Accelerated Moving Multi-Agent Behavior on Two Configurations

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***Abstract*—This paper discusses the coordination of autonomous stochastic moving multi-agents on two kinds of resources consisting of cells; three agents on a line, and three agents on a circle. Each agent on their resources stochastically moves on the cell resources through moving paths with time-lag. Our problem is how to coordinate agents to maximize resource utilization efficiently, where each agent depends on other agents locating the neighbors. We consider a shake for agents in order to increase the resource utilization, and we also call the shake an acceleration. The most highest resource utilization is that every cells are always occupied by agents, i.e. it is desirable to be the fewest expected number of cells not occupied by agents. We show that the resource utilization of the multi-agents becomes higher if every agents have appropriate acceleration. The acceleration depends on the configurations of cell resources, and we can find an optimal acceleration which is to maximize the cell resource utilization depending on their configurations.**

Keywords— **Autonomous Moving Stochastic Multi-Agents, and Resource Utilization.**

I. Introduction

This paper discusses a resource utilization of autonomous stochastic moving multi-agents with time-lag, and their agents stochastically move on the cells through moving paths along finite resources. The resources arrange over a configuration according to transition probabilities in synchronization. We consider two kinds of configurations; three agents on a line, and three agents on a circle. The moving of each agent is restricted so that it depends on the number of other agents on current cells within specific ranged windows, and there is coordination among agents with time-lag depending the window sizes, while the coordination depends on the resource configurations. The interactions among the agents are complicated to analyze the behavior of them, since each agent stochastically moves over the resources autonomously. In addition, the acceleration makes our analysis even more difficult. The paper[22], [21] shows that the stochastic moving multi-agent behavior becomes more stable if every agents arranged on line resources take an acceleration as a shake. The appropriate acceleration makes the coordination of moving-multi-agents maximize. The paper [21] discusses the resource utilization of agents considering the boundary effect on line resources. In this paper, we consider two kinds of cell resource configurations, their agents are accelerated by a shake, and we present their theoretical analysis of cell resource utilization on two kinds of resources. Then, we assume the acceleration of every agents is independent from the locations of resources. Our analysis shows that the resource utilization to maximize them depend on their configurations. Stronger acceleration is necessary in proportion to the number of boundaries in order to increase the efficient use of resources, where the boundaries are cells moving path that is a stop, i.e. a dead end cell of paths.

In our real world, there are a lot of unusual beings with unexpected phenomena that are beyond human understanding. In fact, a multi-agent behavior is one of them, while it is quite difficult to analyze the behavior of multi-agents in general theoretical frameworks, because there are interactions or coordination among agents. Fortunately, letting you perform intensive analysis on your desk, when we discuss the small sizes of configurations.

The studies of complex systems[1] have been expected to explore new unexpected phenomena which are carried by natural or artificial systems. The most attractive one is that the behavior of entire systems does not obvious from a simple combination of each agent behavior. Our stochastic moving multi-agents or multi-objects that take an appropriate average moving speed exactly behave more stable moving or achieve high resource utilization.

We are in need of a simple model with no fat in moving multi-agents for analyzing complex systems. Fortunately, Sen et al.[19] proposed a simple model for analyzing the behavior of moving multi-agents, and Rustogi et al.[17] presented the fundamental results of the former model. Ishiduka et al.[8] also

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introduced a time lag and showed the relationship between time lag and stability in moving multi-agents. The above models are intended to clarify how fast the moving multi-agents fall into a complete stable state, i.e. a hole state in absorbing Markov chain[5], thus the goal is to design a coordinative system which falls into a stable hole in shorter passage time as soon as possible.

On the other hand, in physics, Toyabe et al. [23] experimentally demonstrated that information-to-energy conversion is possible in an autonomous single stochastic moving agent. In other words, the paper presented a solution of Maxwell's devil. The idea is that if an agent goes up the spiral stairs during stochastic movements, it sets the stopper on the stairs so that the agent does not to come down. This approach needs an explicit control that the agent does not come down the spiral stairs. It is a single agent against a multi-agent, and our ultimate goal is to get the energy from stochastic moving multi-agents. In multi-agent models, Hiyama[7] presented the precise theoretical calculation providing the interactions among different types of objects in nucleus. These are realized by stacking a small effect, and also this paper use the stacking of the diffusion in stochastic moving objects.

Our model, Multi-Agent behavior with Time lag and Moving Speed: MATMS, is based on Sen et al.[19] and the developed model with time-lag proposed by Rustogi and Singh[17]. We note that our purpose is different from the papers [19], [17], [8] which try to clarify the relationships between time-lag and stability in multi-agent systems. In other words, their papers try to find the multi-agent configurations satisfying autonomous uniform resource allocation in a shortest passage time. Our model satisfies Markov condition and irreducible so that the states do not depend on the initial states in the limit, and our problem is to find more stable multi-agent states accompanying agent movements. It just likes as a molecule has an energy so that it is always moving while the agents are alive, and it depends on the manner of substances. The paper[22] showed that a stochastic moving multi-agent system, whose the agents move slowly as a whole on average, is more stable than other ones not having average moving speed as a whole theoretically. This paper demonstrates higher resource configuration behavior when we consider the agent locations on resources, that is, boundary effects. We can find the basis of our model in Shilling model[18].

This paper is organized as the following. First, we define our model in Section II. Section III and IV show that there exists a behavior of multi-agents based on theoretical analysis on two configurations: a line and a circle. In the following section, we discuss the related works. Finally, we conclude this paper in Section VI.

II. Stochastic Moving Multi-agent model with time-lag and moving speed

We shall consider two kinds of finite cell resources in Figure 1, and three agents arranged on their cells. All the agents move over the resource consisting of cells $S(i)$, where the resource configurations are a line and a circle. The papers [19], [17] only consider on a circle. All agents run synchronously in discrete time over the resource according to the following transition probabilities $p_{i,j}$ in stochastic manner. In the following, sometimes we are simply expressed as i a resource $S(i)$.

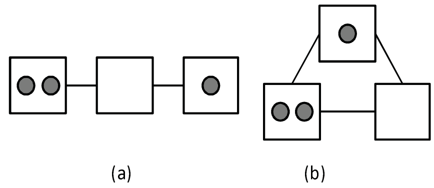

Fig. 1. Two kinds of resources and stochastic moving multi-agents.

First, we define a weight function $f_{i,j}$, $i, j = 1, .., n$ as

$$f_{i,j} = \begin{cases} 1, & i = j \\ 0, & i \neq j, r_i < r_j \\ 1 - \frac{1}{1+\gamma \exp(\frac{move(r_i - r_j, i, j) - \alpha}{\beta})}, & \text{otherwise,} \end{cases} \quad (1)$$

where r_i are the number of agents on i-th cell, and α, β and γ are constants. α is called an "inertia" which is the tendency of an agent to stay in its resource[17], and $move$ is an accelerated function to give average moving speed on either left or right defined by (2) and (3) in later. Rustogi et al. model[17] does not satisfy the condition "irreducible" exactly, while our model satisfies Markov property under the condition not to restrict the moving directions of agents, and the model becomes irreducible.

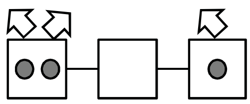

Fig. 2. The model MATMS.

Our model also has an average moving *speed* such as every agents move with an average moving speed s_i $(s_i \geq 1)$ either of left or right directions on the cells, where i are i-th locations of agents. Their agents move along the resources arranged over the line or the circle according to the probability $p_{i,j}$ in stochastic manner, where i and j indicate i-th and j-th cells, respectively. In the case of right average moving speed s_i, the function $move(x, i, j)$, which describes the ratio of imbalance from a cell i to a destination cell j with the difference $x (= r_i - r_j)$ in the numbers of agents on i and j, is defined by

$$move(x, i, j) = \begin{cases} s_i \times x, & i < j, \\ x, & \text{otherwise,} \end{cases} \quad (2)$$

wheres s_i is an average moving speed at i-th cell. On the other hand, for the left average moving, $move$ is defined by

$$move(x, i, j) = \begin{cases} s_i \times x, & i > j, \\ x, & \text{otherwise.} \end{cases} \quad (3)$$

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The function $move(x, i, j)$ is not symmetric with respect to x, and s_i represents the ratio of imbalance at i-th cell in the function $move$. As a special case, $move(x, i, j)$ becomes symmetric with respect to x and the agents do not move on average if $s_i = 1$. The average moving directions are inverted with the same average moving speed if each agent arrives at the leftmost or rightmost cells, i.e. the cells on the boundaries. The average moving speed depends on the agent locations, and each agent moves towards either left or right directions on average independently. Thus, all the agents are randomly choosing the moving directions which are apart from the effect on the left and right boundaries.

A moving transition probability $p_{i,j}$ from a current cell $S(i)$ to a destination cell $S(j)$ is defined by the normalization of $f_{i,j}$ with probability 1 as

$$p_{i,j} = \frac{f_{i,j}}{\sum_k f_{i,k}}, \quad i, j = 1, .., n, \tag{4}$$

based on $f_{i,j}$. Rustogi et al. [17] introduced a window $win(i)$ with a fixed size for analyzing the behavior of multi-agent systems with time-lag. Then, a moving transition probability $p_{i,j}$ from a current cell $S(i)$ to a destination cell $S(j)$ is defined by

$$p_{i,j} = \begin{cases} \frac{f_{i,j}}{\sum_{k \in win(i)} f_{i,k}}, & i = 1, .., n, j \in win(i), \\ 0, & \text{otherwise}, \end{cases} \tag{5}$$

where w is a window size, and $win(i)$ is the set $[i-w, i+w]$. A time delay which is local properties is proportional to the window size w (see [17], [8]).

There are no constraints on the moving of agents such that each cell has a fixed upper limit capacity to occupy agents, while there is another constraint in the model, i.e. the moving transition probability $p_{i,j}$ is 0 if the number of agents on a cell $S(i)$ is less than the number of agents on a destination cell $S(j)$.

Our proposed model, Multi-Agent behavior with Time delay and Moving Speed: MATMS, is similar to the models [17], [8]. The resources in MATMS are arranged over a line $[1, 2, .., n]$ as in [8] or a circle[19], [17], and the wind function $win(i)$ is the set $[i-w, i+w] \cap [1, n]$ if w is a window size. We note that there are two choices on the moving average directions which are either left or right. Suppose an agent moves towards left on average at the previous step. Which is the moving direction at the next step? If we exclude the cases that the agents stay on boundaries, there are two exclusive cases(or a model protocol) for each agent independently: (1) we inherit the directions at the previous steps, i.e. left on average in above, or (2) we randomly select it at each step according to even probability either left or right, i.e. half to half rule for the direction. The second case (2) is suite to Markov property. The first case (1) does not satisfy Markov property so that the systems depend on the initial configurations.

III. Theoretical analysis of 3×3 model on the line

In this section, we present a concrete moving multi-agent such that the multi-agent taking an appropriate average speed achieves higher resource utilization than a multi-agent not taking moving average speed, i.e. every cells are occupied by agents in many cases on average.

Suppose the multi-agent of which the number of cells and agents are 3 together. This is a minimal model to examine a coordination among agents. We first use the parameter values $\beta = 2$ and $\gamma = 1$, and fix the window size w to 1.

Suppose 1, 2 and 3 are their cell names(Figure 1(a)) from left to right. We do not distinguish the names among the agents for the simplicity, and represent it just a. Suppose that the same average moving speed s_1, s_2 and s_3 are both s ($s \geq 1$), respectively, since the resource is symmetric. The moving directions of the agents are randomly selected either left l or right r in half and half at every steps. The multi-agent state is a set of three agent states. $[(a, 1), (a, 2), (a, 2)]$ is an example of the multi-agent states, where a are agents and 1, 2 and 3 are the resources.

An example of the agent moving configuration is represented by $(a, r, 1)$ if the agent a on the cell 1 moves towards right with average moving speed s_b. The multi-agent moving configuration consists of three agent configurations in this minimal model. For an example, $[(a, r, 1), (a, r, 1), (a, r, 1)]$ is a multi-agent moving configuration.

In our minimal model, the directions of average agent moving are stochastically chosen at every steps so that the multi-agent becomes Markov chain. In this setting, there are 10 multi-agent states shown in Figure 3, and we must consider 136 probabilistic transition rules. That is, the number of the states(Figure 3), the state transition rules (Appendix in [22]) and the multi-agent moving configurations (The top items of Appendix in [22]) are 10, 136 and 20, respectively. The illustration of the transition rule (d-3) is shown in Figure 4.

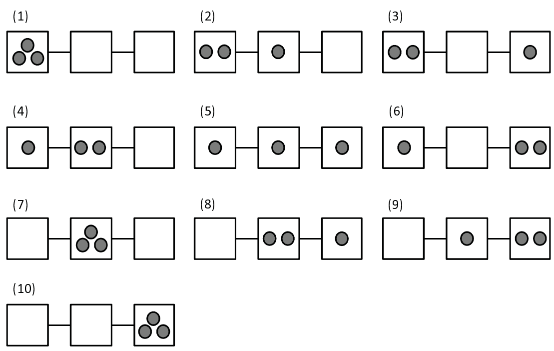

Fig. 3. The states of three agents arranged on the line consisting three cells.

This simple model satisfies Markov condition and it is irreducible, so we easily compute the eigenvectors of the state transition matrix with the size 10×10 using Appendix in [22], and compute the transition probabilities among every states

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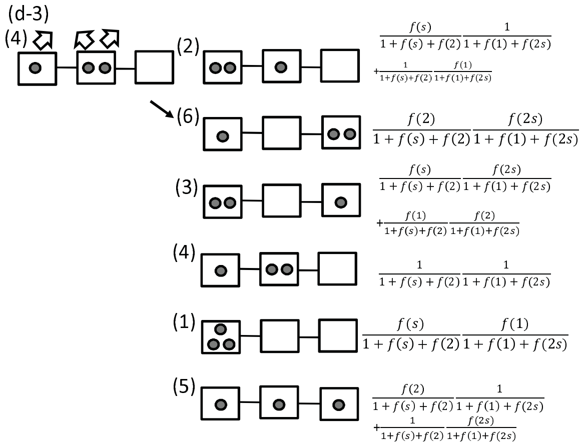

Fig. 4. The details of the transition rules (g-2) of the line, Appendix in [22].

TABLE I
THE PROBABILITIES STAYING THE STATES IN THE CASE $cells = 3$, $agents = 3$, $\alpha = 8$, AND $w = 1$ BASED ON THEORETICAL COMPUTATION OF THE LINE CONFIGURATION.

state	$s = 1$	$s = 7$
(1)	0.0003534811	0.001364175
(2)	0.0879075822	0.021569380
(3)	0.0906589070	0.027278335
(4)	0.0936260239	0.049030678
(5)	0.4540795018	0.793757492
(6)	0.0906589070	0.027278335
(7)	0.0008285100	0.007757373
(8)	0.0936260239	0.049030678
(9)	0.0879075822	0.021569380
(10)	0.0003534811	0.001364175

in the limit by changing the moving speed. The theoretical computational results of the existence probabilities for every states are shown in Table I.

The expected average number m_1 of the cell 1 occupied by agents is given by the following:

$$m_1 = p_1 + p_2 + p_3 + p_4 + p_5 + p_6, \qquad (6)$$

where p_i are the probabilities of the correspondence states i shown in Table I. In other words, m_1 is the average resource utilization on the cell 1.

By similar way, we can compute the expected average numbers m_2 and m_3 of the cells 2 and 3, respectively, occupied by agents:

$$m_2 = p_2 + p_4 + p_5 + p_7 + p_8 + p_9, \qquad (7)$$

$$m_3 = p_3 + p_5 + p_6 + p_8 + p_9 + p_{10}. \qquad (8)$$

The whole expected average number v_m of cells occupied by agents, i.e. the resource utilization over the resource, is given as

$$v_m = (p_1 + p_7 + p_{10}) + 2(p_2 + p_3 + p_4 + p_6 + p_8 + p_9) + 3p_5.$$

TABLE II
THE EXPECTED AVERAGE NUMBER OF THE CELLS OCCUPIED BY AGENTS BASED ON THEORETICAL COMPUTATION OF THE LINE: $cells = 3$, $agents = 3$, $w = 1$.

speed	$s = 1$	$s = 7$
cell 1, mean m_1	0.8240108	0.9202784
cell 2, mean m_2	0.824081	0.942715
cell 3, mean m_3	0.8240108	0.9202784
resource, average vm	2.472103	2.783272

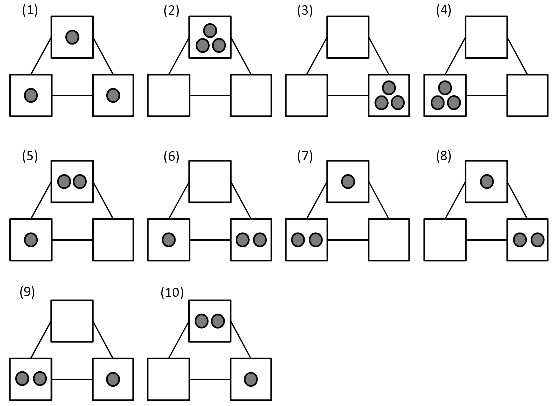

Fig. 5. The states of three agents arranged on the circle consisting three cells.

IV. THEORETICAL ANALYSIS OF 3×3 MODEL ON THE CIRCLE

In this section, we compute the multi-agent behavior without boundary effects on the circle: three cells and three agents shown in Figure 1, and we can compare with and without boundary effects. The number of states and the transition rules are 10 (shown in Fire 5) and 980 (one of them is shown in the Figure 6), respectively.

The resource utilization of the case can compute the ex-

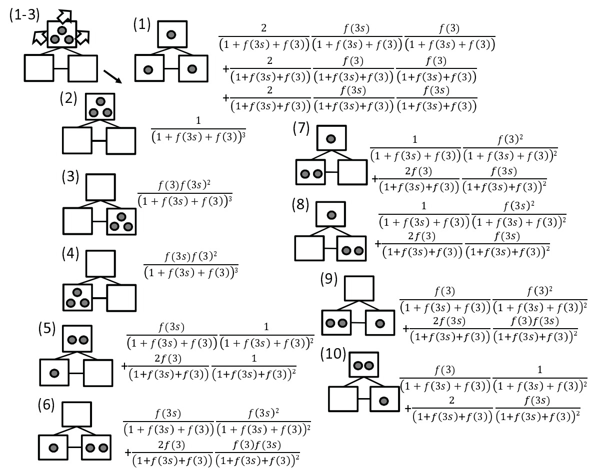

Fig. 6. The details of the transition rules (1-3) in the circle.

TABLE III
THE PROBABILITIES STAYING THE STATES IN THE CASE $cells = 3$, $agents = 3$, $\alpha = 8$, AND $w = 1$ BASED ON THEORETICAL COMPUTATION OF THE CIRCLE CONFIGURATION.

state	$s = 1$	$s = 7$
(1)	0.447229796	0.741141684
(2)	0.0007897928	0.002853046
(3)	0.0007897928	0.002853046
(4)	0.0007897923	0.002853046
(5)	0.0917333748	0.041716530
(6)	0.0917333748	0.041716530
(7)	0.0917333748	0.041716530
(8)	0.0917333748	0.041716530
(9)	0.0917333748	0.041716530
(10)	0.0917333748	0.041716530

TABLE IV
THE EXPECTED AVERAGE NUMBER OF THE CELLS OCCUPIED BY AGENTS ARRANGED ON THE CIRCLE, THEORETICAL COMPUTATION: $cells = 3$, $agents = 3$, $w = 1$.

speed	$s = 1$	$s = 7$
cell 1, mean m_1	0.8149537	0.9108608
cell 2, mean m_2	0.8149537	0.9108608
cell 3, mean m_3	0.8149537	0.9108608
resource, average vm	2.444861	2.732583

pected average numbers m_1, m_2 and m_3 of the cells 1, 2 and 3, respectively, occupied by agents:

$$m_1 = p_1 + p_2 + p_5 + p_7 + p_8 + p_{10}, \quad (9)$$
$$m_2 = p_1 + p_3 + p_6 + p_8 + p_9 + p_{10}, \quad (10)$$
$$m_3 = p_1 + p_4 + p_5 + p_6 + p_7 + p_9. \quad (11)$$

The whole expected average number v_m of cells occupied by agents, i.e. the resource utilization over the resource, is given as

$$v_m = (p_2 + p_3 + p_4) + 2(p_5 + p_6 + p_7 + p_8 + p_9 + p_{10}) + 3p_1.$$

Table V shows that there is an optimal speed to accelerate the MATMS in the circle 3×3 as well as the line configuration (also see [21] on a line case). But, the circle 3×3 is a boundary-less, so the resource utilization becomes low rather than boundary cases in the same given acceleration.

TABLE V
THE RESOURCE UTILIZATION IN THE CASE $cells = 3$, $agents = 3$, $\alpha = 8$, $\beta = 2$, $\gamma = 1$ AND $w = 1$ BASED ON THEORETICAL COMPUTATION OF THE LINE AND THE CIRCLE CONFIGURATIONS.

speed	line v_m	circle v_m
1	2.452544	2.444861
2	2.609133	2.561073
3	2.723913	2.665648
4	2.779706	2.723538
5	2.795896	2.742501
6	2.793445	2.741467
7	2.783272	2.732583
8	2.770548	2.722051
9	2.758217	2.712767
10	2.747893	2.705684

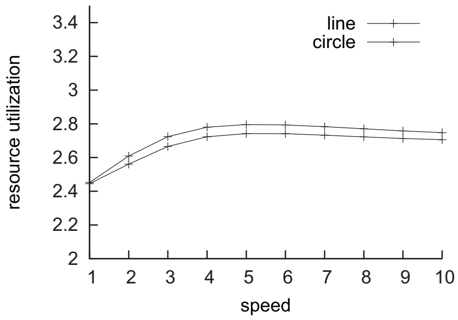

Fig. 7. The expected average number of cells occupied by agents based on theoretical computation in line and circle resource utilization: $cells = 3$, $agents = 3$ and $w = 1$, where $\alpha = 8$, $\beta = 2$, $\gamma = 1$.

V. RELATED WORKS

Sen et al.[19] presented our basic model, and Rustogi et al.[17] also proposed the their extended model with time delay and presented the excellent results. Ishiduka et al.[8] introduced a time lag for the propagation speed explicitly in addition to a window, and showed the relationships between stability and time lags. The similar result had been already obtained[9]. We note that Sen and Rustogi models employ the resources on circles. On the other hand, the resources of Ishiduka model are on a straight line. A straight line of resources are more realistic and natural compares to the circle. How's the boundary effect? How's the circular effect?

There are a lot of discussions on the stability of multi-agents. Chlie et al. [3] tries to find time Markov chains to be stable when its state has converged to an equilibrium distribution. Bracciali et al. [2] presents an abstract declarative semantics for multi-agent systems based on the idea of stable set. Moreau [14] discusses the compelling model of network of agents interacting via time-dependent communication links. Finke and Passino [4] discusses a behavior of a group of agents and their interactions in a shared environment. Lee et al. [10] considers the kinematicd based dynamics-based flocking model on graphs, and the model of the behavior is unstable. They proposed a stable information framework for multi-agents. Mohanarajah and Hayakawa [13] discusses the formation control of multi-agent dynamical systems in the case of limitation on the number of communication channels. Hirayama et al. [6] introduced the distributed Lagrangian protocol for finding the solutions of distributed systems. These papers are intended to control the multi-agent systems in corporative stable states. However, our model is one of the natural models to achieve the coordination without controls and without communication among agents.

VI. CONCLUSIONS

In this paper, we considered a stochastic moving multi-agent model, and presented that the model, Multi-Agent behavior with Time delay and Moving Speed: MATMS, having appropriate average stochastic moving speed become higher resource utilization than ones not having average moving speed. Then, we considered the boundary effects on resources. This shows that each agent needs the moving acceleration to achieve hight resource utilization. The acceleration is a *shake* in this paper. In our model, we considered two kinds of resource configurations, the line and the circle, in moving multi-agents. Then, we showed how are boundary effects related to resource utilization in multi-agents.

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