

A New Method to Channel Estimation in OFDM Systems Based on Wavelet Transform

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ABSTRACT

In OFDM systems, it is necessary to estimate the channel to overcome the distortion caused by channel fading's which can be induced by many phenomena such as: delay spread, multipath effect, mobility and Doppler shift. Most of the channel estimation techniques are proposed in frequency domain using the pilot symbols. One of them which has less complicated is well-known as Least-Squares (LS) method which is widely used in channel estimation but it is more sensitive to noise respected to the other reported techniques. In this paper, a new threshold based method using wavelet decomposition will be proposed which is based on an initial LS estimation technique. The reported simulation results show that the proposed method has better performance compared to the other methods such as Lee Method that has been published recently.

KEYWORDS

OFDM signal, Channel Estimation, Channel State (CS), Least Square (LS), Wavelet Transform, Cyclic Prefix, Channel Impulse Response

1 INTRODUCTION

OFDM is a signaling technique that has been applied widely in wireless communication systems due to its ability to maintain effective transmission and highly efficient bandwidth utilization in the presence of various channel impairments which one of them is frequency-selective fading. In OFDM systems the available spectrum are divided into many orthogonal sub-channels, which are

instantaneously used to data transmission. Also, in this technique the inter-symbol interference (ISI) which is induced due to frequency-selective channels can be reduced by adding the cyclic prefix (CP) [1].

In OFDM systems, channel estimation is necessary to obtain the channel state information (CSI), reducing the bit error rate and also to achieve a distortion less output data. There are various methods to channel estimation such as: with or without a need to parametric models, blind or pilot based methods, frequency and/or time domain analysis, adaptive or non adaptive techniques. Among these mentioned methods, channel estimation in OFDM systems is often done in frequency domain using pilot symbols or training data [2]. The least square and minimum mean-square error (MMSE) are conventional linear channel estimation techniques which are based on pilot arrangement. The LS method is less complicated and simple respect to other methods and consequently is used to channel estimation, but it has a serious drawback which is more sensitive to channel noise. MMSE estimator has better performance than LS method but suffers from a high computational complexity because it requires knowledge of the channel statistics and the signal-to-noise ratio (SNR) [3]. Some different methods have been developed to reduce the complexity and improve the performance of the MMSE estimation such as modified MMSE and singular value decomposition (SVD) [4-5]. In 2006 Noh et al. proposed a method to decomposing the covariance matrix to the

simple and low order sub matrix so that they can decrease the complexity of MMSE method [6]. Hsieh used a comb type pilot arrangement and second order interpolation method to channel estimation [7]. Coleri et al. compared the results of many interpolation techniques to channel estimation with Rayleigh fading such as linear, second order, cubic, low pass filtering and spline interpolation methods [8]. Edfors et al. modified the MMSE and LS methods with assumption that the channel model to be an FIR in which the impulse response duration can't greater than the Guard Interval of an OFDM symbol [9]. Dowler assumed that if the maximum delay caused by channel to be a known parameter so estimation based on DFT method can obtain better results [10]. Minn et al. improved the results of Dowler method by considering a sparse channel model [11]. In [12] Kang et al. proposed a DFT based channel estimation. In their method the effect of channel noise in outside of maximum channel delay are replaced with zero and finally a good estimation were resulted. In 2009 Lee et al. obtained an optimal threshold value based on wavelet decomposition and therefore they could improve the channel estimation [13].

In this paper, we propose a time-domain approach to channel estimation using wavelet decomposition. In this approach, initial channel estimation is calculated by the LS estimator, and then channel coefficients in time-domain are obtained using IFFT transform. It is assumed that the maximum delay caused by channel at most is equal the length of cyclic prefix. In the next step wavelet transform is applied to the obtained coefficients in order to calculate a threshold value [14]. Finally, the estimated coefficients are de-noised and the good estimate is obtained using the calculated threshold. The outline of the remaining of this paper is organized as follows. In section (2), the OFDM system based on pilot channel estimation will be considered. The wavelet transform will be

presented in section (3). The proposed method will be described using wavelet transform in section (4). In section (5), the simulation and obtained results will be considered and finally in section (6) conclusions will be qualified.

2 SYSTEM DESCRIPTIONS

2.1 OFDM Systems

OFDM technique converts a frequency selective channel into a number of frequency nonselective channels by dividing the available spectrum into a number of overlapping and orthogonal narrowband sub channels where each of them sends own data using a subcarrier. A block diagram of OFDM systems is shown in figure (1).

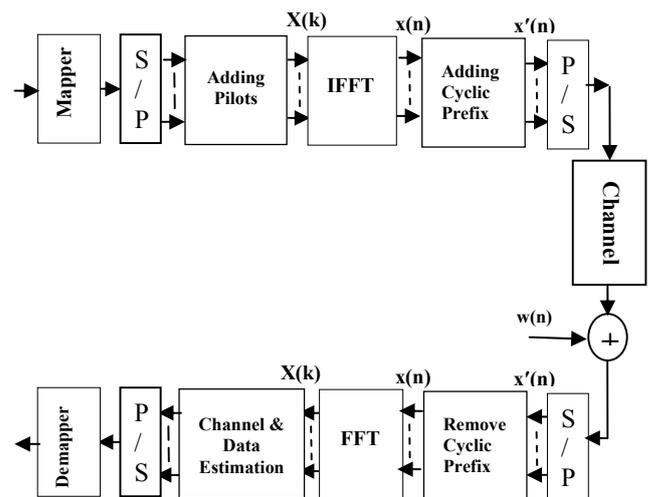


Figure 1. The OFDM System in baseband model

At first, in transmitter the binary inputs are grouped to get an M -ary symbol. According to a predefined baseband modulation such as QPSK and MQAM, the obtained symbols are modulated using a signal mapper subsystem. In the next step, an S/P sub-block converts the serial input symbols to a block data which can be considered as a vector $X=[X_0, X_1 \dots, X_{N-1}]$. The vector size is ' N ' which determine the number of subcarriers in OFDM signal. Any

subcarriers will be modulated by the obtained symbols in data vector using IFFT technique and consequently, the time domain of the OFDM signal are calculated which can be written as equation (1).

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{LN-1} X_k e^{j\frac{2\pi}{LN}kn} \quad 0 \leq n \leq LN-1 \quad (1)$$

Where 'L' is an oversampled factor which can be set to any number as: 2, 4, 8, 16 [15].

To prevent the effect of ISI in OFDM signals, a guard time which well known as cyclic prefix, must be add to the symbol. The equation (2) and figure (2) shows the adding process.

$$x'(n) = \begin{cases} x(N+n) & n = -N_c, -N_c+1, \dots, -1 \\ x(n) & n = 0, 1, \dots, N-1 \end{cases} \quad (2)$$

Where 'N_c' denotes the cyclic prefix length.

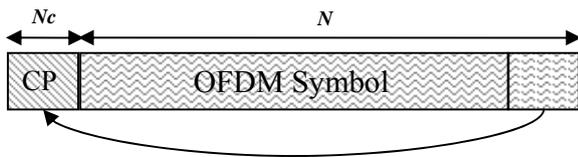


Figure 2. Cyclic Prefix adding process in OFDM symbol

Finally, the obtained OFDM signal is converted to serial form and is transmitted to the receiver side through a frequency selective channel which is often considered as a Rayleigh fading model with additive white Gaussian noise (AWGN). The received signal in the output of the channel can be modeled as equation (3).

$$y'(n) = x'(n) \otimes h(n) + w(n) \quad 0 \leq n \leq N' \quad (3)$$

Where $w(n)$ is the channel noise which is assumed to be AWGN, zero mean and its variance equals σ^2 and $h(n)$ is the impulse response of the channel and $N'=N+N_c-1$. It is assumed that the channel is linear and time invariant in period of an OFDM symbol. After removing the cyclic prefix the above equation can be rewritten as follow:

$$y(n) = x(n) \otimes h(n) + w(n) \quad 0 \leq n \leq N-1 \quad (4)$$

In the frequency domain we can obtain the following equation.

$$Y(k) = X(k)H(k) + W(k) \quad k = 0, \dots, N-1 \quad (5)$$

Where $H(k)$ and $W(k)$ are the Fourier transform of the $h(n)$ and $w(n)$, respectively. In this paper it is assumed that there is no synchronization error.

2.2 Channel Estimation

In any communication systems, channel estimation is a most important and challenging problem, especially in wireless communication systems. Usually, the transmitted signal can be degraded by many detrimental effects such as mobility of transmitter or receiver, scattering due to environmental objects, multipath and so on. These effects cause the signal to be spread in any transformed domains as time, frequency and space. To reducing these effects anyone must estimate the channel impulse response (CIR).

Channel estimation has a long history in single carrier communication systems. In these systems, CIR is modeled as an unknown FIR filter whose coefficients are time varying and need to be estimated [16]. There are many channel estimation methods that can be used in multicarrier communication systems but the especial properties of multicarrier transmission systems give an additional perspective which forces to developing new techniques to channel estimation in wireless communication systems.

In general, channel estimation methods based on OFDM systems can be categorized into two groups as blind and non-blind techniques. In the former, all of the techniques use the statistical behavior of the received signals and therefore, to obtain the accurate CIR a large amount data is required [17]. Finally, the complexity of computations is very high. In the later, to obtain a good estimation of channel, the transmitter sends a collection of data aided as pilots whose are previously known by the receiver. Often, most OFDM based systems as IEEE 802.11a

and hyperLAN2 use pilots in frequency domain in order to sampling the faded channel in frequency domain.

Channel estimation based on pilot arrangement which have been used in many application systems especially wireless communication and power line communication channels can be divided in two main categories as block type and comb type [8]. These arrangements are shown in figures (3) and (4). In these figures ‘ T_c ’ and ‘ B_c ’ parameters denote the time and bandwidth coherency of the channel.

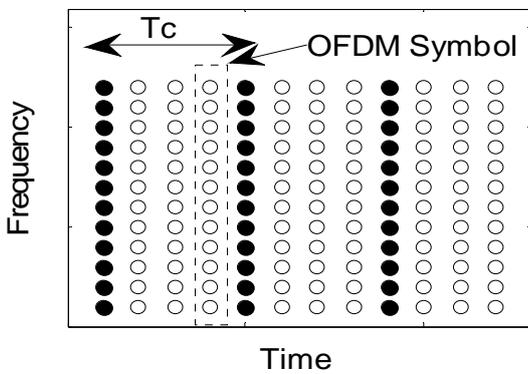


Figure 3. Block type arrangement of OFDM symbols

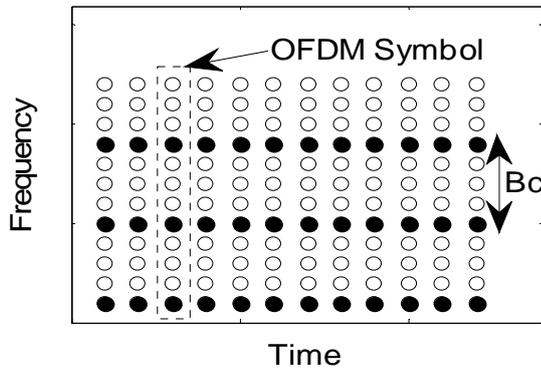


Figure 4. Comb type arrangement of OFDM symbols

According to the type of channel fading, slow or fast fading, anyone can uses the block or comb type arrangements, respectively, to channel estimation. In the block type, an OFDM symbol which contains pilots in all subcarriers, are transmitting periodically which equals the time coherency of the channel which is related to the Doppler effects and in other type the

transmitting OFDM symbols contains data and pilot subcarriers where the pilot spaces must be equal to the frequency coherency which is related to the delay spread caused by multipath effects. In the comb type arrangement when the density of pilots increase then the result of channel estimation will be improve. There are many techniques to channel estimation in both arrangement types as LS, MMSE, DFT based, modified DFT based, Decision Direct and wavelet based methods.

In the comb-type arrangement, pilot symbols are inserted and continuously transmitted over specific pilot sub-channels in all OFDM symbols according to the following equation:

$$X_m = \begin{cases} \text{pilot}, & m = kP \\ \text{data}, & \text{otherwise} \end{cases} \quad (6)$$

Where ‘ P ’ denotes the pilot repetition rate or pilot spaces in OFDM symbol which can be calculate as $P=N/N_c$. Also ‘ N_c ’ indicates the pilot number.

2.3 Least Square Method

This method can be applied in both block and comb type. In later arrangement, in frequency domain, at first the channel output at pilot locations is extracted. In the next step channel estimation can be calculated using the extracted subcarriers which are known to the receiver. The corresponding equation can be written as the following equation.

$$\hat{H}_{LS}(k_p) = \frac{Y(k_p)}{X(k_p)} \quad (7)$$

$$= H(k_p) + W'(k_p), \quad k_p = 1, 2, \dots, N_p$$

Where $W'(k_p) = W(k_p)/X(k_p)$ is the noise component at the estimated channel coefficients in frequency domain and ‘ k_p ’ denotes a subcarrier index at p^{th} pilot. Then to obtaining the channel estimation at the data subcarriers, an interpolation technique is required. There are some interpolation techniques in [8] but linear interpolation is the simple one which can be written as equation (8).

$$\hat{H}(k) = [1 - \frac{(k - k_p)}{L}] \hat{H}(k_p) + \frac{(k - k_p)}{L} \hat{H}(k_p + L). \quad (8)$$

Where ‘ L ’ denotes distance between two adjacent pilot subcarriers.

3 WAVELET DECOMPOSITIONS

Wavelet transforms provide a framework where a signal is decomposed, with each level corresponding to a coarser resolution, or lower frequency band. There are two main groups of transforms, continuous and discrete. Continuous wavelet transform of the signal $x(t)$ can be expressed by (9):

$$x(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) \psi^* \left(\frac{t-b}{a} \right) dt \quad a > 0, b \in \mathbb{R}. \quad (9)$$

Where ‘ a ’ is called the scaling factor and also ‘ b ’ denotes the translation factor and $\psi(t)$ is wavelet function [18]. Although the continuous wavelet transform is simple to describe mathematically, both the signal and the wavelet function must have closed forms, which make it difficult or impractical to apply. The discrete wavelet is used instead so that if we choose the scale ‘ a ’ and position ‘ b ’ based on powers of two, the analysis will be much more efficient and accurate [19]. Any finite energy analog signal $x(t)$ can be decomposed into a coarse approximation represented by scaling functions $\varphi(t)$ and details are represented by wavelet functions $\psi(t)$ as follows:

$$x(t) = \sum_{k=-\infty}^{+\infty} c_k \varphi(t-k) + \sum_{k=-\infty}^{+\infty} \sum_{j=0}^{+\infty} d_{j,k} \psi_{j,k}(t) \quad (10)$$

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k).$$

Where ‘ k ’ and ‘ j ’ are translation and scale parameters, ‘ C_k ’ is scaling coefficient which is used for representation of the low-resolution approximation of the signal and ‘ $d_{j,k}$ ’ is wavelet coefficient which is used for giving detailed information of the signal. Approximated and detailed coefficients can be obtained based on low-pass and high-pass filter respectively. These filter coefficients are calculable according to equation (11):

$$\begin{aligned} \varphi(t) &= \sum_k h_0(k) \sqrt{2} \varphi(2t-k) \\ \psi(t) &= \sum_k h_1(k) \sqrt{2} \psi(2t-k) \end{aligned} \quad (11)$$

Where $h_0(k)$ and $h_1(k)$ are impulse response of low-pass and high-pass filters respectively. Based on these filters, each ideal signal can be expanded as coefficients wavelet. Figure (5) shows related decompositions in two levels.

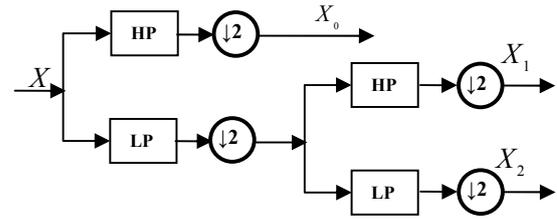


Figure 5. Wavelet decomposition in 2 levels

4 PROPOSED METHOD

Since LS estimator is much sensitive to noise, we introduce a time-domain method to channel estimation which its objective is improvement the performance of LS algorithm. In this paper it is assumed that the pilot spaces are related to the maximum delay caused by the channel. The impulse response of the channel can be calculated using IFFT based on initial LS estimated channel as following equation.

$$\begin{aligned} \hat{h}_{LS}(n) &= IDFT\{\hat{H}_{LS}(k)\} \\ &= \begin{cases} h_1(n) = h(n) + w'(n) & 0 \leq n \leq L-1 \\ h_2(n) = w'(n) & L \leq n \leq L'-1 \\ 0 & L' \leq n \leq N-1 \end{cases} \quad (12) \end{aligned}$$

Where L' can be controlled by the pilot spaces and L can be determined by maximum channel delay. The estimated channel impulse response obtained by LS has most of its energy concentrated on a few first samples because in practice, the channel length is shorter than the IFFT size which can be determined by the cyclic prefix length. In this paper we will consider threshold based channel estimation in wavelet domain. Calculating of the threshold value is very important to noise reduction in channel

estimation. According to [14], the calculated threshold value using wavelet decomposition can be obtained better results than other threshold based techniques.

Since the noise-dominant components of the estimated channel coefficients are in high-frequency part of wavelet decomposition, the detail coefficients can be used to calculate the threshold value [14] as the following equation.

$$\sigma = \frac{\text{median}(|D_i|)}{0.6745} \quad (13)$$

Where ‘ D_i ’ denotes the detail coefficients. By applying the calculated threshold value into channel coefficients according to (14), the noise in the channel will be removed effectively.

$$h_p(n) = \begin{cases} h_{LS}(n), & h_{LS}(n) \geq \sigma \\ 0 & h_{LS}(n) < \sigma \end{cases} \quad (14)$$

Finally by taking the DFT of the obtained channel impulse response, the estimated channel frequency response will be resulted. The proposed algorithm in more details is shown in figure (6).

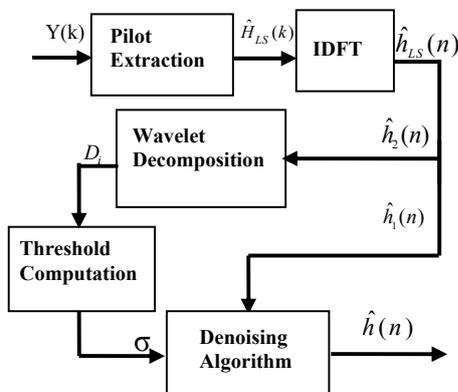


Figure 6. Block diagram of the proposed method

5 SIMULATION RESULTS

In order to evaluate the proposed method it is necessary that a simulation must be taken. The simulation parameters are listed in table (1). In all simulations the block coding and interleavers

are not used and also in order to better evaluation of the proposed method the channel equalization are not considered.

Table 1. Applied parameters to evaluate the proposed method

Parameters	Related Values
OFDM symbol size	N=2048
CP size	$N_c=512$
Baseband modulation	QPSK , 16QAM
Wavelet	Db4
Subcarriers Number	N=2048
Channel length	L=10
Pilot spacing	15, 30
Channel Model	Rayleigh with AWGN

In the following we want to compare the channel estimation results which are obtained using the LS, Lee and proposed methods. The channel model is considered as an FIR filter. The corresponding filter taps is considered as a complex random variable which has Rayleigh distribution in order to model the Rayleigh channel fading. Figures (7) and (8) show bit error rate (BER) of estimation which is obtained by LS, Lee and the proposed methods based on 16-QAM and QPSK modulations, respectively. It is noted that the obtained result in the proposed method is better than Lee and LS methods in overall SNR conditions. It is shown that the proposed method has improvement of about 1-1.5dB compared to the Lee method. Also mean square error (MSE) parameter for more evaluation of proposed method compared with the others is plotted in (9) and (10). In these figures the pilot spacing is considered equal to 30. Figures (11) and (12) are the BER and MSE curves of the LS method, the Lee method and the proposed method in case of 16-QAM with pilot spacing 15 respectively. These figures show that by reducing the number of pilots, Lee method yields poor results compared to the proposed method.

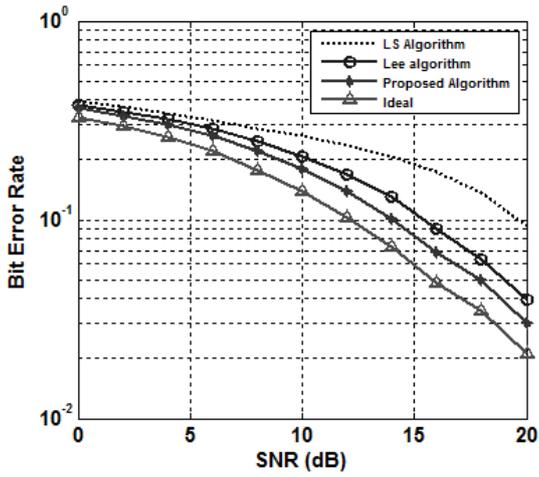


Figure 7. BER Performance comparison with 16QAM

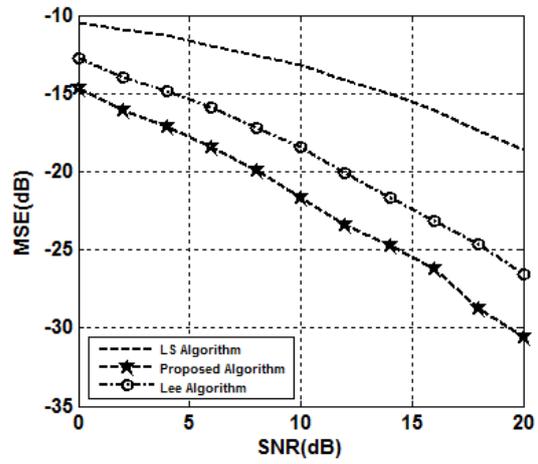


Figure 10. MSE performance comparison with QPSK

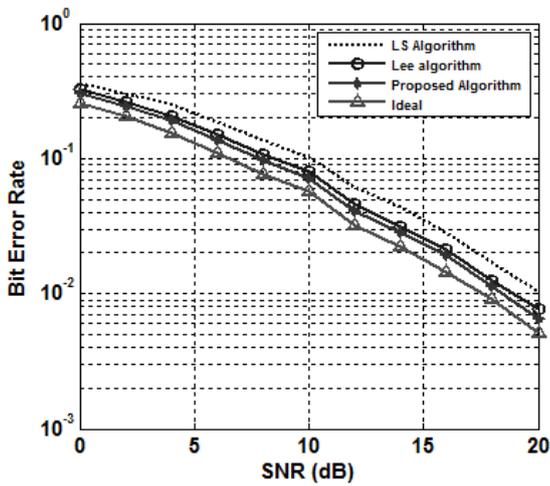


Figure 8. BER performance comparison in case of QPSK

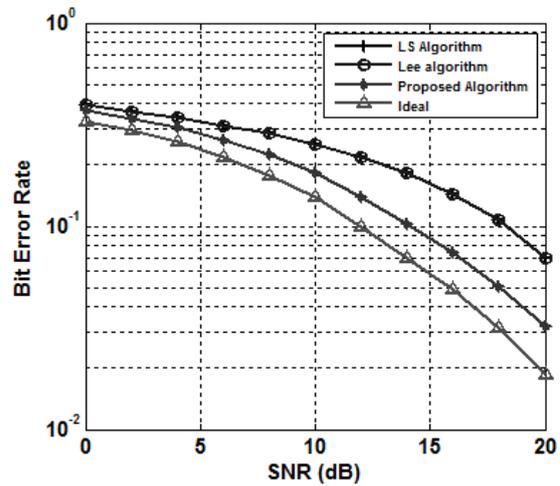


Figure 11. BER performance comparison with 16QAM

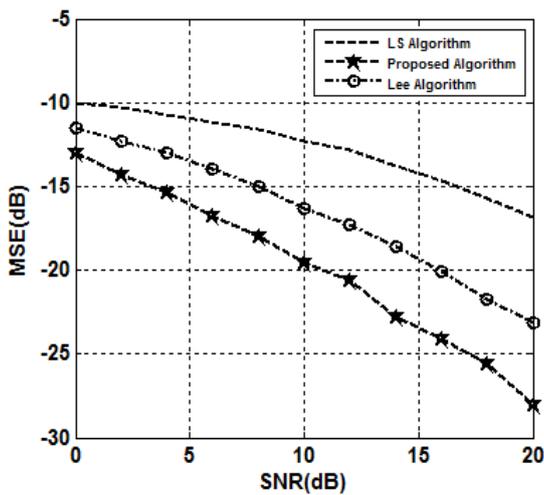


Figure 9. MSE performance comparison with 16QAM

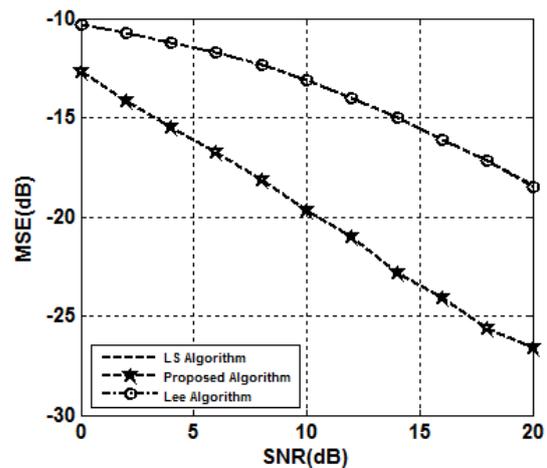


Figure 12. MSE performance comparison with 16QAM

In the following we want to compare the channel estimation results, in case of frequency impulse response, which are obtained using the LS, Lee and proposed methods. Again, the channel model is considered as an FIR filter. The filter taps is considered as a complex random variable which has Rayleigh distribution in order to model the Rayleigh channel fading. Also the channel noise is considered as AWGN with SNR=10dB. The filter has 10 taps.

The obtained results are shown in the figure (13), also in all figures for better comparisons the frequency response of the ideal channel is plotted. As shown in the figures the obtained results in the proposed method are better rather than other methods.

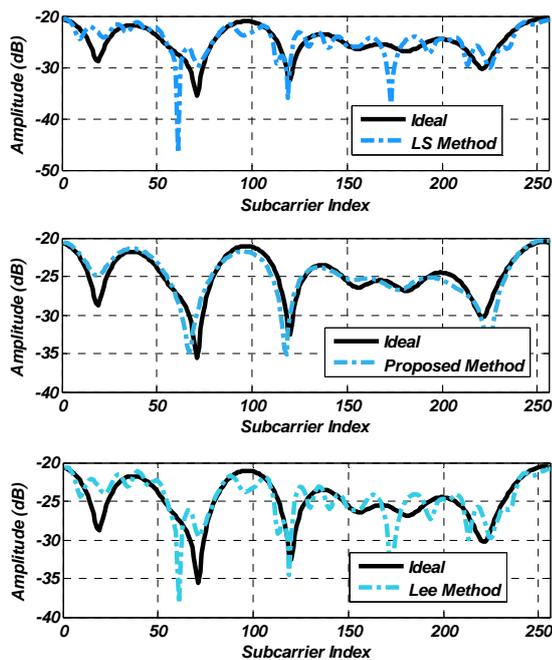


Figure 13. The Estimated Impulse Response of The Channel using the LS, Lee and Proposed Method, SNR=10

6 CONCLUSIONS

In this paper, a new method to estimating the faded channel based on wavelet decomposition is presented. In this method, a threshold based

on wavelet decomposition is used in order to reduce the noise effect. It is assumed that the channel length is related to the maximum delay spread caused by multipath channel. This method is less sensitive to changes in modulation parameters and pilot spacing compared to Lee method that leads to improvement in channel estimation. It is shown that the proposed method has improvement of about 1-1.5dB compared to the Lee method.

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