1+2f(0))}$

(5) $[(a,1),\ (a,2),\ (a,3)],\ \frac{1+2f(0)^2}{(1+f(0))^2(1+2f(0))}$

(6) $[(a,1),\ (a,3),\ (a,3)],\ \frac{f(0)}{(1+f(0))^2(1+2f(0))}$

(7) $[(a,2),\ (a,2),\ (a,2)]$, $\frac{f(0)^2}{(1+f(0))^2(1+2f(0))}$

(8) $[(a,2),\ (a,2),\ (a,3)]$, $\frac{f(0)+f(0)^3}{(1+f(0))^2(1+2f(0))}$

(9) $[(a,2),\ (a,3),\ (a,3)]$, $\frac{f(0)^2}{(1+f(0))^2(1+2f(0))}$

(e-2) $[(a,r,1),\ (a,r,2),\ (a,l,3)]$, $cp_{e2}=\frac{1}{2}$. This case is identical to (e-1).

(f) $[(a,r,1),\ (a,l,3),\ (a,l,3)]$, $cp_f=1$

(4) $[(a,1),\ (a,2),\ (a,2)]$, $\frac{f(2s_b)^2}{(1+f(s_b))(1+f(2s_b))^2}$

(5) $[(a,1),\ (a,2),\ (a,3)]$, $2\frac{f(2s_b)}{(1+f(s_b))(1+f(2s_b))^2}$

(6) $[(a,1),\ (a,3),\ (a,3)]$, $\frac{1}{(1+f(s_b))(1+f(2s_b))^2}$

(7) $[(a,2),\ (a,2),\ (a,2)]$, $\frac{f(s_b)f(2s_b)^2}{(1+f(s_b))(1+f(2s_b))^2}$

(8) $[(a,2),\ (a,2),\ (a,3)]$, $2\frac{f(s_b)f(2s_b)}{(1+f(s_b))(1+f(2s_b))^2}$

(9) $[(a,2),\ (a,3),\ (a,3)]$, $\frac{f(s_b)}{(1+f(s_b))(1+f(2s_b))^2}$

(g-1) $[(a,l,2),\ (a,l,2),\ (a,l,2)]$, $cp_{g1}=\frac{1}{4}$

(1) $[(a,1),\ (a,1),\ (a,1)]$, $\frac{f(3s_c)^3}{(1+f(3s_c)+f(3))^3}$

(2) $[(a,1),\ (a,1),\ (a,2)]$, $3\frac{f(3s_c)^2}{(1+f(3s_c)+f(3))^3}$

(3) $[(a,1),\ (a,1),\ (a,3)]$, $3\frac{f(3s_c)^2f(3)}{(1+f(3s_c)+f(3))^3}$

(4) $[(a,1),\ (a,2),\ (a,2)]$, $3\frac{f(3s_c)}{(1+f(3s_c)+f(3))^3}$

(5) $[(a,1),\ (a,2),\ (a,3)]$, $6\frac{f(3s_c)f(3)}{(1+f(3s_c)+f(3))^3}$

(6) $[(a,1),\ (a,3),\ (a,3)]$, $3\frac{f(3s_c)f(3)^2}{(1+f(3s_c)+f(3))^3}$

(7) $[(a,2),\ (a,2),\ (a,2)]$, $\frac{1}{(1+f(3s_c)+f(3))^3}$

(8) $[(a,2),\ (a,2),\ (a,3)]$, $3\frac{f(3)}{(1+f(3s_c)+f(3))^3}$

(9) $[(a,2),\ (a,3),\ (a,3)]$, $3\frac{f(3)^2}{(1+f(3s_c)+f(3))^3}$

(10) $[(a,3),\ (a,3),\ (a,3)]$, $\frac{f(3)^3}{(1+f(3s_c)+f(3))^3}$

(g-2) $[(a,l,2),\ (a,r,2),\ (a,r,2)]$, $cp_{g2}=\frac{1}{4}$

(1) $[(a,1),\ (a,1),\ (a,1)]$, $\frac{f(3s_c)f(3)^2}{(1+f(3s_c)+f(3))^3}$

(2) $[(a,1),\ (a,1),\ (a,2)]$, $\frac{f(3)^2+2f(3s_c)f(3)}{(1+f(3s_c)+f(3))^3}$

(3) $[(a,1),\ (a,1),\ (a,3)]$, $\frac{f(3)^3+2f(3s_c)^3f(3)}{(1+f(3s_c)+f(3))^3}$

(4) $[(a,1),\ (a,2),\ (a,2)]$, $\frac{f(3s_c)+2f(3)}{(1+f(3s_c)+f(3))^3}$

(5) $[(a,1),\ (a,2),\ (a,3)]$, $\frac{2f(3s_c)f(3)+2f(3)^2+2f(3s_c)^2}{(1+f(3s_c)+f(3))^3}$

(6) $[(a,1),\ (a,3),\ (a,3)]$, $\frac{f(3s_c)^3+2f(3)^2f(3s_c)}{(1+f(3s_c)+f(3))^3}$

(7) $[(a,2),\ (a,2),\ (a,2)]$, $\frac{1}{(1+f(3s_c)+f(3))^3}$

(8) $[(a,2),\ (a,2),\ (a,3)]$, $\frac{f(3)+2f(3s_c)}{(1+f(3s_c)+f(3))^3}$

(9) $[(a,2),\ (a,3),\ (a,3)]$, $\frac{f(3s_c)^2+2f(3s_c)f(3)}{(1+f(3s_c)+f(3))^3}$

(10) $[(a,3),\ (a,3),\ (a,3)]$, $\frac{f(3)f(3s_c)^2}{(1+f(3s_c)+f(3))^3}$

(g-3) $[(a,l,2),\ (a,r,2),\ (a,l,2)]$, $cp_{g3}=\frac{1}{4}$

(1) $[(a,1),\ (a,1),\ (a,1)]$, $\frac{f(3s_c)^2f(3)}{(1+f(3s_c)+f(3))^3}$

(2) $[(a,1),\ (a,1),\ (a,2)]$, $\frac{f(3)^2+2f(3s_c)f(3)}{(1+f(3s_c)+f(3))^3}$

(3) $[(a,1),\ (a,1),\ (a,3)]$, $\frac{f(3)^3+2f(3s_c)^2f(3)}{(1+f(3s_c)+f(3))^3}$

(4) $[(a,1),\ (a,2),\ (a,2)]$, $\frac{f(3s_c)+2f(3)}{(1+f(3s_c)+f(3))^3}$

(5) $[(a,1),\ (a,2),\ (a,3)]$, $\frac{2f(3s_c)f(3)+2f(3)^2+2f(3s_c)^2}{(1+f(3s_c)+f(3))^3}$

(6) $[(a,1),\ (a,3),\ (a,3)]$, $\frac{f(3s_c)^3+2f(3)^2f(3s_c)}{(1+f(3s_c)+f(3))^3}$

(7) $[(a,2),\ (a,2),\ (a,2)]$, $\frac{1}{(1+f(3s_c)+f(3))^3}$

(8) $[(a,2),\ (a,2),\ (a,3)]$, $\frac{f(3)+2f(3s_c)}{(1+f(3s_c)+f(3))^3}$

(9) $[(a,2),\ (a,3),\ (a,3)]$, $\frac{f(3s_c)^2+2f(3s_c)f(3)}{(1+f(3s_c)+f(3))^3}$

(10) $[(a,3),\ (a,3),\ (a,3)]$, $\frac{f(3s_c)f(3)^2}{(1+f(3s_c)+f(3))^3}$

(g-4)$[(a,r,2),\ (a,r,2),\ (a,r,2)]$, $cp_{g4}=\frac{1}{4}$

(1) $[(a,1),\ (a,1),\ (a,1)]$, $\frac{f(3)^3}{(1+f(3s_c)+f(3))^3}$

(2) $[(a,1),\ (a,1),\ (a,2)]$, $3\frac{f(3)^2}{(1+f(3s_c)+f(3))^3}$

(3) $[(a,1),\ (a,1),\ (a,3)]$, $3\frac{f(3s_c)f(3)^2}{(1+f(3s_c)+f(3))^3}$

(4) $[(a,1),\ (a,2),\ (a,2)]$, $3\frac{f(3)}{(1+f(3s_c)+f(3))^3}$

(5) $[(a,1),\ (a,2),\ (a,3)]$, $6\frac{f(3s_c)f(3)}{(1+f(3s_c)+f(3))^3}$

(6) $[(a,1),\ (a,3),\ (a,3)]$, $3\frac{f(3s_c)^2f(3)}{(1+f(3s_c)+f(3))^3}$

(7) $[(a,2),\ (a,2),\ (a,2)]$, $\frac{1}{(1+f(3s_c)+f(3))^3}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;3\frac{f(3s_c)}{(1+f(3s_c)+f(3))^3}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;3\frac{f(3s_c)^2}{(1+f(3s_c)+f(3))^3}$

(10) $[(a,3),\,(a,3),\,(a,3)],\;\frac{f(3s_c)^3}{(1+f(3s_c)+f(3))^3}$

(h-1) $[(a,l,2),\,(a,l,2),\,(a,l,3)],\;cp_{h1}=\frac{1}{4}$

(3) $[(a,1),\,(a,1),\,(a,3)],\;\frac{f(2s_c)^2}{(1+f(2s_c)+f(1))^2}$

(5) $[(a,1),\,(a,2),\,(a,3)],\;2\frac{f(2s_c)}{(1+f(2s_c)+f(1))^2}$

(6) $[(a,1),\,(a,3),\,(a,3)],\;2\frac{f(2s_c)f(1)}{(1+f(2s_c)+f(1))^2}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;\frac{1}{(1+f(2s_c)+f(1))^2}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;2\frac{f(1)}{(1+f(2s_c)+f(1))^2}$

(10) $[(a,3),\,(a,3),\,(a,3)],\;\frac{f(1)^2}{(1+f(2s_c)+f(1))^2}$

(h-2) $[(a,l,2),\,(a,r,2),\,(a,l,3)],\;cp_{h2}=\frac{1}{2}$

(3) $[(a,1),\,(a,1),\,(a,3)],\;\frac{f(2s_c)f(2)}{(1+f(2s_c)+f(1))(1+f(s_c)+f(2))}$

(5) $[(a,1),\,(a,2),\,(a,3)],\;\frac{f(2s_c)+f(2)}{(1+f(2s_c)+f(1))(1+f(s_c)+f(2))}$

(6) $[(a,1),\,(a,3),\,(a,3)],\;\frac{f(2s_c)f(s_c)+f(1)f(2)}{(1+f(2s_c)+f(1))(1+f(s_c)+f(2))}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;\frac{1}{(1+f(2s_c)+f(1))(1+f(s_c)+f(2))}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;\frac{f(1)+f(s_c)}{(1+f(2s_c)+f(1))(1+f(s_c)+f(2))}$

(10) $[(a,3),\,(a,3),\,(a,3)],\;\frac{f(s_c)f(1)}{(1+f(2s_c)+f(1))(1+f(s_c)+f(2))}$

(h-3) $[(a,r,2),\,(a,r,2),\,(a,l,3)],\;cp_{h3}=\frac{1}{4}$

(3) $[(a,1),\,(a,1),\,(a,3)],\;\frac{f(2)^2}{(1+f(s_c)+f(2))^2}$

(5) $[(a,1),\,(a,2),\,(a,3)],\;2\frac{f(2)}{(1+f(s_c)+f(2))^2}$

(6) $[(a,1),\,(a,3),\,(a,3)],\;2\frac{f(s_c)f(2)}{(1+f(s_c)+f(2))^2}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;\frac{1}{(1+f(s_c)+f(2))^2}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;2\frac{f(s_c)}{(1+f(s_c)+f(2))^2}$

(10) $[(a,3),\,(a,3),\,(a,3)],\;\frac{f(s_c)^2}{(1+f(s_c)+f(2))^2}$

(i-1) $[(a,l,2),\,(a,l,3),\,(a,l,3)],\;cp_{i1}=\frac{1}{2}$

(4) $[(a,1),\,(a,2),\,(a,2)],\;\frac{f(s_b)^2f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(5) $[(a,1),\,(a,2),\,(a,3)],\;2\frac{f(s_b)f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(6) $[(a,1),\,(a,3),\,(a,3)],\;\frac{f(s_c)}{(1+f(s_b))^2(1+f(s_c))}$

(7) $[(a,2),\,(a,2),\,(a,2)],\;\frac{f(s_b)^2}{(1+f(s_b))^2(1+f(s_c))}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;2\frac{f(s_b)}{(1+f(s_b))^2(1+f(s_c))}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;\frac{1}{(1+f(s_b))^2(1+f(s_c))}$

(i-2) $[(a,r,2),\,(a,l,3),\,(a,l,3)],\;cp_{i2}=\frac{1}{2}$

(4) $[(a,1),\,(a,2),\,(a,2)],\;\frac{f(s_b)^2f(1)}{(1+f(s_b))^2(1+f(1))}$

(5) $[(a,1),\,(a,2),\,(a,3)],\;2\frac{f(s_b)f(1)}{(1+f(s_b))^2(1+f(1))}$

(6) $[(a,1),\,(a,3),\,(a,3)],\;\frac{f(1)}{(1+f(s_b))^2(1+f(1))}$

(7) $[(a,2),\,(a,2),\,(a,2)],\;\frac{f(s_b)^2}{(1+f(s_b))^2(1+f(1))}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;2\frac{f(s_b)}{(1+f(s_b))^2(1+f(1))}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;\frac{1}{(1+f(s_b))^2(1+f(1))}$

(j) $[(a,l,3),\,(a,l,3),\,(a,l,3)],\;cp_j=1$

(7) $[(a,2),\,(a,2),\,(a,2)],\;\frac{f(3s_b)^3}{(1+f(3s_b))^3}$

(8) $[(a,2),\,(a,2),\,(a,3)],\;3\frac{f(3s_b)^2}{(1+f(3s_b))^3}$

(9) $[(a,2),\,(a,3),\,(a,3)],\;3\frac{f(3s_b)}{(1+f(3s_b))^3}$

(10) $[(a,3),\,(a,3),\,(a,3)],\;\frac{1}{(1+f(3s_b))^3}$