

## Spectrum Analysis of the Suspension Dynamics Measured by MEMS Inertial System

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### ABSTRACT

The current paper compares the spectrum methods to estimate the suspension dynamics measured by the MEMS inertial data acquisition system which is installed on the vibration stand. The analysis is accomplished by different type of Wavelet transformation functions and Short Time Fourier Transform (STFT) to compare the time and frequency resolution of the results.

### KEYWORDS

MEMS inertial sensor, suspension dynamics, spectrum analysis

### 1 INTRODUCTION

Vibration analysis is an important consideration when an applied load is not constant (static), inducing unstable modes of vibration (resonance) which result in a shortened service life and cause unexpected failures. The frequency analysis process involves of the dynamics determining the vibration severity, identifying frequencies and patterns, associating the peaks and patterns with mechanical or electrical components, forming conclusions and, if necessary, making recommendations for repair. Also the estimation of the resonant frequency, amplitude (strength) and spectrum (signature) of the suspension accelerations allow active or passive monitoring of vehicle suspension elements.

But the time domain and frequency domain analysis are inadequate to fully describe the nature of non-stationary signals. A variety of methods for obtaining the energy density of a function, simultaneously in the time and frequency have

been devised, most notably Short Time Fourier Transform (STFT) [1,2], Wigner-Villes Transform (WVD) [2], Wavelet Transform [3], Linear decimation [4], Frequency Response Functions (FRF) [5], Power spectrum density (PSD) [6], Auto-Regressive Model [7].

The current paper discusses the comparison results of the applied Short Fast Fourier Transform and Wavelet transformation to detect resonant frequencies and their amplitudes in the suspension dynamics. It is shown that the choice of the spectrum analysis parameters show great influence over the time – frequency distribution of the modal frequency amplitudes.

### 2 SPECTRUM ANALYSIS METHODS

The wavelet analysis usually is preferred due to its better frequency and time resolution towards Fourier transform. The transformation coefficients are calculated according to the equation [8]:

$$W_{(x)}(a,b) = a^{-\frac{1}{2}} \int_{-\infty}^{\infty} \psi\left(\frac{t-b}{a}\right) x(t) dt \quad (1)$$

The wavelet transformation uses a wavelet function  $\psi\left(\frac{t-b}{a}\right)$  as a base and variables  $a$  (scale) and  $b$  (position). The wavelet functions may be considered as windows, which parameters depend on the signal. The time-frequency resolution of the windows is estimated by the equation [9]:

$$\Delta t \Delta \omega \geq \frac{1}{4\pi} \quad (2)$$

The choice of the wavelet function specifies the approach to this boundary. This task could not be solved in a simple way and there are lots of

methods to solve it. One of the possible ways is described at [10].

The output signal is processed on the bases of the wavelet packets which use the similar signal decomposition as a fast wavelet transform, but the details are decomposed to two parts. In this way we obtain a complete binary tree and deeper analysis may be accomplished. The level decomposition choice is based on the signal parameters or other specific criteria such as entropy of the calculated coefficients [11].

The decomposition level is based on the evaluation of the statistic parameters of the decomposition coefficients. Usually the decomposition level is chosen on the basis of the Gaussian noise testing model, because the signal model is based on the additive white Gaussian noise (WGN). The comparison method for the decomposition level estimation is proposed at [11]. It is based on the comparison of the energy of the entropy of the decomposition level coefficients to the noise with similar length and given distribution law. The statistic parameters which are used for the decomposition procedure are pointed to standard deviation, signal energy and correlation coefficient of the decomposition coefficients.

In this paper we suggest to use the last criteria [12]:

$$R_i(j) = \frac{\sum_{t=1}^{n-1} (F_j(t) - \overline{F_j(t)}) (F_j(t+1) - \overline{F_j(t)})}{\sum_{t=1}^n (F_j(t) - \overline{F_j(t)})^2}, \quad (3)$$

where  $\overline{F_j(t)}$  - approximated signal average value of the corresponding decomposition level  $j$

If the signal length is equal to  $n$ , then there are  $(n-1)$  correlation coefficients. The optimal decomposition level is defined by the minima of the  $R_i(j)$  value.

There are several methods for the threshold selection mainly developed by Donoho and Johnstone [13-15]. The different threshold values may be written according to the equation  $th = \delta \lambda$ , where  $\delta$  - the valuation of the noise level. Some of the criteria of the selection of the  $\lambda$  value are known as VisuShrink [16,17], SureShrink, BayesShrink [18], [19], minimax [15].

### 3 EXPERIMENTAL STUDY AND RESULTS

The experimental car is lifted on the vibration stand type BOGE-AFIT ShockTester, which excites the platform to 16 Hertz, and then shuts off. The ensuing platform vibrations decay at a rate that infers the performance of damper. The data acquisition system based on inertial MEMS sensor is located on the vibration stand and measure the vehicle suspension response. The vibration amplitude and frequency depend from the suspension design and condition. The test car is Fiat Bravo with installed Macpherson strut front suspension and its position on the vibration stand is shown at Figure 1.

The inertial MEMS sensor reads the data with a sampling frequency of 40Hz to satisfy the Nyquist criteria.



Figure 1. Vehicle situation on the vibration stand

The second frequency analysis is realized on the basis of a Short Fast Fourier Transform. The window has a rectangular shape with different size (from  $N=32$  to  $N=128$  samples) and 16 overlapped samples. The time – frequency distribution is shown at Figure 2.

The vibration signal is analyzed by MATLAB routine, which contains lots of functions for the signal processing analysis [20], and the results of the spectrum analysis are shown at the following figures:

- Figure 2 – Time – frequency distribution according to STFT using different window sizes

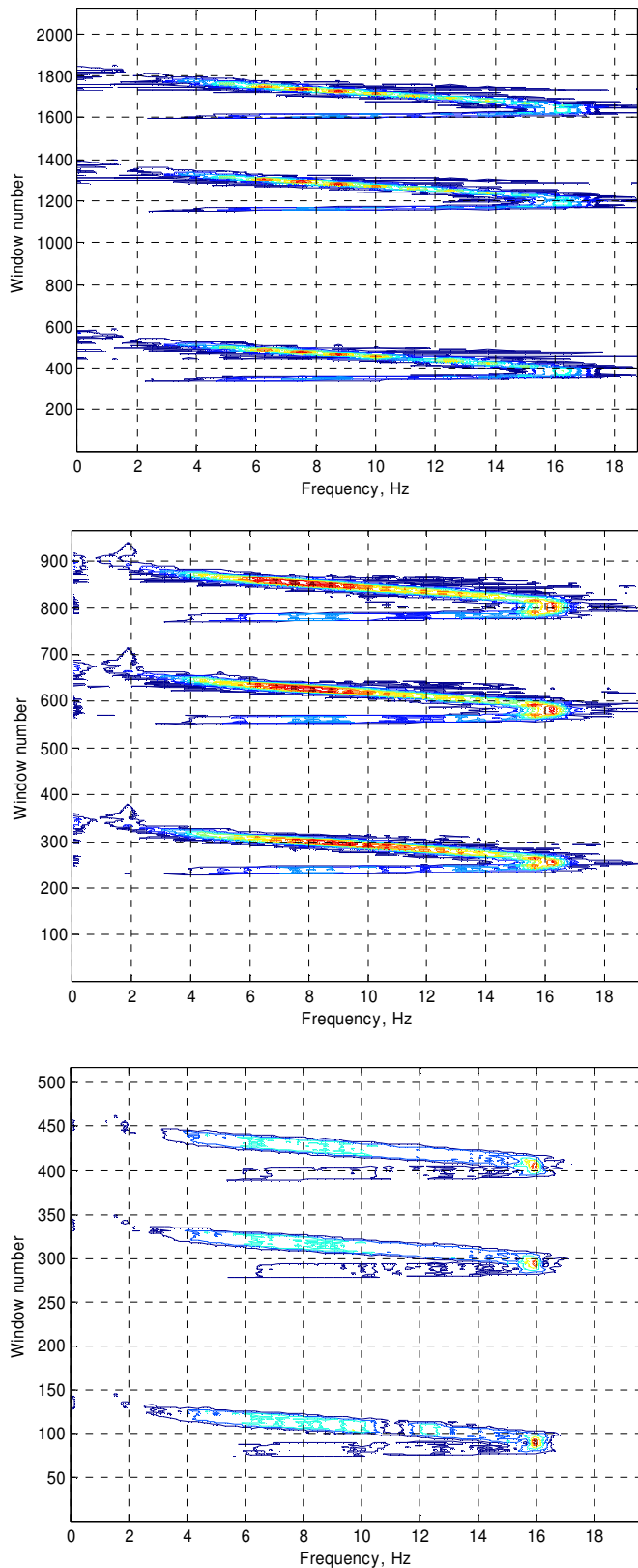


Figure 2. Time-frequency distribution of the oscillations using STFT and different window size  $N=32, 64$  and  $128$  respectively

- Figure 3 - Time - pseudo-frequency distribution using continuous wavelet transformation. The conformity between pseudo - frequencies and the scales is shown at Table 1.
- Figure 4 - Time - frequency distribution according to wavelet analysis using Daubechies family packets (db3) and 4-th level of decomposition
- Figure 5 - Time - frequency distribution according to wavelet analysis using Daubechies wavelet (db3) and 5-th level of decomposition

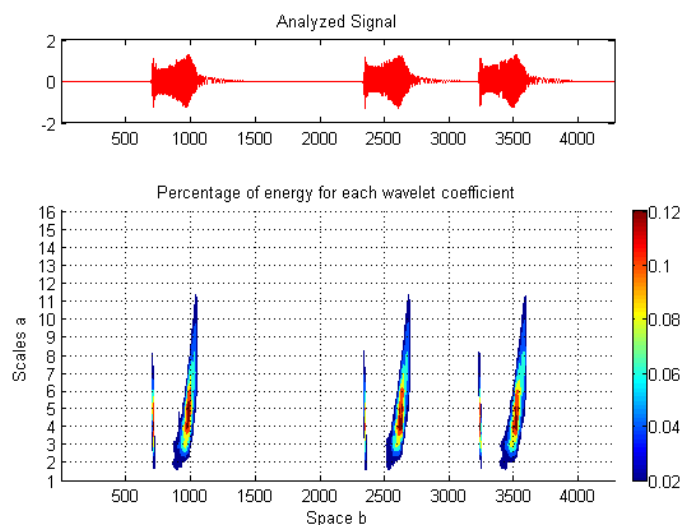


Figure 3. Time-pseudofrequency distribution using continuous wavelet transformation

Table 1. Conformity of pseudo-frequency and scales

Pseudo-frequency, Hz	Scales a
26,67	1
13,33	2
8,89	3
6,67	4
5,33	5
4,44	6
3,81	7
3,33	8
2,96	9
2,67	10
2,42	11
2,22	12
2,05	13
1,90	14
1,78	15
1,67	16

- Figure 6 - Time – frequency distribution according to wavelet analysis using Symlets wavelet (sym2) and 4-th level of decomposition
- Figure 7 - Time – frequency distribution according to wavelet analysis using Coiflets wavelet (coif8) and 4-th level of decomposition.
- Figure 8 - Time – frequency distribution according to wavelet analysis using DMeyer wavelet (dmey) and 4-th level of decomposition

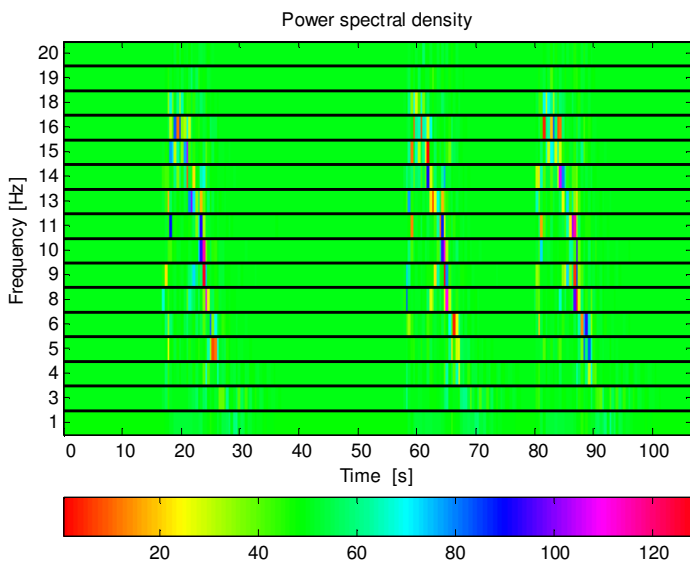


Figure 4. Time-frequency distribution using db3 wavelet packets /4-th decomposition level/

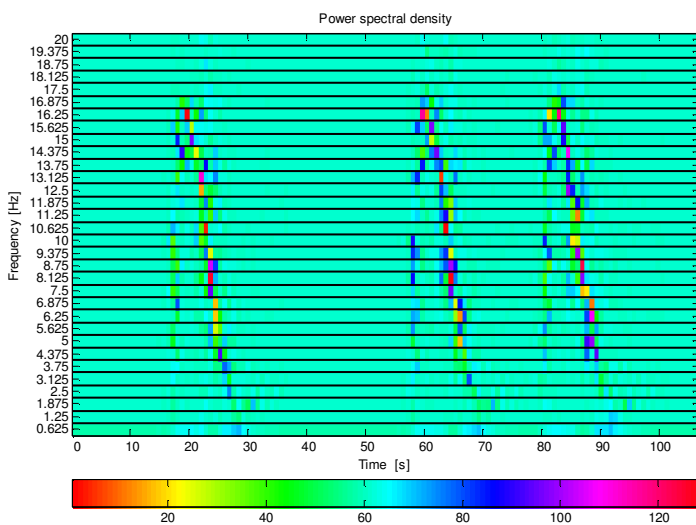


Figure 5. Time-frequency distribution using db3 wavelet packets /5-th decomposition level/

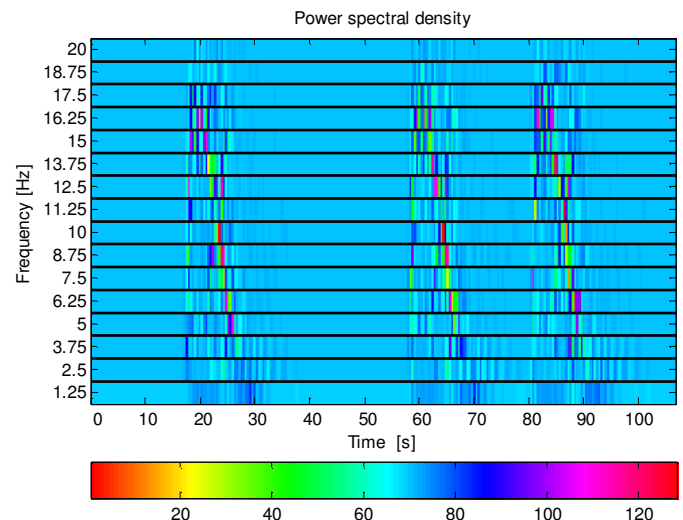


Figure 6. Time-frequency distribution using sym2 wavelet packets /4-th decomposition level/

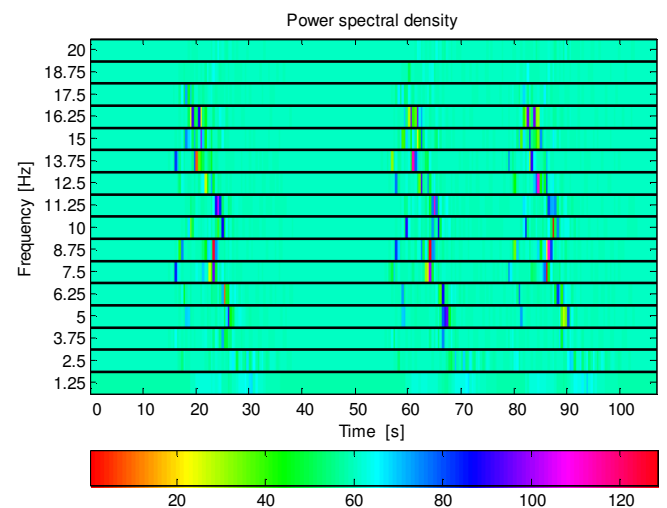


Figure 7. Time-frequency distribution using coif8 wavelet packets /4-th decomposition level/

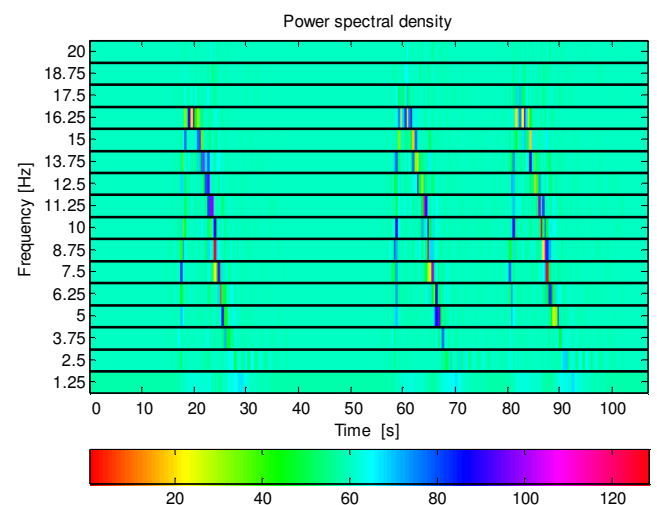


Figure 8. Time-frequency distribution using dmey wavelet function /4-th decomposition level/

- Figure 9 – 3D (Time – Frequency – Amplitude) distribution according to STFT using optimal window size  $N=64$
- Figure 10 - 3D (Time – Pseudo-Frequency – Amplitude) distribution according to wavelet analysis using Daubechies wavelet (db3) and optimal 5-th level of decomposition.

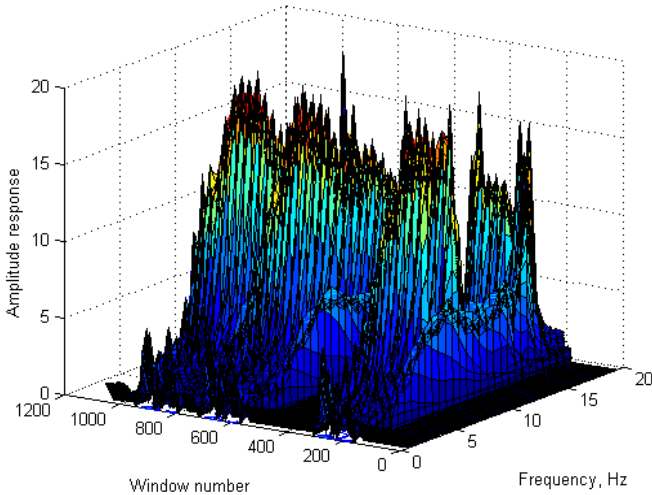


Figure 9. 3D representation of the spectrum using STFT

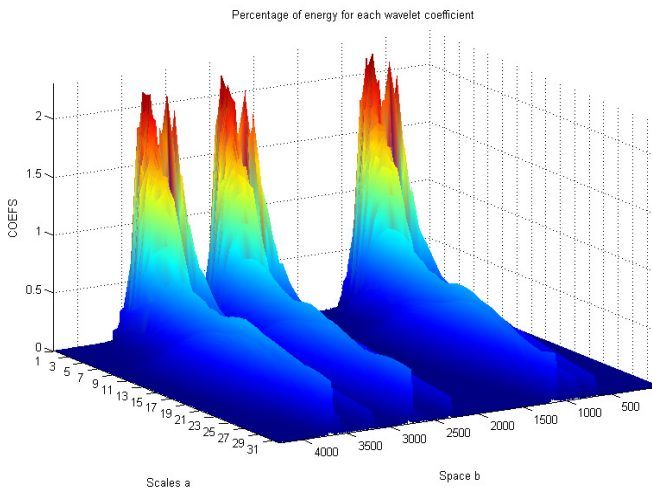


Figure 10. 3D representation of the spectrum using db3 wavelet packets /5-th decomposition level/.

The results show that if the window size is very small (up to 32 points – first picture, Figure 2) the frequency resolution is too small and the both spectrum peaks (resonance frequency of the left and right damper) appear as one. If the frequency resolution is increased too much i.e. the window size is increased from 64 to 128

points (second and third picture, Figure 2) than the suspension resonance peaks are too small to be separated and remain only the spectrum peak at the stand vibration frequency (16Hz). Therefore, there is an optimal window size, which includes the total number of points with the selected frequency and if the window size is greater than the optimal value, than the analyzed window contains signal and noise or a signal with more than one predominant frequency.

The continuous wavelet transform (CWT) shows the similar result as STFT (Figure 2), but the frequency scale is not distributed evenly and the neighbouring frequency peaks cannot be recognized due to the low frequency resolution.

The results from the spectrum analysis based on wavelet transform shows that not only the different wavelet families produce different results, but also the spectrum response depends from the decomposition level.

If the decomposition level is lower (Figure 4) the spectrum peaks are not well defined, but as it is increased to 5 according to the equation (3) than the peaks are well visible and the resonant frequencies may be identified successfully.

In the same time the comparison of the results produced by the wavelet families (sym2 – Figure 6, coif8 – Figure 7 and dmey – Figure 8) shows that the worst case is obtained by the Symlets wavelet family while the best case may be seen at Figure 8 using DMeyer wavelet family. The other wavelet families included in the MATLAB Wavelet Toolbox represent similar results shown at Figure 7.

Therefore the comparison of the 2D spectrum pictures shows that the Short Fast Fourier Transform is the most expedient tool to obtain the attenuation time because the attenuation line is well visible (Figure 2) while the wavelet transform is more appropriate to detect the resonant frequencies in the suspension dynamics due to the better frequency resolution (Figure 5) and the possibility to analyze abrupt changes in signal.

This situation is also proofed by the 3D spectrum analysis shown at Figure 9 and Figure 10 which represent the STFT and db3 wavelet packets spectrum.



In time-frequency signal analysis the classical Fourier transform analysis is inadequate because Fourier transform of a signal does not contain any local information which is the major drawback of the Fourier transform. The choice of the window size is also a critical problem in the Fourier transform due to the time and frequency resolutions are tied and the better time resolution conducts to the poor frequency resolution and vice versa. The wavelets give a better signal representation using multiresolution analysis, with balanced resolution at any time and frequency.

#### 4 CONCLUSION

The current paper discusses the spectrum analysis methods to obtain the dynamic characteristics of the vehicle suspension such as attenuation time and resonant frequency detection. It is shown that the wavelet transform could be used successfully to solve the selected task but the results depend from the used wavelet function and the decomposition level.

The best results are obtained using Daubechies wavelet (db3) and 5-th level of decomposition.

#### 5 ACKNOWLEDGMENT

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