Application of the $4 \times 4$ Butler Matrix Consisting of Tapered-Coupled-Line Directional Couplers in an Ultra-Broadband Multiport Reflectometer

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ABSTRACT

A novel application of a $4 \times 4$ Butler matrix consisting of $0/180^\circ$ tapered-coupled-line directional couplers in an ultra-broadband multiport reflectometer is presented. The performed analysis reveals, that such a Butler matrix provide an advantageous inner power distribution, which allows to obtain a good accuracy of complex reflection coefficient measurement. Due to the higher number of available ports in comparison to the classic solutions, the proposed system allows for a great variety of configurations, leading to increase of the measurement accuracy. The proposed multiport reflectometer has been verified experimentally in a frequency range $1 – 8$ GHz by measurements of several reflective elements. The results show a good agreement with the ones obtained with the use of a commercial vector network analyzer.

KEYWORDS

multiport reflectometer, multiport measurement technique, reflection coefficient, Butler matrix, tapered-coupled-line couplers.

1 INTRODUCTION

The multiport measurement technique is considered as an attractive alternative for vector network analyzers, due to its simplicity and incomparably lower costs. Such a measuring system consists of a linear passive multiport network, which ensures a proper power division, microwave signal source, and several power meters [1,2]. Due to the suitable calibration procedure, all imperfections of the utilized elements can be taken into account, therefore, their impact on the measurement accuracy can be significantly decreased or even eliminated [3,4]. Moreover, the requirements related to the precision of utilized elements are relaxed. All these features caused, that the multiport-based measurement systems have found a wide range of application, including biological research [5], microwave receivers [6], or microwave diversity imaging [7]. The rapid development of the multiport measurement technique has resulted in a variety of multiports, starting from the simplest four-ports [8], to even ten-ports [9] ensuring a higher measurement accuracy. However, still the six-ports are considered to be the most popular. Recently, it has been shown in [10], that a classic $4 \times 4$ Butler matrix consisting of four quadrature directional couplers can also serve as a multiport in such measuring systems.

Figure 1. Schematic diagram of the proposed multiport reflectometer with a single $4 \times 4$ Butler matrix utilizing tapered-coupled-line directional couplers [11].
In this paper a new application of a $4 \times 4$ Butler matrix consisting of $0/180^\circ$ tapered-coupled-line directional couplers in multiport reflectometry is presented. The performed theoretical analysis has shown, that such a Butler matrix serving as a multiport network provides an advantageous power distribution allowing for precise measurement of complex reflection coefficient. Moreover, the proposed solution features a large number of possible system configurations. Each of them can provide different inner power division, and therefore, can be considered as the utilization of other multiport, which allows to increase the measurement accuracy. The proposed multiport reflectometer has been verified experimentally by measurements of a set of one-port reflective elements over a wide frequency range $1 – 8$ GHz. The results show good agreement with the ones obtained with the use of a commercial vector network analyzer, proving the usefulness of the proposed measurement system.

### 2 THEORETICAL ANALYSIS

The multiport reflectometers allow to measure a complex reflection coefficient by simple power measurements. However, to ensure a proper performance, the power incident to each power meter has to be a sum of two signals. The first one is related to the measured reflection coefficient, whereas the second one is a reference signal. In the proposed system it has been achieved by connecting one-port that is to be measured and a reflective one-port element to two of the output ports of the utilized $4 \times 4$ Butler matrix, as shown in Fig. 1. If the matrix is fed by one of its input port, the power is distributed to the output ports and it is reflected towards input ports, therefore to the remaining input ports the power meters have to be connected. Assuming an ideal $4 \times 4$ Butler matrix one can derive the relation between the measured power and the reflection coefficient of the one-port connected to the measuring port as follows:

$$
P_{\text{imgr}} = \frac{P_{\text{imgr}}}{P_{\text{REF}}} = \left| s_x \Gamma s_{mg} + s_r \Gamma C s_{mr} \right|^2.
$$

(1)

where $p$ – normalized power, $P$ – measured power, $P_{\text{REF}}$ – reference power measured at port #8, $i$ – number of port to which power is supplied (#1 - #4), $m$ – number of port at which the power is measured (ports #1 - #4), $g$ – number of measuring port (port #5 - #8) and $r$ – number of port with connected reflective element having reflection coefficient $\Gamma_C$ (port #5 - #8) and $s_{xx}$ are the scattering parameters of the utilized $4 \times 4$ Butler matrix consisting of four $0/180^\circ$ tapered-line directional couplers given by:

$$
S = \frac{1}{2} \begin{bmatrix}
0 & 0 & 0 & 0 & e^{-j\pi/2} & e^{j\pi/2} & e^{j\pi} & e^{-j\pi} \\
0 & 0 & 0 & 0 & e^{j\pi/2} & e^{j3\pi/2} & e^{-j\pi} & e^{j\pi} \\
0 & 0 & 0 & 0 & e^{j\pi/2} & e^{j\pi} & e^{j\pi} & e^{-j\pi} \\
0 & 0 & 0 & 0 & e^{j\pi} & e^{j\pi} & e^{-j\pi} & e^{-j\pi} \\
0 & 0 & 0 & 0 & e^{j\pi/2} & e^{j\pi} & e^{j\pi} & e^{-j\pi} \\
0 & 0 & 0 & 0 & e^{-j\pi} & e^{-j\pi} & e^{-j\pi} & e^{-j\pi} \\
0 & 0 & 0 & 0 & e^{-j\pi} & e^{-j\pi} & e^{-j\pi} & e^{-j\pi} \\
0 & 0 & 0 & 0 & e^{-j\pi} & e^{-j\pi} & e^{-j\pi} & e^{-j\pi}
\end{bmatrix}
$$

(2)

Analyzing (1) one can notice, that the general formula for the measured power is the same as in case of the classic $4 \times 4$ Butler matrix consisting of quadrature directional couplers [10]. However, the $S$-matrix of the Butler matrix containing $0/180^\circ$ tapered-line directional couplers is different than in [10], therefore the power distribution in the proposed system has to be investigated. The analysis can be performed with the use of a well-known geometric description of the multiport measurement technique, in which, the measured complex value is an intersection of three circles on a complex plane. To verify the usefulness of a multiport in such measurements, its circle centers distribution has to be analyzed. Utilizing (1) and (2) one can derive the circle centers distribution obtainable in the presented measuring system:

$$
c_{\text{imgr}} = \frac{S_{rr}S_{mr}}{S_{gg}S_{mg}} \Gamma_C
$$

(3)

It can be observed, that the proposed reflectometers allows to obtain a high number of connection combinations. Assuming, that as a reflective element an ideal short-circuit is used, the possible circle center distributions are presented in Table 1 and Table 2.
Table 1. Distribution of circle centers for different choice of measuring port and port with short-circuit, when $4 \times 4$ Butler matrix is fed from port #1 or #2 and $\Gamma_c = -1$.

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<thead>
<tr>
<th>Number of port with short-circuit</th>
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Table 2. Distribution of circle centers for different choice of measuring port and port with short-circuit, when $4 \times 4$ Butler matrix is fed from port #3 or #4 and $\Gamma_c = -1$.

<table>
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<tr>
<th>Number of port with short-circuit</th>
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<td>Number of measuring port</td>
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It is seen, that the proposed measuring system features circle center distributions being similar to these obtained with the use of $4 \times 4$ Butler matrix consisting of $0/180^\circ$ tapered-line directional couplers [10]. The only difference is that entire distributions are rotated by $90^\circ$, which has no impact on the general measurement accuracy, since the mutual locations of particular circle centers are preserved. Furthermore, it must be emphasized that, similarly to the system presented in [10], some combinations shown in Table 1 and Table 2 contain only two different circle centers, hence they do not allow for unambiguous measurement and cannot be utilized.

3 EXPERIMENTAL RESULTS

The considerations presented in Section 2 are related to an ideal $4 \times 4$ Butler matrix consisting of $0/180^\circ$ tapered-line directional couplers. However in a practical realizations such a Butler matrix exhibits additional features, i.e. imperfect isolations, return losses and amplitude/phase imbalance. All these imperfections deteriorate the circle centers, affecting the measurement conditions. However, the well-known calibration procedures can take into account these parameters and, as long as circle centers feature reasonable distribution, the measurement can be accurate.

For the experimental verification the proposed measuring system has been built with the use of the $4 \times 4$ Butler matrix containing tapered-coupled-line 3-dB $0/180^\circ$, which has been previously shown in [11]. The picture of the inner laminate of utilized Butler matrix and its exemplary $S$-parameters of the are shown in Fig. 2. and Fig. 3 respectively.

Figure 2. Picture of the inner laminate layer on which the traces of the $4 \times 4$ Butler matrix utilized in measurements were etched [11].

As it is seen the utilized Butler matrix operates within the frequency range starting from 2 GHz. The amplitude imbalance is greater than $\pm 1$ dB and the isolations as well as return losses are about 10 dB. Such parameters have distinct impact on power distribution, therefore, significant deterioration of the circle center distribution should be expected.
Figure 3. Measured S-parameters of the \(4 \times 4\) Butler matrix containing tapered-coupled-line 3-dB \(0/180^\circ\) utilized in measurements: transmission coefficients (a), return losses (b) and isolations (c).

In the proposed measuring system utilizing the described \(4 \times 4\) Butler matrix four USB power meters PWR-8FS produced by Mini-Circuits have been used. Their measurement frequency range is from 0.001 to 8 GHz with accuracy being equal to \(\pm 0.15\) dB. The developed system is fed by 3\textsuperscript{rd} port of the Butler matrix, the reflected power is measured at ports \#1, \#2 and \#4, port \#5 is the measuring port, to port \#6 a short-circuit as a reflective element is connected, the reference power is measured at port \#8 and port \#7 is not used. Such reflectometer has been calibrated with the procedure described in [12] within the frequency range 1 – 8 GHz. The frequency range has been, therefore, extended in comparison to [11] and it now covers 3 octaves. The calibration results are presented in Fig. 4 and Fig. 5. It can be observed that the obtained circle center distribution significantly differs from the theoretical ones, which is caused by distinct imperfection of the utilized Butler matrix, as it has been mentioned in Section 2. Nevertheless, within the investigated frequency range the circle center distribution allows for a proper measurement.
Figure 4. Circle center distribution resulting from the performed calibration procedure within frequency range 1 – 8 GHz: circle centers on a complex plane (a), magnitude of circle centers (b) and phase of circle centers (c).

Figure 5. Magnitude of the reflection coefficient seen at the measuring port.

Figure 6. Measured magnitude of the reflection coefficient of shorted attenuators. Solid lines indicate the measurement results obtained with the use of the proposed system, dashed lines present the reference values obtained with Agilent N5224A VNA.

Figure 7. Measured phase of the reflection coefficient of shorted attenuators. Solid lines indicate the measurement results obtained with the use of the proposed system, dashed lines present the reference values obtained with Agilent N5224A VNA.

5 CONCLUSIONS

In this paper a novel, ultra-broadband multiport reflectometer has been presented, in which the role of power division network is performed by the 4 × 4 Butler matrix consisting of tapered-coupled-
The presented theoretical analysis has shown that such a multiport features advantageous power division in a wide frequency range. Moreover, the proposed measuring system features great flexibility, since there is a great number of possible system configurations. Each of them ensures the different power division, which has the same effect as the application of a different multiport. The presented multiport reflectometer has been experimentally verified over a wide frequency range from 1 to 8 GHz, which is wider than the primarily assumed bandwidth of the utilized Butler matrix. The obtained results show good agreement with reference values, proving the usefulness of the proposed reflectometer.

5 ACKNOWLEDGEMENT

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6 REFERENCES


