

## **Robust Control of a Permanent Magnet Synchronous Machine (PMSM) Using Internal Model Control**

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### **ABSTRACT**

This paper deals with the synthesis of an Internal Model Control (IMC) applied to the current and speed of a drive permanent magnet synchronous machine. The designed IMC controller enhances significantly the closed loop performance of the speed and greatly simplifies the conception process. Limits of control parameters that ensure the training stability are derived. The experimental results were carried out on a permanent magnet synchronous machine in order to show the effectiveness of IMC controller design.

### **KEYWORDS**

Internal Model Control, PMSM, Feedback controller, robust control, Uncertainties.

### **1 INTRODUCTION**

The permanent magnet synchronous machine is progressively becomes the most ideal candidate for high industrial performance training since it has a simple structure, high power density and energy. In addition its maintenance is not expensive and minimal. This is due to brushless rotor construction. Nevertheless the precision of the parameters in the engine model directly impacts the drive characteristics owing to nonlinearities and uncertainties when using conventional control methods. We think that the internal model control, which has been proposed by Garcia and Morari in 1982 in an attempt to design an optimal feedback controller for process control systems, can close

this gap by providing as good a performance as can be obtained by an optimal control. It is one of the model predictive control methods based on the predictive output of the plant model. Its major advantage is that the internal-model control (IMC) is a powerful technique to control the nonlinear and uncertain systems [1], [2],[3], [4],[5], which do not overly depend on the accuracy of motor model. Moreover it is a strongly robust control method, and can be applied in the presence of model uncertainties parameter fluctuations and external disturbances and can efficiently compensate the nonlinearity and uncertainties of the plant [6].

Furthermore the closed-loop stability is achieved simply by selecting a stable IMC controller. Likewise, closed-loop performances characteristics are linked directly to controller parameters, which make one-line tuning of the IMC controller very practical. Several papers have been introduced on the theoretical view of IMC where the relation between robust control and Yula parameterization for stable model has been studied.

The paper is organized as follows: in section 2 the modeling of the permanent magnet synchronous machine is briefly presented. Section 3 deals with the calculation of the stator currents controllers. In sections 4 the principle of the internal model control was presented and used for the design of the regulator of the speed loop, in section 5, experimental results validating the designed drive are presented

## 2 PERMANENT MAGNET SYNCHRONOUS MACHINE MODEL

The mathematical stator d, q equations of the PMSM in the rotor frame are:

$$\begin{aligned} L_d \frac{dI_d}{dt} &= -R_s I_d + L_q \omega_r I_q + v_d \\ L_q \frac{dI_q}{dt} &= -R_s I_q - L_d \omega_r I_d - \Phi \omega_r + v_q \\ J \frac{d\omega_r}{dt} &= p(c_e - c_r) - f_c \omega_r \\ J \frac{d\omega_r}{dt} &= p^2 [(L_d - L_q)I_d + \Phi]I_q - f_c \omega_r - pc_r \end{aligned} \quad (1)$$

Thus the electromagnetic torque is given by:

$$c_e = p[(L_d - L_q)I_d + \Phi]I_q \quad (2)$$

$$J \frac{d\omega_r}{dt} = p^2 [(L_d - L_q)I_d + \Phi]I_q - f_c \omega_r - pc_r \quad (3)$$

where:

$v_d, v_q$ : stator winding d, q axis voltage respectively

$i_d, i_q$ : stator winding d, q axis current respectively

$\omega_r$ : rotor speed

$\Phi$ : constant magnet flux linkage produced by permanent magnet rotor

$c_e$ : electromagnetic torque

$c_r$ : external load torque

$p$ : number of pole pairs

$R_s$ : stator resistance

$L_d, L_q$ : stator winding d, q -axis inductance respectively

$J$ : moment of inertia

$f_c$ : damping coefficient

## 3 CURRENT and SPEED CONTROLLERS STRUCTURE

The analysis of the system of equations (1), allows us to notice that the model is strongly coupled and thus nonlinear. Indeed, the d-q axis currents depend at the same time on  $I_{q-d}$  and  $\omega_r$  respectively. Moreover, the electromagnetic

torque is a function of  $I_d$  and  $I_q$ . The compensation of these nonlinearities is done via the cancellation of the direct current component ( $I_d=0$ ). This is due to the fact that for a smooth pole machine, the best choice of its operation is achieved for a value where the internal angle is equal to  $\frac{\pi}{2}$ . Physically, this strategy reduces the stator current to the only component  $I_q$  [7]. The control structure applied to the PMSM is given by the diagram below where the d-q axis currents are controlled by PI controllers

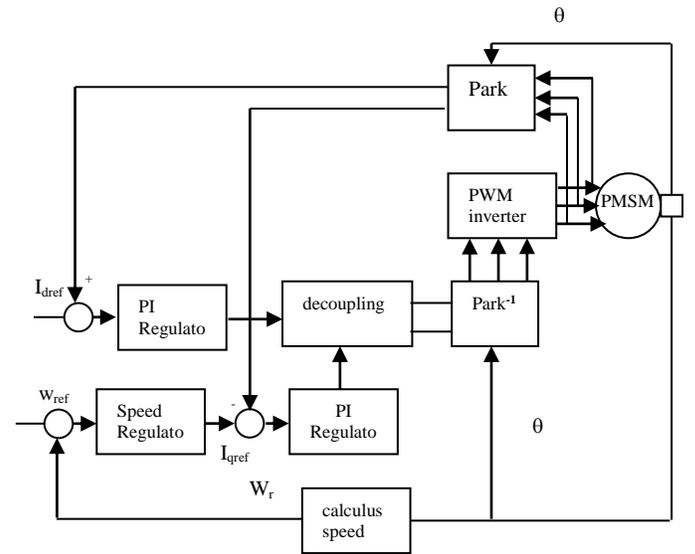


Figure 1: block scheme of control structure

The transfer functions of the stator currents PI controllers are given by [8]:

$$C(s) = K_p + \frac{K_i}{s} \quad (4)$$

$K_p$  and  $K_i$  are deduced via the compensation of the dynamic real pole of the system with the zero introduced by the controller  $C(s)$ . Thus, the controller parameters are given by:

$$K_p = K_i \cdot \frac{L_d}{R_s} \quad (5)$$

$$K_i = \alpha \frac{R_s^2}{L_d} \quad (6)$$

where  $\alpha$  characterizes the rapidity of the current loop and represents to the ratio between the open and the closed loop dynamic.

In this paper, we concentrate our study on the development of speed controller.

#### 4 IMC STRUCTURE of SPEED CLOSED – LOOP

##### A) IMC Principle

The Internal Model Control (IMC) is based on some representation of the process to be controlled. Indeed, if the control scheme to be designed relies on an exact model of the process, then the above control is achievable. Contrariwise, if the process model is not invertible and the system is affected by unknown disturbances, the IMC has the potential to achieve perfect control [9]. Its basic diagram structure is illustrated in figure (2)

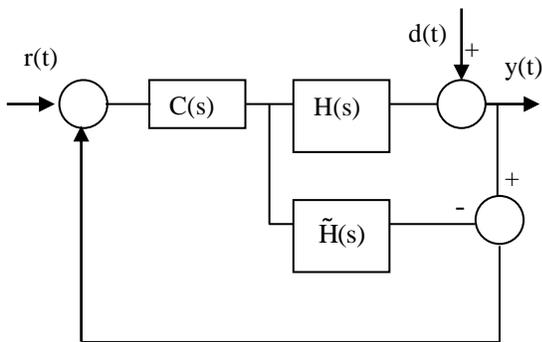


Figure 2: IMC structure of speed loop

$H(s)$  denotes the process,  $\tilde{H}(s)$  denotes process internal model transfer function and  $C(s)$  denotes internal model controller. Note that the IMC block diagram in figure (2) can be transformed to a standard closed loop structure as follow:

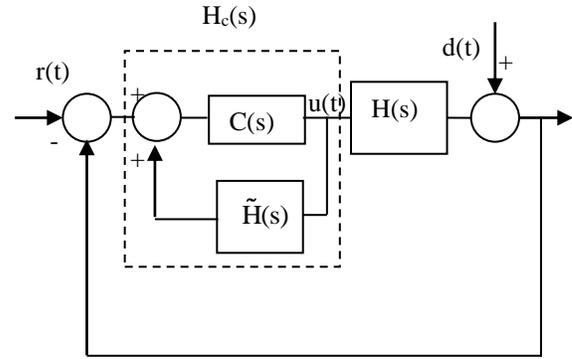


Figure 3: Standard control structure of speed loop

Where

$$H_c(s) = \frac{C(s)}{1 - \tilde{H}(s)C(s)} \quad (7)$$

The basic procedure for IMC is the following [9]:

- 1) Adopt a perfect internal model that is to say  $H(s) = \tilde{H}(s)$ .
- 2) Adopt that the controller and its steady state gain are equal to the process model inverse  $C(s) = \tilde{H}(s)^{-1}$  and its inverse gain  $C(0) = \tilde{H}(0)^{-1}$  in order to obtain a zero steady state error.

If the process is a minimum phase one must assume the following procedures

- a- we set  $H(s) = \tilde{H}_-(s)\tilde{H}_+(s)$  where  $\tilde{H}_-(s)$  and  $\tilde{H}_+(s)$  denote the non minimum and minimum phase part respectively of  $H(s)$ .
- b- The controller  $C(s) = \tilde{H}_-(s)L(s)$  where  $L(s)$  denotes a low-pass filter of the

$$\text{form: } L(s) = \frac{1}{(\beta s + 1)^n}$$

Notice that  $L(s)$  is chosen in order to obtain a proper controller  $C(s)$ .  $\beta$  is the time filter constant.

Let us now consider the performance of IMC from the input  $u$  and the disturbance  $d$  to the output  $y$ . The transfer function of IMC closed-

loop is:

$$y = \frac{HC}{1+C(H-\tilde{H})}r + \frac{1-\tilde{H}C}{1+C(H-\tilde{H})}d \quad (8)$$

Assuming that  $H(s) = \tilde{H}(s)$  is stable and proper, therefore the closed-loop stability is ensured by the stability of controller  $C(s)$ .

If the process is a non-minimum phase, the output is:

$$y = LH_+r + (1-LH_+)d \quad (9)$$

One can notice that the output response is determined by the IMC filter.

## B) Process Model

As we mentioned in section 2,  $I_d = 0$ , if we work in vacuum, then the response time of the regulators of current is neglected, and using (2) and (3) we deduce that the dynamic model is reduced to:

$$\tilde{H}(s) = \frac{p\Phi}{Js + f_c} \quad (10)$$

As soon as the process model is a first order system, we choose the IMC filter  $L(s) = \frac{1}{\beta s + 1}$ .

Then the IMC controller is of the following form:

$$C(s) = \tilde{H}^{-1}(s)L(s) = \frac{Js + f_c}{p\Phi} \frac{1}{\beta s + 1} \quad (11)$$

Referring to (10), the equivalent standard controller becomes:

$$H_c(s) = \frac{C}{1-\tilde{H}C} = \frac{Js + f_c}{p\Phi\alpha s} \quad (12)$$

A comparison with the classic PI controller that implies adjustment of the two parameters  $K_p$  and  $K_i$ , the setting problem of the designed controller  $H_c(s)$  is reduced to the regulation of parameter  $\beta$  which characterize the closed-loop bandwidth. This, not only simplifies the design procedure of the controller, but also extends the performance of speed-loop [10].

The transfer function of IMC closed-loop is:

$$y = \frac{H(s)C(s)}{1+C(s)(H(s)-\tilde{H}(s))}r + \frac{1-\tilde{H}(s)C(s)}{1+C(s)(H(s)-\tilde{H}(s))}d \quad (13)$$

By replacing  $C(s)$  by its expression given by (11), we obtain a first order closed-loop IMC:

$$y = L(s)r + (1-L(s))d \\ = \frac{1}{\beta s + 1}r + \frac{\beta s}{\beta s + 1}d \quad (14)$$

In our study  $r$  denotes the speed reference and  $d$  denotes the load torque which is considered as the main external disturbance.

## 5 EXPERIMENTAL RESULTS

The experimental results of the IMC control structure was conducted on a test bench composed of two identical synchronous permanent magnet machines. Each one is equipped with an incremental encoder; one was used as a motor and the other as a load whose data are given in table 1.

TABLE I. PARAMETER OF THE PMSM.

Motor rated power	1KW
Rated current	6.5A
Pole pair number (p)	2
d-axis inductance $L_d$	4.5mH
q-axis inductance $L_q$	4mH
Stator resistance	0.56 $\Omega$
Motor inertia $J$	2.08.10 <sup>-3</sup> Kg.m <sup>2</sup>
Friction coefficient $f_c$	3.9.10 <sup>-3</sup> Nm.s.rad <sup>-1</sup>
Magnet flux constant $\Phi$	0.064wb

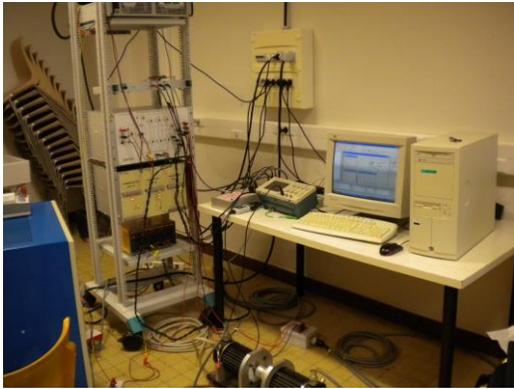
The proposed control algorithm was performed using *MATLAB/SIMULINK* software, and compiled on *DSPACE 1104* card. In the *SIMULINK* solver, the *Euler's* method with a sampling period of *0.1 ms* is used for the control system discretization. The chosen sampling period for the current loop is *10e<sup>-4</sup> s* and the one for the speed loop is *3.10e<sup>-4</sup> s*. The

parameters of the *PI* regulator used in d-q current loops are the same for  $I_d$  and  $I_q$ :

$$K_{pdq} = 5.7; K_{idq} = 722, \beta = 0.03$$

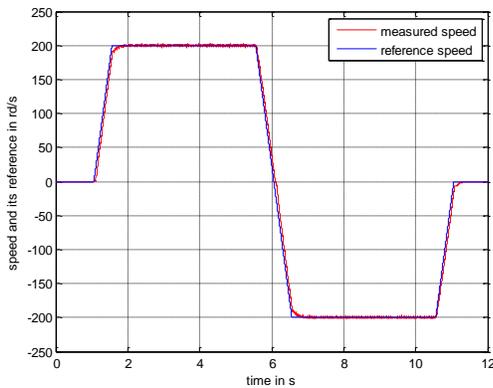
$\beta$  is chosen small enough to improve the load disturbance rejection performance.

The test bench is given by figure (4)



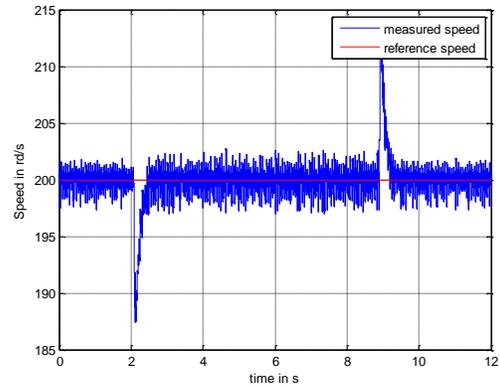
**Figure 4:** Photo of the test benchmark

The speed response given by the IMC method is shown in figure (5). We can notice that this response has an overshoot null with a 0.24 s setting time



**Figure 5:** Speed response and its reference

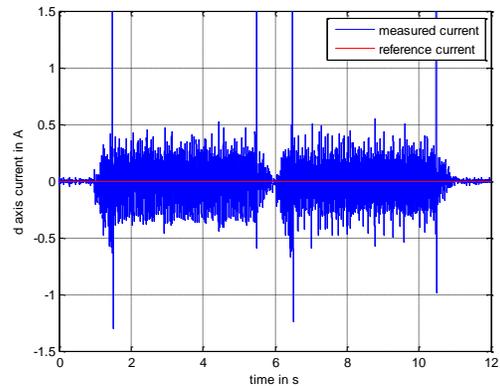
To improve the load torque disturbance reject performance a load torque  $c_r=2\text{Nm}$  is applied at  $t=2.2\text{s}$  and  $t=9\text{ s}$  as shown in figure (6)



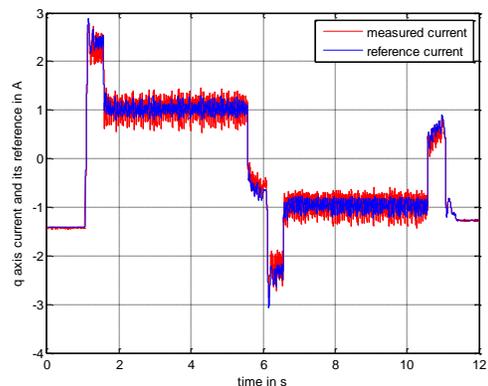
**Figure 6:** Load torque applied on speed response

Figure (7) shows the d-axis current; we notice that this current keeps its reference which is set at zero. Figures (8) show the efficiency of the current corrector.

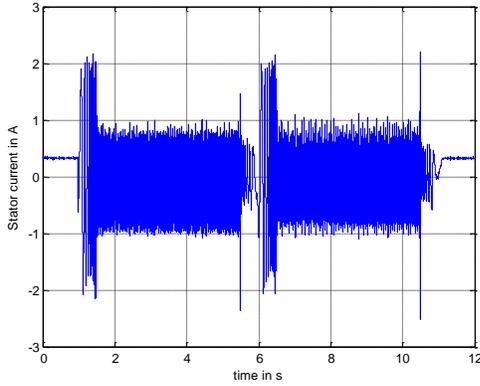
The measured current  $I_q$  tracks well the reference current obtained by the speed regulator. Figure (9) gives the shape of stator phase current



**Figure 7:** d-axis current evolution and its reference

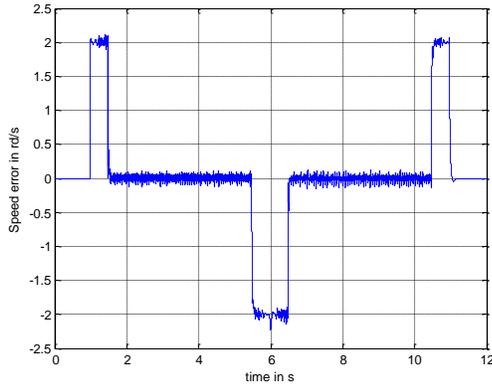


**Figure 8:** q-axis current evolution and its reference

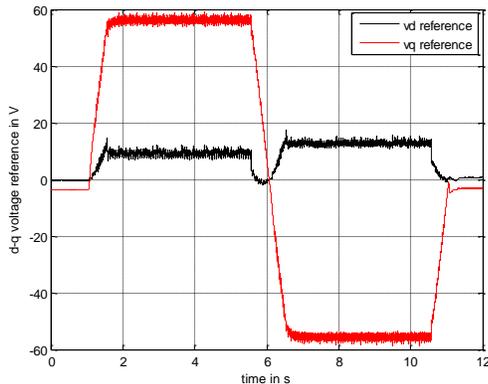


**Figure.9:** Stator phase current

Figure (10) gives a speed error between measured and reference one. In figures (11) we present the reference of d-q-axis voltage.



**Figure.10:** Speed error between the reference and measured speed



**Figure 11:** d-q-axis voltage references

In these figures, it is noted that the dynamics speed is not affected by external disturbance,

Indeed, despite of the noise which is present in the stator currents, the effectiveness of the control method based on the internal model and its good robustness opposite the variations of the internal parameters or the errors of modeling influencing the operation of the process

## 6 CONCLUSIONS

In this paper, we developed, in a first time, a classic algorithm which employed the well known PI controllers for d-q axis currents. Their design is based on the principle of pole compensation. In a second time, and in order to ameliorate the dynamic performance of the PMSM speed, we implemented a controller based on the internal model control. This controller has proved its ability and load disturbance rejection. Experimental results are satisfactory.

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