PAPR Reduction of OFDM Signal with SPS-SLM Technique and Analysis of BER Performance through AWGN Channel

Md. Salim Raza¹ and Md. Shahjahan Ali²
¹Dept. of Electrical & Electronic Engineering, Hamdard University Bangladesh, Munshiganj-1510
²Dept. of Electrical & Electronic Engineering, Islamic University, Kushtia-7003, Bangladesh
¹E-mail: salimaeece.iu@gmail.com ²E-mail: jahanshah.iu@gmail.com

ABSTRACT
Orthogonal Frequency Division Multiplexing (OFDM) has been currently under intense research for broadband wireless transmission due to its robustness against multipath fading. However, implementation of the OFDM system entails several difficulties. One of the major drawbacks is high Peak-to-Average Power Ratio (PAPR) which leads to increase Bit Error Rate (BER) due to the nonlinearity of the high power amplifier. The Selected Mapping Technique (SLM) is one of the most popular techniques to reduce PAPR of OFDM signal. With SLM Technique, side information bits are required to recover original data, which lead to increase the rate of data loss. In this paper, a specific set of sequential phase sequence (SPS) has been developed to perform SLM technique along OFDM transceiver without requiring side information. The SPS based SLM (SPS-SLM) technique is able to reduce almost same PAPR compared to the conventional SLM technique. Moreover, the BER performance has been studied considering different number of subcarriers as well as modulation orders.

KEYWORDS
Orthogonal Frequency Division Multiplexing (OFDM), Selected Mapping technique (SLM), Sequential Phase Sequence SLM (SPS-SLM), Peak to Average Power Ratio (PAPR), Additive White Gaussian Noise (AWGN), Complimentary Cumulative Distribution Function (CCDF), Bit Error Rate (BER).

1 INTRODUCTION
Orthogonal Frequency Division Multiplexing (OFDM) is an effective technology for high speed wireless communication because it uses multicarrier modulation scheme in which each subcarrier is used in orthogonal form. The orthogonal subcarrier offers high spectral efficiency and the multicarrier modulation scheme considerably eliminates the problem of multipath fading [1],[2]. As a result, the OFDM technique provides high speed data communication in wireless medium and, has been inevitably selected in many wireless communication standards such as Digital Video Broadcasting (DVB) system and worldwide interoperability for microwave access (WiMAX) [3]. In comparison with the single-carrier system, the OFDM system combines all independent subcarrier signals at the final stage of the OFDM transmitter. Therefore, the combined signals may cause great Peak to Average Power Ratio (PAPR). It is the key challenge of the OFDM system due to non-linearity effects of power amplifier at the final stage of the OFDM transmitter. To remove the problem of high PAPR, Linear high frequency power amplifier is required at the OFDM transmitter. So it is required to research on the PAPR with its distribution and reduction technique in the OFDM system [4],[5]. Recently, complementary Cumulative Distribution Function (CCDF) is conventionally used to express distribution of the PAPR, and different theories and hypotheses have been developed for the distribution of the PAPR of the OFDM signal [6],[7],[8].

In this paper, we have proposed the model of new phase sequence which is called sequential phase sequence (SPS). The proposed SPS-SLM based OFDM system has shown better performance in reducing PAPR of OFDM signal and no need side information (SI) to recover original data at OFDM receiver.

The remainder of this paper is organized as follows: The OFDM system model has been presented in section 2, section 3 represents PAPR reduction technique, section 4 describes the proposed sequential phase sequence SPS and the SPS-SLM based OFDM system, section 5
and Section 6 contains the simulation results and conclusion respectively.

2 OFDM SYSTEM MODEL

The basic model of OFDM transmitter and receiver is presented as shown in figure 1. In OFDM system, frequency bandwidth $B$ is divided into $N$ non-overlapping orthogonal subcarriers of bandwidth $\Delta f$, where $B = NA_f$. In OFDM transmitter, serial input data stream is divided into frames of $N_f$ bits. These $N_f$ bits are arranged into $N$ groups, where $N$ is the number of subcarriers [9].

![Basic OFDM Transceiver Model](image)

Figure 1. Basic OFDM Transceiver Model

The number of bits in each of the $N$ groups determines the constellation size for that particular subcarrier. For example, if all the subcarriers are modulated by QPSK then each of the groups consists of 2 bits, if 16-QAM modulation is used where each group contains 4 bits. Let us denote a block of $N$ frequency domain subcarriers as a vector $X = [X_0, X_1, X_2, X_3, \ldots, X_{N-1}]^T$. In the time domain, via an IFFT operation we obtain $x = [x_0, x_1, x_2, x_3 \ldots, x_{N-1}]^T$. Thus, the time domain sampled sequence is expressed in the following equation [10].

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad (1)$$

Every time domain symbol $x(n)$ corresponding every subcarrier is combined together by P/S converter which output is discrete time signal that is converted into continuous form by D/A converter. Finally, this continuous time domain OFDM signal is transmitted through power amplifier after filtering. The OFDM receiver just executes reverse operation of the OFDM transmitter and finally recovers every original transmitted data block. One of the major problems of OFDM transmitting signal is high PAPR because it is formed by the sum of several sinusoidal signals. The high PAPR of the OFDM signal that leads to power inefficiency in RF section of the transmitter and increased complexity in the analog to digital (A/D) and digital to analog (D/A) Converter. So it is a very challenging job in OFDM system to adopt low PAPR in the OFDM signal. The conventional definition of the PAPR for the OFDM symbol in the time domain $x$ may be expressed as [11]:

$$\text{PAPR} = \frac{P_{\text{peak}}}{P_{\text{average}}} = \frac{\text{Max}[|x(n)|^2]}{E[|x(n)|^2]} \quad (2)$$

The value of PAPR can be measured in decibel form in the following expression:

$$\text{PAPR}(\text{dB}) = 10 \log_{10} \frac{\text{Max}[|x(n)|^2]}{E[|x(n)|^2]} \quad (3)$$

3 PAPR REDUCTION TECHNIQUES

The PAPR reduction techniques have been proposed in the literature [12],[13],[14]. It should be noted that most of the methods are based on the same idea of selecting the time domain signal to be transmitted from a set of different representations with the constraint of minimization of the PAPR which would degrade the performance of system [15],[16]. Nevertheless, the PAPR reduction methods can be classified into distortion-less and distortion techniques. Here, only two distortion techniques such as Clipping and Companding, and two distortion-less techniques such as Partial Transmit Sequence (PTS) and Selective Mapping Scheme (SLM) will be discussed in below.

3.1 DISTORTION BASED PAPR REDUCTION TECHNIQUES

The schemes that introduce spectral re-growth belong to distortion based category. These
techniques are the most straightforward PAPR reduction methods. This is one of the simplest ways to reduce PAPR in an OFDM system, but as we will see below that the simplicity in the approach brings an advantage along with several disadvantages.

3.1.1 Clipping

The clipping is one of the simplest distortion based techniques to reduce the PAPR of the OFDM signal. It reduces the peak of the OFDM signal by clipping the desired level but both in-band distortion and out-of-band radiation are created in transmitting OFDM signal. To limit out-of-band radiation and PAPR, Jean Armstrong proposed iterative clipping and filtering scheme [17].

3.1.2 Companding

Companding is another effective distortion based PAPR reduction technique in the OFDM system [18]. The µ-law companding based distortion scheme was proposed by Wang et al. to reduce the PAPR of OFDM signal [19]. And in this scheme, the average power of the OFDM signal is increased after companding but peak power remains unchanged. As a result, the commanding scheme significantly reduces the PAPR of the OFDM signal and improves the BER performance.

3.2 NON DISTORTION BASED PAPR REDUCTION TECHNIQUES

Non-distortion PAPR reduction schemes do not distort the shape of the OFDM signal and therefore no spectral re-growth takes place. Two more distortion-less PAPR reduction techniques namely partial transmit sequence (PTS) and selective mapping (SLM) are also conventionally used to reduce PAPR of OFDM signal [20].

3.2.1 Partial Transmit Sequence (PTS)

The idea of partial transmit sequences (PTS) algorithm is to divide the original OFDM sequence into several sub-sequences, and for each sub-sequence, multiplied by different weights until an optimum value is chosen. If the number of sub blocks increases then it increases not only computational complexity for selecting the optimum set (provide least possible PAPR) of phase sequence but also increases the amount of Side Information(SI) to be conveyed to the receiver. The involvement of SI decreases data rate in OFDM system.

3.2.2 Selective Mapping Scheme (SLM)

Selective mapping is considered as a promising technique for PAPR reduction because it does not produce distortion yet maintain the system performance to a great extent [21],[22]. In this scheme, data block are firstly converted into several independent blocks and the block with lower PAPR is sent, in which converting process involves multiplying data sequences with generated random phase sequences. The selected index is called side-information index which must also be transmitted to allow recovery of the data block at the receiver side. SLM leads to the reduction in data rate. In this method, main complexity occurs in recovering the side information which has the highest importance because it is used to recover the original data signal. If SI gets corrupted then entire OFDM symbol block can be damaged and error performance of SLM OFDM system degrades severely. The SLM techniques with different approaches are quite effective to reduce PAPR of OFDM signal either considering side information bit or not [23],[24],[25].

4 PROPOSED SPS-SLM TECHNIQUE

There are no basic differences between SPS-SLM and conventional SLM technique, The SPS-SLM technique can only be distinguished from conventional SLM by the characteristic of Sequential Phase Sequences (SPS) that are mathematically represented with equations of 4 to 6.

In the SPS-SLM based OFDM transmitter, the random binary data is considered as input data sources that are converted into symbolic form according to the modulation technique. After that all serial symbolic data are divided into number of block with N data symbol which is equal to number of subcarrier that was taken for OFDM
transmitter. Every data block that contains N serial data symbols are converted into N parallel data symbol using serial to parallel (S/P) converter. Now N parallel symbol in every data block is converted into N complex form symbol using mapping (such as M array QAM or PSK, etc). The model of SPS-SLM based OFDM Transmitter has been presented as shown in figure 2. Let us consider, the block of N frequency domain symbol as a vector $X_n = [X_0, X_1, X_2, \ldots, X_{N-1}]^T$, and $X_n^{(u)} = [X^{(u)}_0, X^{(u)}_1, X^{(u)}_2, \ldots, X^{(u)}_{N-1}]^T$ where $n=0, 1, 2, 3, \ldots, N-1$ and N represents IFFT point as well as number of subcarriers. After that Each block is multiplied (element by element multiplication) by phase sequences vector $P^{(u)} = [P^{(u)}_0, P^{(u)}_1, P^{(u)}_2, \ldots, P^{(u)}_{N-1}]^T$. Where $k=0, 1, 2, 3, \ldots, N-1$; $u=1, 2, 3, \ldots$, $U$ and $U$ represents maximum number of phase sequence. In this research work, The $P^{(u)}$ is called Sequential Phase Sequence (SPS) whose characteristic is defined by the following equation:

$$P^{(u)} = P_k^{(u)} = [\exp(j \phi^{(u)}_k)]^T \quad (4)$$

$$P_k^{(u)} = \exp \left[ j \left\{ 2\pi k (u-1)/D + 2\pi (u-1)/D \right\} \right] \quad (5)$$

$$D = N \ast \left[ (U/2) + 2 \right] + (U/2) \quad (6)$$

In this paper, D is also called as “Phase Divider”. After multiplication of $X_n^{(u)}$ and $P^{(u)} = P_k^{(u)}$, can be expressed by following equation:

$$X_n^{(u)} = [P^{(u)}, \ast, X_n^{(u)}] \quad (7)$$

Now frequency domain symbol vector $X_n^{(u)}$ is transformed into a time domain symbol vector $x_n^{(u)}$ by IFFT operation. Mathematically it can be denoted as:

$$x_n^{(u)} = \text{IFFT} \left[ X_n^{(u)} \right] \quad (8)$$

Finally, only one data vector is selected from $x_n^{(u)}$ with minimum PAPR and transmitted after passing through (S/P), (D/A) and Filter.

The model of SPS-SLM based OFDM receiver is shown in figure 3. At first, the receiver receives OFDM signal that passes through the filter, A/D converter and P/S converter which provides the time domain symbol vector $x_n^{(u)}$.

![Figure 2: SPS-SLM based OFDM Transmitter](image)

The time domain symbol vector $x_n^{(u)}$ is converted into frequency domain symbol vector using FFT operation. Mathematically it can be written as the following equation:

$$X_n^{(u)} = \text{FFT} \left[ X_n^{(u)} \right] \quad (9)$$

Now the number of different anti-sequential phase sequence (SPS) can be denoted by $-P_k^{(v)}$, which is multiplied with frequency domain symbol vector $X_n^{(u)}$ element by element. Mathematically, it can be written as the following equation:

$$X_n^{(u)} = -P_k^{(v).}\ast X_n^{(u)} \quad (10)$$

Where $-P_k^{(v)} = -\left[ \exp \left\{ -j \phi^{(u)}_k \right\} \right]^T = \exp \left\{ -j \left\{ 2\pi k (u-1)/D + 2\pi (u-1)/D \right\} \right\}$ and $v = u = 1, 2, 3, \ldots, U$.

In notation $X_n^{(u)}$, if the value of $v$ is equal to the value of $u$ then we get $X_n^{(u)} = X_n$ which is selected by decision making device in SPS-SLM based OFDM receiver.

The complex frequency domain symbol vector $X_n$ is converted into a normal symbol vector according to modulation that has already been taken at the OFDM transmitter. Now this vector or data block passes through P/S converter which
gives the output data block of serial N symbol and so on.

\[ X(t) \rightarrow \text{Filter} \rightarrow X(t) \rightarrow \text{A/D Converter} \rightarrow \text{S/P Converter} \rightarrow x_n^{(u)} \]

\[ x_n^{(u)} \rightarrow \text{N-point FFT Circuit} \rightarrow \text{Select Original Complex data block which was created by mapping at transmitter} \rightarrow X_n \]

\[ X_n \rightarrow \text{De-Mapping} \rightarrow \text{P/S Converter} \rightarrow \text{Original binary data} \rightarrow \text{Destination} \]

Figure 3. SPS-SLM based OFDM Receiver

Finally, every serial symbol in every data block is converted into binary serial data which are actually same as transmitted binary data at the OFDM transmitter.

5 RESULTS AND DISCUSSION

Many factors are considered to analyses performance of OEDM system such as number of subcarriers as well as IFFT/FFT point, number of phase sequences, modulation scheme, type of channel, BER and etc. The performance study is generally done by changing one of the above parameters keeping others unchanged. In this paper, 128 and 256 subcarriers have been taken for 4QAM and 16QAM modulation schemes and for each case, the effect of changing phase sequences on PAPR reduction and the BER performances considering AWGN channel have been observed. In order to evaluate the performance of the system, a MATLAB programming has been performed considering random bit information.

5.1 PAPR Performance

The PAPR performance of the OFDM signal is represented as shown in figure 4 considering 128 subcarriers and 4QAM modulation scheme. In the figure, it is observed that the PAPR of OFDM signal is about 10.5 dB corresponding CCDF of \(10^{-3}\) without SPS-SLM. PAPR of OFDM signal is 10.2dB, 9.9dB, 8.7dB and 8.6dB for number of phase sequences 2, 4, 8 and 16 respectively. The maximum PAPR is reduced by 1.9dB.

Figure 4. PAPR performance of OFDM signal for 128 subcarriers and 4QAM modulation scheme

Figure 5. PAPR performance of OFDM signal for 128 subcarriers and 16QAM modulation scheme
Figure 5 shows the PAPR performances of OFDM signal for 128 subcarriers and 16QAM modulation scheme. Here it is observed that PAPR of OFDM signal is about 10.5 dB corresponding CCDF of $10^{-3}$ without SPS-SLM. PAPR of OFDM signal is 10.5dB, 10.2dB, 8.5dB and 8.2dB for number of phase sequences 2, 4, 8 and 16 respectively. The maximum PAPR is reduced by 2.3dB.

Figure 6 presents the PAPR performance of OFDM signal for 256 subcarriers and 4QAM modulation scheme. In the figure, it is observed that the PAPR of OFDM signal is about 10.8 dB corresponding CCDF of $10^{-3}$ without SPS-SLM. The PAPR of the OFDM signal is 10.5dB, 9.9dB, 9.2dB and 9.0dB for number of phase sequences 2, 4, 8 and 16 respectively. The maximum PAPR is reduced by 1.8dB.

In figure 7, it is observed that the PAPR of the OFDM signal is about 10.9 dB corresponding CCDF of $10^{-3}$ without SPS-SLM for 256 subcarriers and 16QAM modulation scheme. The PAPR of the OFDM signal is 10.6dB, 10.4dB, 9.0dB and 8.8dB for number of phase sequences 2, 4, 8 and 16 respectively. The maximum PAPR is reduced by 2.1dB.

5.2 BER Performance

The AWGN channel model is widely used in studying of OFDM system. In this model there is only linear addition of white noise with a constant spectral density and Gaussian distribution of amplitude. The model does not consider fading and frequency selectivity, interference. Although it is not much suitable for most of the terrestrial links yet being used for providing simple and controlled mathematical models to study the basic behavior of a system in the absence of the above mentioned factors. In this simulation, MATLAB function has been used to simulate BER performance of OFDM signal considering two parameters such as number of subcarriers and modulation scheme. In order to analyses performance of BER of OFDM signal, numbers of subcarriers are fixed but modulation orders are varied at a specific case. It is to be remembered that the term “NS” has been used instead of number of subcarrier on figure 8 and figure 9. Figure 8 presents the BER performance of the OFDM signal for 4QAM versus 16QAM modulation scheme with 128 subcarriers. The red line indicates 128 subcarriers with 4QAM modulation scheme and green line indicates 128 subcarriers with 16QAM modulation scheme. It has been observed that the BER of the OFDM signal is about $10^{-5}$ for 4QAM modulation scheme at 12 SNR in dB but same BER is measured for 16QAM modulation scheme at 19 SNR in dB.
Figure 9 presents the BER performance of the OFDM signal for 4QAM verses 16QAM modulation scheme with 256 subcarriers. The red line indicates 256 subcarriers with 4QAM modulation scheme and green line indicates 256 subcarriers with 16QAM modulation scheme. By observing, we have seen the BER of the OFDM signal is about $10^{-5}$ for 4QAM modulation scheme at 11.9 SNR in dB but same BER is measured for 16QAM modulation scheme at 19.1 SNR in dB.

### 6 CONCLUSION

This paper contained the model and working principle of proposed SPS-SLM technique in which new model of phase sequence called sequential phase sequence (SPS) has been developed where no side information needs to be sent. Finally, it has been measured the performance of PAPR and BER through the AWGN channel of the OFDM signal with the SPS-SLM technique. In this study, maximum 16 phase sequences have been used to improve the PAPR performance of the OFDM signal. For 128 subcarriers, maximum PAPR is reduced up to 2.3dB when 16QAM modulation scheme is used but only 1.9dB PAPR can be reduced when 4QAM modulation scheme is used. And for 256 subcarriers, the highest PAPR is reduced up to 2.1dB under 16QAM modulation scheme but 1.8dB PAPR can be reduced under 4QAM modulation scheme. The BER of the OFDM signal is about $10^{-5}$ for 128 subcarriers with 4QAM and 16QAM modulation scheme at SNR of 12dB and 19dB respectively. Near to same results are measured for 256 subcarriers with 4QAM and 16QAM modulation scheme. Analyzing the above results, it has been observed that the SPS-SLM technique shows the better performance for 16QAM modulation scheme over 4QAM modulation scheme. So it can be concluded that the SPS-SLM technique not only solves the problem of side information bits as well as system complexity but also works effectively under higher order QAM modulation scheme which leads to increase data rate in the wireless communication system where the SPS-SLM based OFDM system is chosen.

### REFERENCES


