Analysis and Modeling of Symmetric Slab Dielectric Structures to Solve Electrical and Magnetic Transverse Modes

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ABSTRACT
Optics devices have recently witnessed considerable improvement by fabricating them in terms of the change in integrated optical circuits' properties. Previously, the optical devices were adequate to propagate electromagnetic fields, whereas they are not appropriate for present integrated optics devices because the optical devices (e.g. glass lenses) were very large compared to the wavelength. This paper concerns with solving the guided modes. It is useful to start with a simple case of slab structure. The symmetric dielectric planar waveguides are the simple case to demonstrate the mechanisms of solving the TE-Mode and TM-Mode of optical structures. The optical wavelength is selected to be $1\mu m$ which provides the appropriate propagation constant modes (β) . This gives an effective refractive index which meets the condition of the field confinement in the core region. By including the active semiconductor lasers, the optical structures have experienced a significant improvement since they operate at sub-wavelength compared to the conventional structures. Simulations are carried out using MATLAB with Simulink tools to study the fields' behavior.

Keywords: propagation constant mode, effective refractive index, thickness, semiconductor lasers, modal gain.

1 INTRODUCTION
The fundamental element in the technology of integrated photonic is the optical waveguide. Waveguides can be introduced as an optical structure that enables the confinement of light within its boundaries depending on the theory of total internal reflection [1]. They are also known as a physical devices which guide the electromagnetic waves in the form of an optical signal. It is therefore possible to bound the optics signal by a system contaning various media, which can form the optical waveguides and surrounded by another media with refreactive indexe lower than that of core layer. The performace of this structure is basically based on the phenemian of the total internal reflection as shown in figure(1) [2].

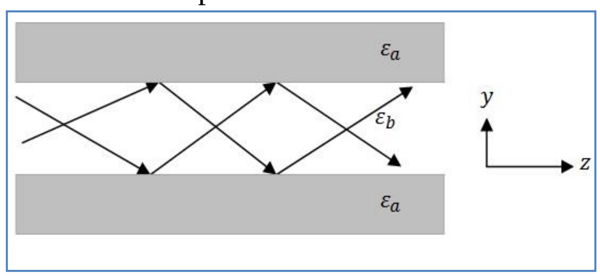
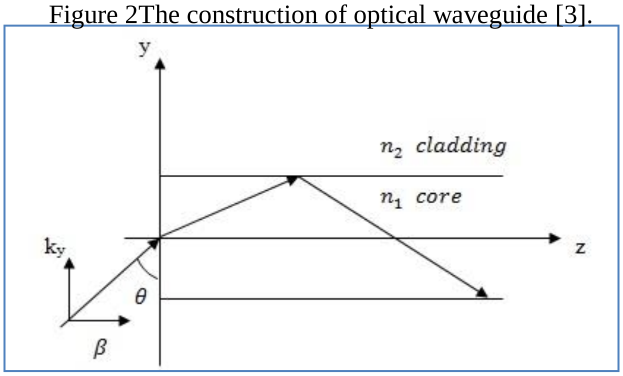


Figure 1 Total internal reflection [1].

Moreover, The optical waveguides can be classified in terms of mode structure , for instance, a single mode and multi-mode [1]. The principle means of solving waveguides theory can be derived by using Maxwell's equations since the simple configuration of the optical waveguides consists of three layers (substrate, core and cladding). Therefore, several mechanisms have been improved for the optical waveguides modeling to treat the field comprising both guided and radiated modes. The main mechanism of utilizing optical waveguide is based on the confinement of the optical waves within the core region as shown in figure (2) [3].

This phenomenon can be obtained when the total internal reflection (TIR) occurs. According to Snell's law, the beam has a curvature towards the normal pattern during its movement from lower to higher refractive index of the material [1] &[2]. At critical angle, the beam has a reflection by angle 90°; afterward the beam is totally reflected back to the core.

Figure 2The construction of optical waveguide [3].



Total internal reflection is also another concept which should be considered. It demonstrates how to confine the field within the desirable region, especially in the case of confining the field in the outmost layers [4].The formulation of Snell's law is:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad (1)$$

Then, to obtain the (TIR), $\theta_2 = 90°$, and $\theta_1 = \theta_{critical}$.

$$\theta_{critical} = \sin^{-1}\frac{n_2}{n_1} \quad (2)$$

When the incident angle exceed $\theta_{critical}$ an *evanescent wave* occurs along the interface, which is decayed exponentially in the second media. This wave can be bounded tightly towards the interface and named as surface wave which satisfies the boundary conditions at the interface [2].

2 DIELECTRIC SLAB WAVEGUIDE

To solve the propagation modes in the dielectric slab waveguide structures, it is more advantageous to begin with the simple case of having a symmetric slab waveguide. This is therefore considered to be more helpful to understand the guided modes of the waveguide structures shown in the following figures. Figure (3) introduces the whole model of the structure, while figure (4) shows a simple structure of slab waveguide which is the basic stage of demonstrating the field distribution in terms of both TE-Mode and TM-Mode cases. The assumption is made to have TE- Mode; meant electric field does not exist in the trend of the wave travel. The propagating of the electromagnetic wave is assumed to be in Z-direction. In this case, all fields must be tangential at the interface $(y = w/2)$ to satisfy the boundary conditions [2].

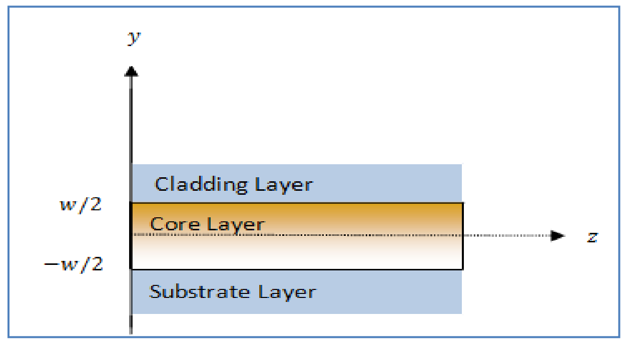


Figure 3 Dielectric slab structures [2].

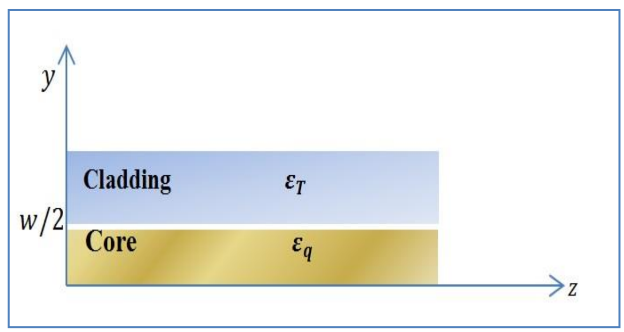


Figure 4 A schematic of a symmetric step-index slab waveguide [2].

2.1 Electric & Magnetic Modes Modeling Equations

The analysis of dielectric slab waveguide will consider the symmetric property, therefore; one interface is tested instead of whole structure. As it is shown in figure (4), the thickness at $(y = \frac{w}{2})$ represents the interface between the core and cladding layers where all fields must be continuous. This condition is achieved when

the propagation constant mode (β) is similar for both layers at the interface [2]. The mathematical method of solving this structure can be derived as follows.
Assume a plane wave that travels through the lossless dielectric medium. Its travelling plane is in y-z. Therefore, there is no field variation in x-direction. All Fields $F(\,y,\,z)$ must satisfy electromagnetic wave equation: [2]
For core region where $y < \left|\frac{w}{2}\right|$ the electromagnetic wave equation is

$$\frac{\partial^2\, F_q(y,z)}{\partial y^2} + \frac{\partial^2\, F_q(y,z)}{\partial z^2} + k_q^2 F_q(y,z) = 0 \qquad (3)$$

For cladding region where $y > \left|\frac{w}{2}\right|$

$$\frac{\partial^2\, F_T(y,z)}{\partial y^2} + \frac{\partial^2\, F_T(y,z)}{\partial z^2} + k_T^2 F_T(y,z) = 0 \qquad (4)$$

According to that, both TE modes and TM modes are possible to be solved.
In TE-Mode,
$E_z = 0$, and $\frac{\partial}{\partial_x} = 0$. Then, $\frac{\partial}{\partial_y} E_y = 0$.
The field at the interface must match all z values at $(y = w/2)$ to achieve the tangential field components and $[\beta = \beta_q = \beta_T]$ must be the same in all regions.

$$E_{qx}(y,z) = E_{qx}(y)e^{-j\beta z} \qquad (5)$$

$$\frac{d^2}{dy^2}\, E_{qx}(y) - \beta^2\, E_{qx}(y) + k_0^2\varepsilon_q\, E_{qx}(y) = 0 \qquad (6)$$

Where, ε_q : is the permittivity of the core layer

$$\frac{d^2}{dy^2}\, E_{qx}(y) + (k_0^2\varepsilon_q - \beta^2)\, E_{qx}(y) = 0 \qquad (7)$$

Suppose $k_q^2 = (k_0^2\varepsilon_q - \beta^2)$

$$\frac{d^2}{dy^2}\, E_{qx}(y) + k_q^2\, E_{qx}(y) = 0 \qquad (8)$$

The solution of this equation is

$$E_x(\mathrm{y}) = A_q e^{-j(k_q y)} + B_q\, e^{j(k_q y)} \qquad (9)$$

In the core region the wave equation will be simplified to the following expression in a symmetric configuration as shown in figure

$$E_{xq}(y) = A_q \cos(k_q y) + B_q \sin(k_q y) \qquad (10)$$

Since the attention is given to one interface, it is possible to select one part of the above equation. The first part is chosen because of symmetry [2]

$$E_{xq}(y) = A_q \cos(k_q y) \qquad (11)$$

The magnetic field can be derived from Maxwell equation

$$-jw\,\mu_0 H_q(y) = \begin{pmatrix} u_x & u_y & u_z \\ 0 & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ E_{Xq} & 0 & 0 \end{pmatrix}$$

$$-jw\,\mu_0 H_{qx} = 0$$

$$-jw\,\mu_0 H_{qy} = -\,(0 - \frac{\partial}{\partial_z} E_{Xq})$$

$$H_{qy}(y) = \frac{\beta}{\omega\mu_0} E_{Xq} = \frac{\beta}{\omega\mu_0} A_q \cos(k_q y)$$

$$-jw\,\mu_0 H_{qz} = (0 - \frac{\partial}{\partial_y} E_{Xq})$$

$$H_{qz}(y) = \mathrm{j}\frac{\mathrm{kq}}{\omega\mu_0} A_q \sin(k_q y)$$

For top layer

$$E_x(\mathrm{y}) = A_T e^{-j(k_T y)} + B_T e^{j(k_T y)} \qquad (12)$$

In the top layer the field should decay in order to achieve the light confinement and obtain the maximum field distribution in the core region. Therefore, it is essential to assume $k_T = j|k_T|$,

$$E_{Tx}\,(\mathrm{y}) = A_T e^{|k_T|y} + B_T e^{-\,|k_T|y} \qquad (13)$$

The incident amplitude (A_T) is considered to be zero, and then the second part will provide field decaying. The reason of having $A_T e^{|k_T|y} = 0$ to achieve the light confinement; otherwise, the field will spread to propagate away from the desirable area and become impossible to have field confinement.

$$E_{Tx}\,(\mathrm{y}) = B_T e^{-\,|k_T|y} \qquad (14)$$

$$-jw\,\mu_0 H_T(y) = \begin{pmatrix} u_x & u_y & u_z \\ 0 & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ E_{Xq} & 0 & 0 \end{pmatrix}$$

$$-jw\,\mu_0 H_{Tx} = 0$$

$$H_{Ty}\,(y) = \frac{\beta}{\omega\mu_0} B_T e^{-\,|k_T|y}$$

$$H_{Tz}(y) = \frac{j|k_T|}{\omega\mu_0} B_T e^{-\,|k_T|y}$$

By deriving the electric field and magnetic field in both layers, it is possible to apply the boundary conditions at the interface (y=+w/2).

$$E_{qz}\left(y = +\frac{w}{2}\right) = E_{Tz}\left(y = +\frac{w}{2}\right)$$

$H_{qz}\left(y = +\frac{w}{2}\right) = H_{Tz}\left(y = +\frac{w}{2}\right)$

Then;

$$A_q \cos(k_q \frac{w}{2}) = B_T e^{-|k_T|\frac{w}{2}} \quad (15)$$

$$j\frac{kq}{\omega\mu_0} A_q \sin(k_q \frac{w}{2}) = \frac{j|k_T|}{\omega\mu_0} B_T e^{-|k_T|\frac{w}{2}} \quad (16)$$

Dividing eq. (16) by eq. (15), the result will be,

$k_q \frac{w}{2} \tan(k_q \frac{w}{2}) = \frac{w}{2} \mid k_T \mid$

Let $u = k_q \frac{w}{2}$ and $v = \frac{w}{2} \mid k_T \mid$ then,

$$u\tan(u) = v \quad (17)$$

Since $k_q^2 = k_0^2\, \varepsilon_q - \beta^2$ and $k_T^2 = k_0^2\, \varepsilon_T - \beta^2$

$k_T = j|k_T|$, then $-|k_T|^2 = k_0^2\, \varepsilon_T - \beta^2$

After substitution, the equation will be reformed as follow:

$$\beta^2 = k_0^2\, \varepsilon_q - k_q^2 = k_0^2\, \varepsilon_T + |k_T|^2 \quad (18)$$

$|k_T|^2 + k_q^2 = k_0^2\, \varepsilon_q - k_0^2\, \varepsilon_T$

Where k_0 , ε_q and ε_T are given, which are used to find k_q , k_T .

$u = \frac{w}{2}\; k_q$, $v = \frac{w}{2} \mid k_T \mid$

$Q = \frac{W}{2}(|k_T|^2 + k_q^2) = k_0^2\, \varepsilon_q - k_0^2\, \varepsilon_T$

$$Q = \frac{w}{2} k_0^2 \sqrt{\varepsilon_q - \varepsilon_T} \quad (19)$$

Now, it is possible to find the propagation constant mode in terms of u and v,

$$\beta^2 = \left(\frac{2}{W}\right)^2 (v^2 + u^2) \quad (20)$$

$$\begin{cases} u\tan(u) = v \\ Q^2 = u^2 + v^2 \end{cases} \quad \text{For symmetric } TE\text{-Mode} \quad (21)$$

In case of TM mode, the formula of the propagation wave for both fields will be as follows:

$H(y,z) = u_x H_x(y,z)$

$E(y,z) = u_y E_y(y,z) + u_z E_z(y,z)$

The same principle is applied to obtain the TM symmetric modes which results in,

$$\begin{cases} u\tan(u) = \frac{n_2^2}{n_3^2} v \\ Q^2 = u^2 + v^2 \end{cases} \quad \text{For symmetric TM –Mod} \quad (22)$$

These are the main equations of obtaining the propagation constant mode (β) and the effective refractive index which play an important role in the design of optical devices.

2.2 Numerical Solution for (TE-Mode) Symmetric Slab Waveguide

The numerical method of solving slab waveguide in this paper is based on the mathematical calculation that is presented in detail in the previous section and then simulated by using Matlab. The test is made to examine the propagation modes for slab symmetric structure in terms of searching for a desired value of the propagation constant mode (β) . Afterward, it is possible to find the other parameters to obtain the bound mode in the structure shown in figure (5) [2].

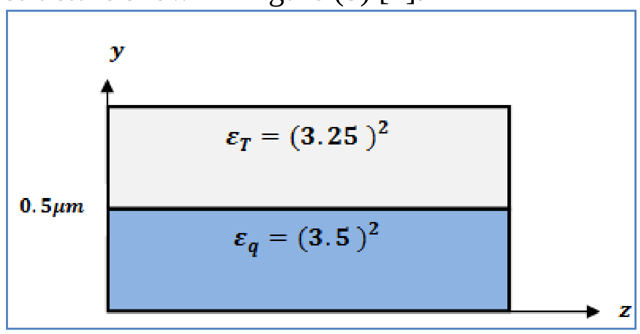


Figure 5 Symmetric slab waveguide ($\varepsilon_q > \varepsilon_T$)

As it is demonstrated in figure (5), a simple slab waveguide (symmetric) is introduced with parameters as follows:

- The permittivity of the core region *is* $\varepsilon_q = (3.5)^2$, this therefore gives refractive index is $n_q = 3.5$, where $n_q = \sqrt{\varepsilon_q}$.
- The permittivity of the cladding (top) region is $\varepsilon_T = (3.25)^2$,$n_T = 3.5$
- The free space wavelength is $\lambda = 1\mu m$.
- The thickness is $d = \frac{w}{2} = 0.5\mu m$.

The numerical analysis begins with equation (17). Mathematically, this equation has no solution; therefore it is only solved by the following way:

$u\tan(u) = v$. Substituting v in$Q^2 = u^2 + v^2$

$Q^2 = u^2 + (u\tan(u))^2$, Q $= \frac{w}{2} k_0 \sqrt{\varepsilon_q - \varepsilon_T}$.

The final formula of this equation is

$$f(u) = u + \mathrm{Q}\cos(\mathrm{u}) \quad (23)$$

Since the equation (23) has no a mathematical solution, consequently; the only solution is to search for the value of (u) at $f(u) \approx 0$. This can be achieved by giving a range to u and search for the value of $f(u)$. When the $f(u)$ value equals to or close to zero, in this case the corresponding value of (u) is selected to be used in this simulation to find the other parameters. The following figure (6) shows the curve changes from negative to positive sign when $f(u) \approx 0.0119$ at value of $u = 1.26$

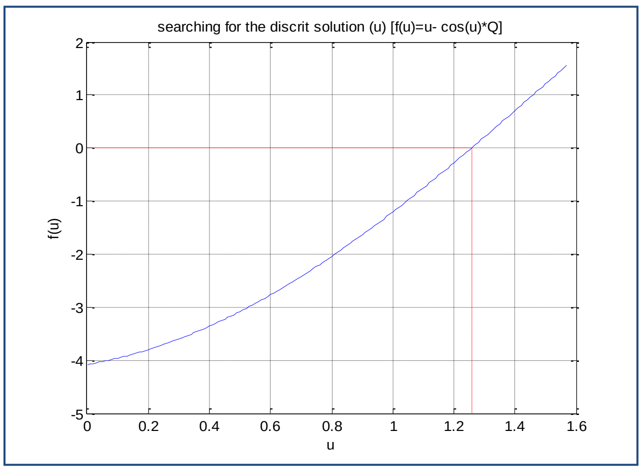


Figure 6 The value of the discrete solution $(f(u) = u - cos(u)Q)$.

According to the value of (u), all other parameters in this structure can be obtained. The main condition to solve a dielectric waveguide structure is to find the propagation constant mode (β) as introduced in equation (18).

This is the propagation constant mode for the core layer, and also the cladding layer. It must be equal for both layers to ensure the field is matched at the interface. To obtain the propagation constant mode, the normalized propagation constant parameters such as $k_{0,}\ k_q and\ k_T$ must firstly be calculated.

Since,

$$k_0 = \frac{2\pi}{\lambda}\ ,\ \ v = \sqrt{Q^2 - u^2}\ \ ,\ k_q = \frac{u}{d}\ ,\ k_T = \frac{v}{d}\ \ and\ \ n_{eff} = \frac{\beta}{k_0}.$$

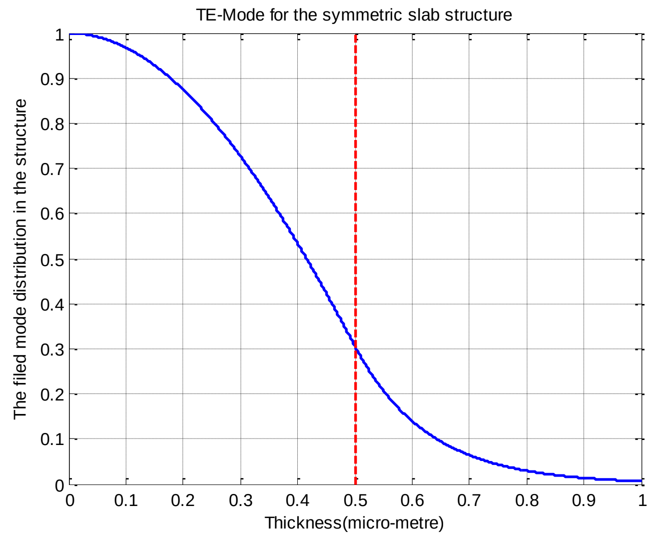


Figure 7 Field distributions in TE-Mode with core thickness (0.5μm).

The second condition is to calculate the effective refractive index. The validation of this test is to have the value of n_{eff} within the interval between the two of core and cladding indexes. In this test, the value of n_{eff} must satisfy the following relation.

$$n_T < n_{eff} < n_q$$

Figure (7) shows the field distribution for symmetric structures. The y-axis shows the field mode distribution, while x-axis represents the thickness of the layer. For this simulation the thickness d is given 0.5 μm at where the red dashed line demonstrated. The dashed red line represents the interface between the core and top layer.

The above figure assures the condition of optical structures, since there is an exponential decay toward the interface and most field amount bounded in the core layer.

In case of the thickness $d = 0.5\mu m$, the propagation constant mode equals$(21.8463\mu m^{-1})$. This gives effective refractive index of(3.4769). It is a sufficient result to achieve the bound mode in the core layer, since n_{eff} lays between the refractive indexes of core and outmost layer and it is closer to refractive index of the core than that of the cladding one.

To test the strength of the bound mode, the structure is modified to have various thicknesses of the core region. As it is represented in the table (1),

Table 1 Results of TE-Mode for a symmetric dielectric slab waveguide ($\varepsilon_q = (3.5)^2, \varepsilon_T = (3.25)^2$)

TE-Mode						
$d(\mu m)$	u	Q	k_q	k_T	$\beta(\mu m^{-1})$	n_{eff}
0.5	1.26	4.0810	2.5200	7.7633	21.84	3.47
0.4	1.2	3.2648	3	7.5908	21.78	3.46
0.2	0.95	1.6324	4.7500	6.6376	21.47	3.41

The modification is made on figure (8) to have thicknesses $0.4\mu m$, $0.2\mu m$ respectively. These result in having propagation constant modes $(21.7856\ \mu m^{-1})$ and $(21.4720\mu m^{-1})$ with effective refractive indexes $(3.4673)\ and\ (3.4174)$ respectively. From these results, it is pointed out that the increase in the thickness enhances the mode to be stronger and more bounded. Therefore, when the thickness is $0.5\mu m$, the effective refractive index (n_{eff}) has a value which is the closest to the refractive index of the core region (n_q).

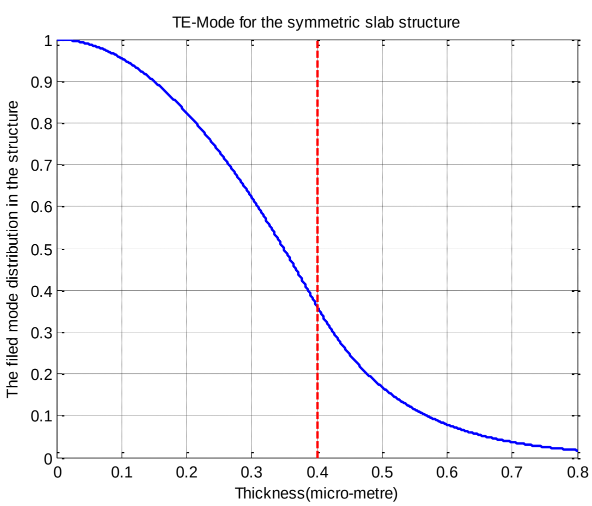


Figure 8 Field distributions in TE-Mode with core thickness ($0.4\mu m$).

Figure (7) and figure (8) show a decaying in the field from the cladding layer towards the interfaces that are represented in red dashes.
In Figure (7) the field has more confinement than that in figure (8) because of the thickness. Not only the thickness can enhance the field, but also the refractive index and the material of the layers can play a significant role for modifying the optical structure. All of the results above are presented in terms of having symmetric TE-Mode.
It is therefore possible to simulate the whole structure shown in figure (3) to prove that the transverse electric field profile is symmetric as shown in Figure (9). It shows the field mode distribution of the whole structure evaluates the validity of the symmetry case that is assumed and derived in figures above. The both dashed red lines show the thickness width of core layer between (-0.5) and (0.5). This figure ensures the valid condition of optical structures, where the field experiences decay for both the substrate and cladding (top) layers, while more field bounded in the core (desired) region. This can be obtained if and only if the following assumptions applied:

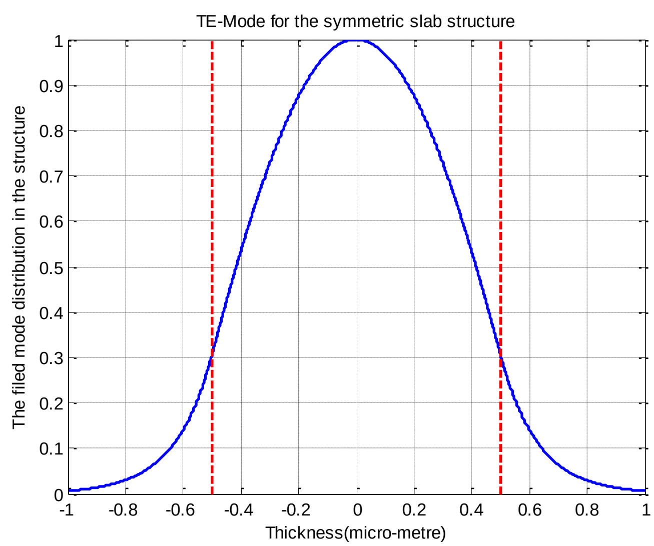


Figure 9 Field distribution of whole lab structure

1. Necessity of surrounding the high index (core) layer, where majority of the

radiation energy is confined, by low refractive index (cladding) media.

2. Propagation mode β must be resulted from integrating total internal reflection and constructive interference. This can be performed by invoking the tangential components continuity of the electric and magnetic fields at the interfaces.

2.3 Numerical solution for (TM-Mode) symmetric slab waveguide

It is essential to clarify that there is no magnetic field propagates along the direction of the wave travel (z- direction) in case of TM-Mode. The same slab waveguide structure in figure (5) is used to solve TM-Mode [2]. In this case the simulation steps are the same; however, equation (21) is used instead.

$$f(u) = u\sqrt{1 + \frac{\varepsilon_T}{\varepsilon_q}\tan(u)} - Q$$

The results are also obtained according to the variation the thickness of the core such as at 0.5 μm and 0.4 μm as shown in both figures (10) & (11).

The equation above cannot mathematically be solved, and then the possible method is to change the value of(u). This value is selected when$f(u) \approx 0$

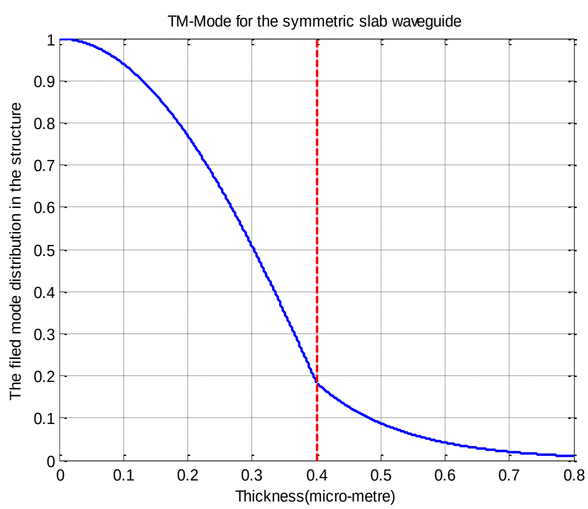


Figure 10 Field distributions in TM-Mode with core thickness (0.4µm).

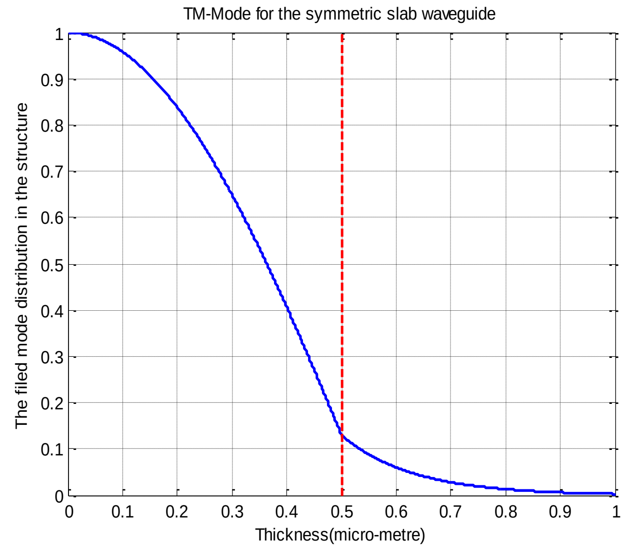


Figure 11 Field distributions in TM-Mode with core thickness (0.4µm).

Table 2 Results of TM-Mode for a symmetric dielectric slab waveguide ($\varepsilon_q = (3.5)^2, \varepsilon_T = (3.25)^2$)

TM-Mode						
$d(\mu m)$	u	Q	k_q	k_T	$\beta\ (\mu m^{-1})$	n_{eff}
0.5	1.44	4.08	2.89	7.631	21.79	3.46
0.4	1.38	3.26	3.46	7.391	21.71	3.45

In case of TM- Mode, table (2) shows the results based on two thicknesses 0.5 μm and 0.4 μm. Both of them meet the condition of having field confinement and an effective refractive indexes lay between the core and cladding indexes. The results achieved from TE-Mode and TM-Mode demonstrates that the thickness has a vital impact on the strength of the field, since there is a forward proportional between the thickness and field strength. This is represented in the figures (10) and (11).

3 OPTICAL STRUCTURE WITH ACTIVE CORE

The best candidate material is recently used in the application of the optical circuits is Gallium Arsenide GaAs. This compound consists of a

combination of Gallium and Arsenide [5]. It has played a major role since was being used in the diodes and then in the integrated electronic circuits [5]. The main reason of using this compound is because it works efficiently in the ultra- high frequencies. Furthermore, it produces less noise which makes it beneficial in case of having weak signal amplification. According to its physical properties, it is placed instead of using silicon to manufacture advanced digital integrated circuits [6].
The major goal is to design a single mode index guided of Gallium Arsenide *(GaAs)* to work at room temperature$(300 kelven)$. This is an active core which is encapsulated by cladding called Aluminium Gallium Arsenide *(Al_x Ga_{1-x} As)* [7]. The variable (x) is defined as a number that varies between 0 and 1 referring to the mixture between *GaAs* and *Al As*. It can be seen that Al_x Ga_{1-x} *As* is utilized as material depends on the heterostructure devices. This provides more confinement of the electrons towards *GaAs* area [7]. Assuming $x = 0.25$, the target is to calculate the propagation modes to hit the lasing threshold. Figure (12) shows the symmetric slab with an active core of semiconductor. This kind of structure has a tiny active core thickness.

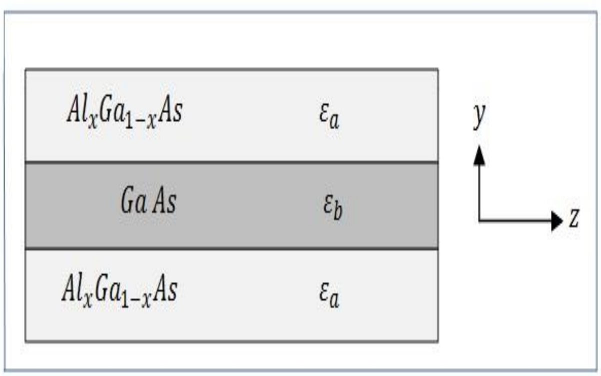


Figure 12 Active (Ga As) semiconductor structure [7].

The simulation is conducted in case of having both an active mode with a real permittivity as well as an active mode with complex permittivity

3.1 Simulating Semiconductor Laser Core with A Real Permittivity

To test this structure, there is a need to define the initial values which are as follows:

- The permittivity of the cladding $\varepsilon_a = (3.5)^2,\ n_a = 3.5$.
- In case of $\varepsilon_b = (\,n_a + \Delta n)^2$, as Δn^2 is very small variation ,then $\varepsilon_b = {n_a}^2 + 2\,n_a \Delta n$. $\varepsilon_a = (3.5)^2 + 2(3.5)(0.62x)$, where x=0.25.
- The wavelength is $\lambda = 0.8\mu m$ and the thickness is 0.1 μm.

The procedure of solving this device is the same as those in the ordinary waveguide. In this case, TE-Mode is tested using the equation (18) to find the propagation constant mode. The results are achieved as in the following table since the thickness is considered be$(0.1\ \mu m)$.

Table 3 Results of TE-Mode for a symmetric active core slab waveguide ($\varepsilon_a = (3.5)^2, \varepsilon_b =\ 13.3350$)

TE-Mode						
$d(\mu m)$	u	Q	k_b	k_a	β	n_{eff}
0.1	0.65	0.818	6.5	4.69	27.934	3.556

The implementation is extended to include the optical structures with an active core region instead. This makes the fabrication more efficient for introducing a device with more lasing and better field confinement. The useful selection is to use semiconductor laser with heterostructure devices. From figure (12), the $(GaAs)$ compound is used as active medium encapsulated by$(Al_x\ Ga_{1-x}\ As)$. The permittivity of the cladding is given. However, to derive the permittivity of the core, the value of the parameter x must be taken into account because it represents the percentage of mixing $GaAs$ and $Al\ As$, in this simulation $x = 0.25$. From table (3) the propagation constant β=27.9343 which is greater than those in the dielectric waveguide. That gives $n_{eff} =$

3.5567 which is a suitable result to provide stronger mode within the active region as shown in figure (13).

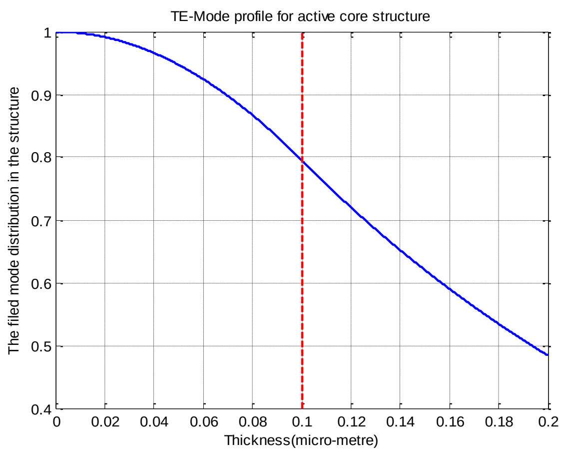


Figure 13 Field distributions in TE-Mode with active core thickness (0.1μm).

For more realistic paradigm, the modeling is improved to consider the material gain and modal gain to determine the confinement factor of the lasing action. To achieve that, it is more efficient to modify the structure with complex permittivity. The material gain is always given to determine the imaginary refractive index of the core. Once it is obtained, the complex permittivity can be calculated.

3.2 An Active Core Semiconductor Lasers with A Complex Permittivity

In this stage of the modification, it becomes obvious that there is a considerable possibility to achieve the optical gain. That is because the present structure is introduced with a very small perturbation in the core permittivity. Therefore, it is important to introduce the definition of the material gain and the modal gain since the relation between them results in the confinement factor.

Modal gain is defined as a fundamental aspect of modeling or shaping the optical amplifiers and semiconductor lasers. The ratio of modal to material gain can simply be achieved. This relation is known as a gain factor [8]. The following equation introduces the confinement factor that is defined as the ratio of the optical power in the active zone to the power of the whole structure.

$$\Gamma \text{ (confinement factor)} = \frac{\int_{-d}^{d} |s(z)| dz}{\int_{-\infty}^{\infty} |s(z)| dz} \quad (24)$$

Where, the numerator part of the equation represents the expected poynting vector in the particular region (active core region) while, the denominator defines the poynting vector of whole structure. Based on the derivation of the electromagnetic plane waves, the optical gain can be obtained from the poynting vector (s_z) since the refractive index is complex [9].

$$n = n_r + jn_i \ , \quad \text{and} \quad k_o = \frac{2\pi}{\lambda}$$

$$s_z = \frac{1}{2}|A|^2 e^{-2(k_o n_i)z} \ \frac{k_o n_r}{w \ \mu_0} \quad (25)$$

Using this equation to establish the modification for the material permittivity in terms of the optical gain or loss, since

$$s_0 = \frac{1}{2}|A|^2 \ \frac{k_o n_r}{w \ \mu_0} \ \text{and} \ g = \pm 2(k_o n_i) \quad (26)$$

After calculating the case of having an active core semiconductor laser with purely real permittivity, now the attention is being shifted to the active region with complex permittivity. This is more crucial step due to including the material gain to calculate the imaginary part of the permittivity. The following calculation demonstrates the procedure of obtaining the complex permittivity for the active core region [9].

$$\varepsilon_c = \ \varepsilon_b + j \ \varepsilon_i \quad (27)$$

Since ε_i is the small variation used for device modification, where $\varepsilon_i = \ {n_i}^2$.

The material gain is always given which is generated from the stimulated emission of moving the electrons from the lower to upper layer. In this case, $g = 200cm^{-1}$

$n_i = \frac{g}{2k_o}$, where,$\lambda = 0.8\mu m$, then $k_o = \frac{2\pi}{\lambda}$

After calculating the imaginary part of the refractive index, the complex permittivity becomes ($\varepsilon_c = 13.3350 + j0.0093$). It can be seen that the very small variation which provides the modification for all other parameters.

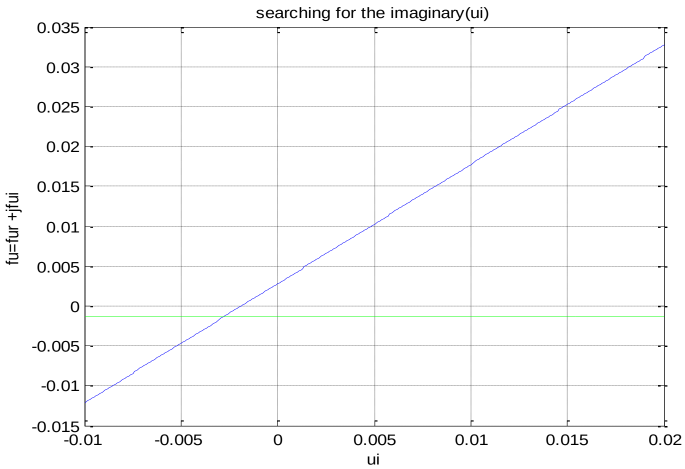


Figure 14 Searching for the discrete solution of u_i ($u_i = -0.0025$).

Since, $g = 200cm^{-1}$,$\lambda = 0.8\mu m$, then $k_o = \frac{2\pi}{\lambda} = 7.8540x10^{-4}cm^{-1}$
$n_i = \frac{g}{2k_o} = 0.0013$.
Then, $\varepsilon_c = (n_a + j\, n_i)^2 = 13.3350 + j0.0093$
This fabrication uses the same principles as in the dielectric and active structures starting with searching for discrete solution (u_i). In this simulation the imaginary part is provided in figure (13) since $u_i \approx -0.0025$. Then, the complex discrete solution is $u_c = u_r + ju_i = 0.65 - j0.0025$. The propagation constant mode is 27.9342-j0.0056. To calculate the confinement factor, the first step is to find the modal gain,
$\alpha = 2\beta_i$, where
$\beta_i = 0.0056$, $G = \alpha\, z = 151cm^{-1}$
Γ (confinement factor) = $\frac{151}{200} = 0.5750$
This is the confinement factor for achieving lasing action and the result is an appropriate percentage seems to be acceptable although for better reflection the confinement factor should be 0.3.
However, the metals that are used in optical structure are lossy because of their physical properties. Due to their properties, their permittivity becomes negative at the optical frequencies. This phenomenon leads to discovering a more suitable surface used at the interface to minimize the amount of loss and provide more lasing named surface plasmons. In this case the electromagnetic waves travels close to the interface.

4 DISCUSSIONS OF RESULTS

It is generally seen that solving waveguide structure is based on the field profile confinement in the core region in conjunction with a decaying for the field form outermost layers towards the surface. According to the results, there is a noticeable improvement in the bound mode based on the improvement of the optical waveguide modification. Starting with a simple case of dielectric slab waveguide, by applying the boundary conditions, all fields must be continuous at the interface. The solution of these structures is based on the transcendental equations (21) to obtain the dispersion relations $u = Q\cos(u)$ where Q contains the free space wavelength and the permittivity of the core and cladding.
As a result, calculating propagation constant mode (β) is a fundamental step to modify the device. If (β) is obtained, the effective refractive index can be simply achieved based on the relation between them. The results show that the n_{eff} lays between both indexes of the core and the cladding. According to table (1) the values of n_{eff} as a function of the thickness variation are 3.4769, 3.4673 and 3.4174, which emphasis that the n_{eff} exists at any value between 3.25 and 3.5 for the core and cladding. That means it becomes possible to obtain the bound mode as represented in the figures (7) and (8) which are related to TE-Mode of the symmetric dielectric slab waveguide.
The same procedures and conditions are applied for TM-Mode symmetric dielectric slab waveguide. The variation of the core's thickness impacts on the strength of the bound mode. As a result, due to the increase in the

thickness, the mode becomes stronger. In this paper the test is made by assuming the thicknesses $0.5\mu m$, $0.4\mu m$ and $0.2\mu m$. When the thickness is $0.5\mu m$ the effective refractive index n_{eff} has a result that closer to the refractive index of the core compared to the other thicknesses as shown in table (1).

The results that are interpreted above have led to the desire for knowing the quantity of the gain in the structure. For this reason, the semiconductor laser is introduced to the structure. In this case, the structure consists of a thin active core region which is a compound of gallium arsenide coating by($Al_x\, Ga_{1-x}\, As$). The value of propagation constant increases to result in approximately $27\mu m^{-1}$ compared to above $21\mu m^{-1}$in dielectric slab waveguide. This in turn enhances the confinement of the bound mode. However, to achieve the modal gain, it was useful to fabricate the structure to a complex permittivity instead. Therefore, the modal gain is obtained from the imaginary part of the propagation constant mode. The material gain is given and modal gain can be achieved based on the amount of the power in the core, therefore, the confinement factor can be obtained. In this test the result of the confinement factor is 0.5750.However, based on the other studies, the confinement factor should be around 0.3 to achieve better lasing.

5 CONCLUSIONS

To sum up, according to the appropriate results that have been obtained, there is a gradual improvement for fabricating the optical integrated circuits. The modification is conducted to begin with the implementation of the dielectric slab waveguide and then considering the active medium. These structures are the platform for designing and developing the optical circuits. The result confirmed the validation of optics field profiles. All figures shows that the most field is obligated to travel through the core layer rather than the outmost layers. This can only be applicable if the refractive index for the core layer greater than the surrounded one. However, it was strongly recommended to consider the demand for miniaturizing the optical devices which is based on the power consumption and the type of the metal used. The purpose behind that is some materials are not suitable at optical frequencies due their physical properties. The direction towards smaller optical devices with less power consuming indicates that the tangential and ray (geometrical) optics are not applicable. That is because the metal at the optical frequencies behaves as plasma based on the free electron and holes of the materials. This is similar to the ionosphere layer at the radio frequencies. For this reason, the heterostructure compound is the most appropriate structure to be used to validate the achievement of laser and confining the field within the desirable area.

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