

# LMS-based Equalization in Filter Bank Multicarrier Wireless Communication Systems

Mashhoor Al Tarayrah and Qasem Abu Al-Haija

*King Faisal University, Department of Electrical Engineering,  
Al-Ahsa 31982, P.O. Box 380, Saudi Arabia*

[Mtarayrah@kfu.edu.sa](mailto:Mtarayrah@kfu.edu.sa) ; [Qalhaija@kfu.edu.sa](mailto:Qalhaija@kfu.edu.sa)

## ABSTRACT

Multicarrier systems give many attractive characteristics for high data rates wireless communications. Filter bank multicarrier systems (FBMC) provide some advantages more than the traditional orthogonal frequency division multiplexing (OFDM) with cyclic prefix (CP). The most obvious difference between the two techniques is in frequency selectivity. In this paper, we will design adaptive channel equalization algorithms for FBMC system with offset QAM modulation (OQAM). Our proposed algorithms will be optimized based on well-known cost functions, which is the mean-squared error (MSE) criterion and it is based on least-mean-square (LMS) algorithm. Thus we will have an adaptive equalizer with lower complexity because of applying the equalizer to each sub-carrier. We have used in our simulation practical channel information based on the International Telecommunications Union ITU Standards. In this paper, we aim to optimize and evaluate the convergence characteristic curves of LMS equalization algorithm per-subcarrier.

**Keywords:** Channel Equalization, Filter Bank Multicarrier Systems, OQAM, LMS.

## I. INTRODUCTION

Wireless communications needs to support high data rate, with high quality to transmit data, thus requires a wide transmission bandwidth. Increasing the transmission rate generally converts the communication channels into a frequency selective one. Frequency selectivity appears in the form of intersymbol interference (ISI) that results from the generated multipath effect [1]. The following schematic illustrates how intersymbol interference is generated when the signal bandwidth is large (this is the case for high-bit rate signals). The condition for frequency selectivity

that the signal bandwidth ( $B_T$ ) is much larger than the channel coherence bandwidth ( $\Delta f_c$ ), i.e.,  $B_T \gg \Delta f_c$

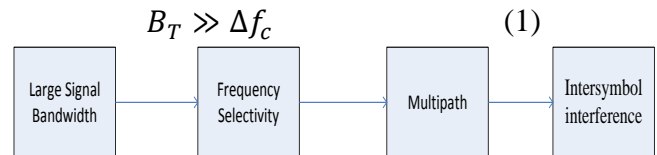


Figure 1: ISI generating in large signal bandwidth

Multicarrier systems provide many attractive properties for high rate wireless communications. A major advantage of the multicarrier approach is its robustness to the multipath effect, and therefore to ISI where multicarrier modulation splits the large-bit rate incoming sequence into several parallel lower-bit rate sequences. The number of parallel sequences can be adjusted such that the wireless channel becomes frequency non-selective. Filter bank based multicarrier (FBMC) systems offer a number of benefits over conventional multicarrier systems based on Orthogonal Frequency Division Multiplexing (OFDM). The most obvious difference between the two techniques is frequency selectivity. OFDM exhibits large ripples in the frequency domain. In contrast, FBMC divides the transmission channel of the system into a set of sub-channels, where any channel overlaps only with its immediate neighbors [2].

Generally, the channel equalization plays a major role in enhancing the performance of communication systems such as FBMC system which uses simple sub-channel equalization, where equalizers take the form of FIR filters with very small numbers of taps. It should be emphasized that using multicarrier modulation leads to substantial simplification of the required channel equalizers; due to its ability to combat the



multipath/ISI effects of the frequency selective wireless channel [3].

In last years, many publications have been investigated based on filter bank transceivers. In [4], Lin and Phoong work on distortion-free unstructured filter bank transceivers. Instead of imposing ISI-free systems, in [5], Vaidyanathan study the use of so-called principal component filter banks (PCFB) to solve the bit rate maximization problem. In [4] and [5] both transceivers they have essential limitations that make their use in practice unavailable. In [6], T. H. Stitz proposed a detailed analysis of synchronization and channel estimation strategies for FBMC based on scattered pilots. Special problems relating to using scattered pilot schemes based in the FBMC are highlighting. The implementation of channel parameter estimation and redresses are successfully in the frequency domain, in the method of a subchannel-wise, which is attractive in spectrally graceful and cognitive radio scenarios.

In [7], the authors prove that the filter bank could be used to repress possible narrowband interferences in the signal band and mitigate the out-of-band interferences effectively. This is done in flexible way, thus permitting adjusting the user signal bandwidths and divides the signal band for different users, without needing the precise time-synchronization of the different user signals. In [1], there are a lot of solutions to solve the channel equalization in FBMC systems. Most of them are not morbidity because of that they purpose at equalizing the frequency response of the sub-channel, work either at the symbol rate or using adjacent sub-channels. Since equalizing the frequency response does not allow to direct control of the remaining interference at the output signal, working at the symbol rate will suffer from irrevocable aliasing in the signal where utilizing adjacent sub-channels will increase the complexity.

In [9], the authors prove that the filter bank could be used to repress possible narrowband interferences in the signal band and mitigate the out-of-band interferences effectively. This is done in flexible way, thus permitting adjusting the user signal bandwidths and divides the signal band for

different users, without needing the precise time-synchronization of the different user signals.

In [11] T. Ihalainen, Y. Yang and M. Renfors shows that by using oversampled analysis bank, the subcarrier equalizers will operate individually of each other. where the channel estimates using a frequency sampling based approach is used to calculate the equalizer coefficients. They assume two different types of subband equalizers and they are a 3-tap complex FIR filter, and a cascade of a linear-phase FIR filter as amplitude equalizer and an allpass-filter as phase equalizer. The authors prove that it is possible to combine the frequency response at each frequency point where used in the equalizer design. This results give the subband equalizer solutions for each antenna.

In [12] T. H. Stitz proposed a detailed analysis of synchronization and channel estimation strategies for FBMC based on scattered pilots. Special problems relating to using Scattered pilot schemes based in the FBMC are high lighting. The implementation of channel parameter estimation and redresses are successfully in the frequency domain, in the method of a subchannel-wise, which is attractive in spectrally graceful and cognitive radio scenarios. T. Ihalainen, Y. Yang and M. Renfors have developed filter bank is based on the receiver signal processing for frequency domain equalization of single-carrier signals in [3], and for FBMC in [1]. The same low-complexity linear equalizer structure was for subcarrier equalizers in the multicarrier case and for subband equalizers in the SC-FDE case.

The authors of [13], [14] have employed an effective implement of OQAM-FBMCs through using the polyphase decomposition, which is known as modified DFT filter banks (MDFT) [15].

In [1], there are a lot of solutions to solve the channel equalization in FBMC systems. Most of them are not morbidity because of that they purpose at equalizing the frequency response of the sub-channel, work either at the symbol rate or using adjacent sub-channels. Since equalizing the frequency response does not allowed to a direct control of the remaining interference at the output signal. Then working at the symbol rate will produce suffer from irrevocable aliasing in the

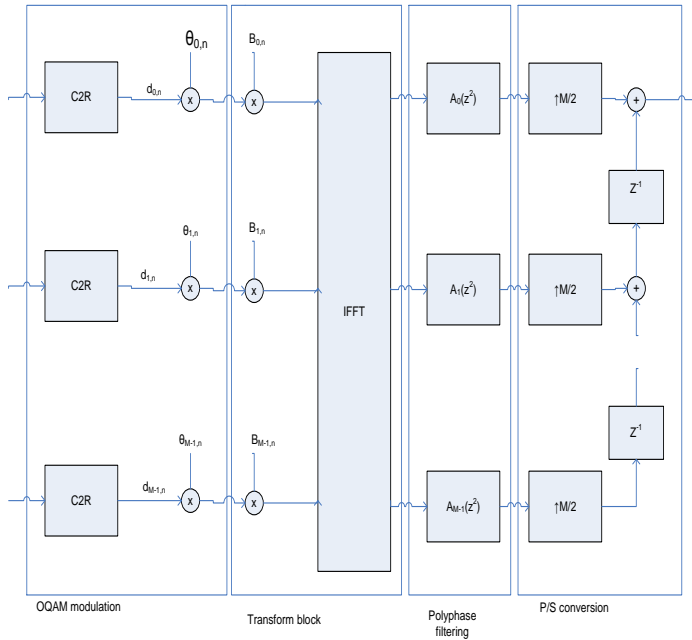


signal where utilizing adjacent sub-channels will increase the complexity.

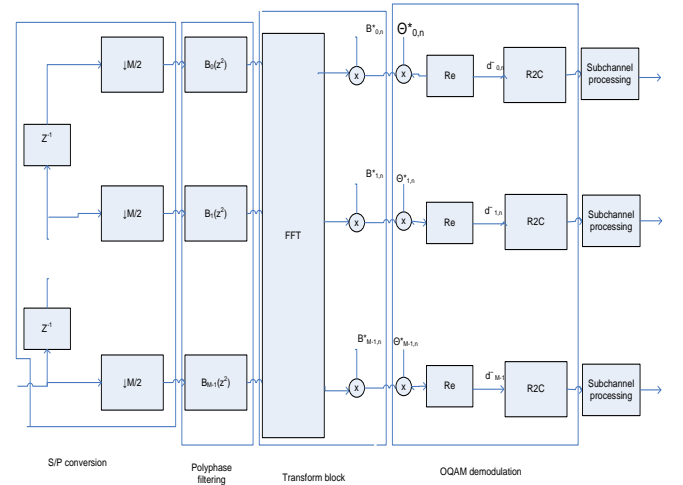
In this paper, we propose to design adaptive channel equalization algorithms for FBMC system with offset QAM modulation which are optimized based on the well-known cost functions such as Mean-Squared Error (MSE) criterion. We propose to design LMS equalizer to every subcarrier, thus we can use different step size values for each subcarrier.

## II. SYSTEM MODEL

In this paper, FBMC system will be used according to the figures 2 and 3 which contains the following main processing blocks: OQAM pre-processing, Synthesis filter bank (SFB), Analysis filter bank (AFB), OQAM post-processing and LMS per-subcarrier equalization. Linear Time Varying (LTV) channel model which is based on AWGN channel and has physical characteristic like multipath propagation is assumed in this paper. We will focus on a specific prototype filter length that is ( $L_p = KM$ ), an extra delay  $z^{-D}$  has to be introduced on the SFB or AFB input, here depends on  $L_p$ , that's  $L_p = KM + 1 - D$ , so in our model the extra delay is  $z^{-1}$ .



**Figure 2:** Polyphase filter bank structures Synthesis filter bank (SFB).



**Figure 3:** Polyphase filter bank structures Analysis filter bank (AFB).

### A. OQAM SIGNAL

Offset in OQAM express the time shift of half the inverse of the sub-channel spacing between the real and imaginary parts of a complex symbol. The data will be sent on the real and the imaginary part of samples alternatively. In OQAM, The throughput rate will stay as in QAM modulation as used in OFDM systems, but without inserting the guard time [4].

To represent OQAM signals we can start from a QAM signal constellation. First let us assume the QAM symbol is  $c_{m,l}$  as a complex quantity, and then  $c_{m,l}$  can be expressed in the form

$$c_{m,l} = c_{m,l}^R + j c_{m,l}^I \quad (2)$$

where  $c_{m,l}$  carries the information to be transmitted over sub-channel  $m$  during the  $l_{th}$  frame of bits. And  $c_{m,l}^R$  is the real part and  $c_{m,l}^I$  is the imaginary part of  $c_{m,l}$ .

### B. OQAM PREPROCESSING

Preprocessing block converts the QAM symbols into OQAM. To convert QAM symbols into OQAM, firstly, a simple complex to real conversion required. We must know that the conversion will be different for even and odd



subchannels as shown in Table 1. this conversion increases the sample rate by 2 [3,8].

Secondly, the conversion is followed by multiplication  $\theta_{m,n}$  sequence.

where

$$\theta_{m,n} = j^{m+n} \quad (3)$$

$n$  is discrete time variable that runs at twice the rate of  $l$ .

The pattern of real and imaginary samples must follow the sign of the  $\theta_{m,n}$  sequences. Here  $\theta_{m,n}$  sequences can be as follows as an example

$$\theta_{m,n} = \begin{cases} 1, j, 1, j, 1, \dots & \text{for } n \text{ is even} \\ j, 1, j, 1, j, \dots & \text{for } n \text{ is odd} \end{cases}$$

After the OQAM preprocessing, the input signals are either pure real or pure imaginary.

The OQAM symbols will be denoted by  $\beta_{m,n}$ , OQAM symbols are given by:

$$\beta_{m,n} = \theta_{m,n} d_{m,n} \quad (4)$$

**Table 1:** OQAM symbol  $\beta_{m,n}$  for even/odd values of  $m$  &  $n$

	Even $n$	Odd $n$
Even $m$	$c_{m,n/2}^R$	$c_{m,(n-1)/2}^I$
Odd $m$	$c_{m,n/2}^I$	$c_{m,(n-1)/2}^R$

### C. SYNTHESIS FILTER BANK

In SFB, A P/S converter will be consists of upsamplers with  $M/2$  and delay chain. Type-1 polyphase filters are expressed as  $a_k[m] = p[k + mM]$ . Because of the length of the prototype  $L_p = KM$ . Then

$$\beta_{k,n} = (-1)^{kn} (-1)^{kK} e^{j\frac{\pi k}{M}} \quad (5)$$

- The output signal of SFBs  $[m]$  is complex-valued. we can express the discrete-time baseband signal at the output of the SFB of an FBMC transmitter based on OQAM modulation as

$$s(m) = \sum_{k=0}^{M-1} \sum_{n=-\infty}^{\infty} d_{k,n} \theta_{k,n} g_k(m - nM/2) \quad (6)$$

where

$$\theta_{k,n} = j^{(k+n)}$$

$$d_{k,n} = (-1)^{kn}$$

And

$M$  is number of subcarriers.

$d_{k,n}$  is the real-valued symbols at the  $k$ th subcarrier during the  $n$ th symbol interval.

$g_k(m)$  are shift-invariant impulse responses of the SFB channel filters.

### D. ANALYSIS FILTER BANK

In AFB a S/P converter will be consists of down samplers with  $M/2$  and delay chain,

Type-2 polyphase filters are expressed as  $b_k[m] = a_{M-1-k}[m] = p[M - 1 - k + mM]$ .

Then AFB output is

$$\hat{\beta}_{k,n} = (-1)^{kn} e^{(-j\frac{2\pi k}{m}(\frac{L+1}{2}))} \quad (7)$$

And because the length of the prototype filter is  $L = KM$ . Then

$$\begin{aligned} \hat{\beta}_{k,n} &= (-1)^{kn} (-1)^{kK} e^{(-j\frac{\pi k}{M})} \\ \hat{\beta}_{k,n} &= \beta_{k,n}^* \end{aligned} \quad (8)$$

here the extra delay is merged to S/P converter.

### E. OQAM POST-PROCESSING

In the post-processing operation there are 2 steps, firstly, the real part should be taken after multiplication by  $\theta_{m,n}^*$  sequence. The second operation is real-to-complex conversion, where two successive real-valued symbols (with one multiplied by  $j$ ) form a complex-valued symbol  $\hat{c}_{m,l}$ . This conversion decreases the sample rate by a factor 2 [3,8].

### F. LMS PER-SUBCARRIER EQUALIZATION

Sub-channel equalization in this paper is based in MSE criterion, and the LMS algorithm is used as adaptive equalizer, here a per-subcarrier equalizer



works at  $T/2$ , where  $T$  is the symbol duration. One FIR equalizer per subcarrier is used to mitigate ISI and ICI that results from the frequency selective channel and to improve the symbol decisions. The LMS equalizer is used in the minimization of the mean square error (MSE) between the desired output of the equalizer and the actual output equalizer.

In this paper we will apply the equalizer to the real and imaginary parts for each sub-carrier individually. The filter weights are updated as in the below equation. LMS can be calculated iteratively by

$$\hat{d}_k(n) = w_N^T(n) y_N(n) \quad (10)$$

$$e_k(n) = x_k(n) - \hat{d}_k(n) \quad (11)$$

$$w_N(n+1) = w_N(n) - \mu_u * e_k^*(n) y_N(n) \quad (12)$$

Now to compute the mean square error  $|e_k|^2$  at instance time  $k$

$$MSE = E[e_k^* e_k] \quad (13)$$

### III. SIMULATION RESULTS

Simulations illustrating the performance of sub-channel equalizers was evaluated by computer simulation as shown in the flowchart Below. Performance was tested with FBMC consisting  $M = 1024$  sub-channels and the overlapping factor  $K = 4$  based on OQAM modulation. Prototype filter designed with the sampling frequency technique ( $L = K * M$ ) coefficients.

In design the prototype filter we will use simple technique which is called frequency sampling technique, and it is presented with the following parameters

$$L = 4096, M = 1024, K = 4.$$

We start the design by determination of  $L$  desired values  $H(k/L); 0 \leq k \leq L-1$  in the frequency domain by

$$\begin{aligned} H(0) &= 1 \\ H(1/L) &= 0.97195983; \\ H(2/L) &= \sqrt{2}/2; \\ H(3/L) &= 0.23514695; \\ H\left(\frac{k}{L}\right) &= 0. \text{ for } 4 \leq k \leq L-1 \end{aligned}$$

Then, the prototype filter coefficients are obtained by inverse DFT as

$$h(m) = 1 + 2 \sum_{k=1}^{K-1} (-1)^k H(k/L) \cos(2\pi km/L). \quad \text{for } 1 \leq m \leq L-1$$

where

$$h(0) = 0$$

Finally, The convergence characteristics of LMS algorithm were performed. In the simulation results  $N$  represents the equalizer taps and  $\mu_u$  represents the step size of the equalizer.

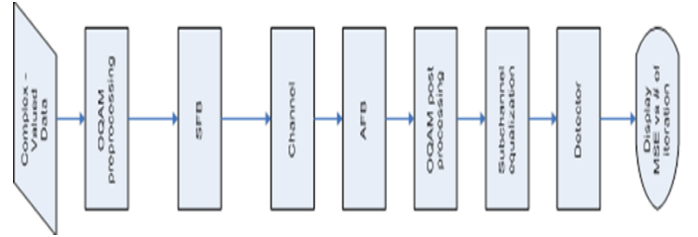


Figure 4: Flowchart of Subchannel equalization based on FBMC (Read Left – To- Right)

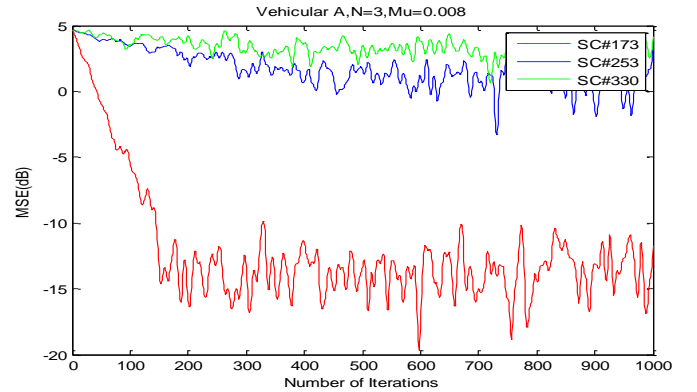


Figure 5 : Comparison of MSE vs Number of iteration for different subcarriers,  $\mu_u=0.008$

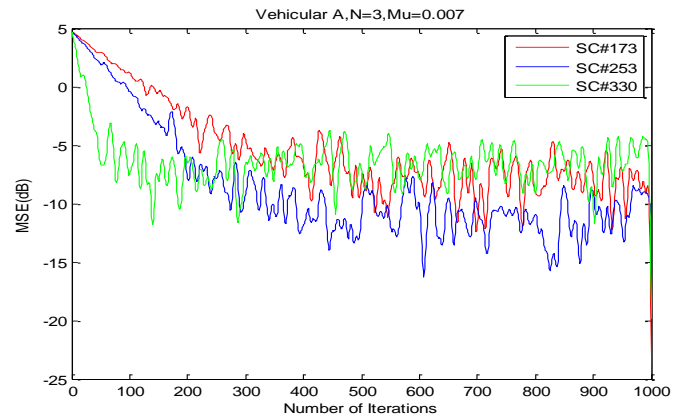


Figure 6 : Comparison of MSE vs Number of iteration for different subcarriers,  $\mu_u=0.007$



Figures 5 and 6 illustrates the comparison in the convergence characteristic curves of LMS algorithm of vehicular A channel models for subcarrier numbers 173, 263 and 330. With different step size values, 3 tap equalizer and 12 dB SNR.

We can see that for subcarrier number 173 with step size 0.008 converges faster to -15 dB than with step size  $M_u = 0.007$  which converges to -10 dB, here we can say that the subcarrier number 173 with  $M_u = 0.008$  is doing well more than  $M_u = 0.007$ . and it is clear for subcarrier number 330 with step size 0.007 converges faster to -10 dB than with step size  $M_u = 0.008$  which converges to 4 dB, here we can say that the subcarrier number 330 with  $M_u = 0.007$  is doing well more than  $M_u = 0.008$ .

we can notice that the same subcarrier will have a good characteristic curve at certain step size value and a bad characteristic curve at another step size. At the same time another subcarrier will be in opposite way with the same step size value in the first subcarrier. And that is because of frequency selectivity which give each subcarrier different correlation matrix which should the step size to be taken.

#### IV. CONCLUSION

Adaptive channel equalization algorithms for FBMC systems with offset QAM modulation have been designed and evaluated in this paper. Due to the simplicity of LMS, we have designed an adaptive LMS algorithm to optimize the meansquared error (MSE) criterion where LMS per-subchannel equalizer operates as fractionally spaced (T/2) equalizer which aims to avoid irrevocable aliasing of the subchannels. Also, we have optimized and evaluated the convergence curve and complexity of LMS equalization algorithms as well as applied the equalizer to ITU channel model VEH A with different numbers of equalizer step sizes and subcarriers. The simulation results showed that because of frequency selectivity, every sub-channel will have a different optimum LMS equalizer step sizes based on the correlation matrix to every sub-channel.

#### V. REFERENCES

- 1) T. Ihalainen, Y. Yang and M. Renfors, "Filter Bank Based Frequency-Domain Equalizers with Diversity Combining" *IEEE International Symposium on Circuits and Systems*, pp. 93-96, 2008.
- 2) P. Siohan, C. Siclet, and N. Lacaille, "Analysis and Design of OFDM/OQAM Systems Based on Filterbank Theory" *IEEE TRANSACTIONS ON SIGNAL PROCESSING*, VOL. 50, NO. 5, pp. 1170-1183, MAY 2002.
- 3) T. Ihalainen et. al., "Channel equalization in filter bank based multicarrier modulation for wireless communications" *EURASIP Journal on Advances in Signal Process.*, 2007.
- 4) Y.-P. Lin and S.-M. Phoong, "ISI-free FIR filterbank transceivers for frequency-selective channels," *IEEE Trans. Signal Process.* Nov. 2001; 49(11): 2648-2658.
- 5) P. Vaidyanathan, Y.-P. Lin, S. Akkarakaran, and S.-M. Phoong, "Discrete multitone modulation with principal component filter banks," *IEEE Trans. Circuits Syst.* Oct. 2002; 49(10): 1397-1412.
- 6) M. A. Tzannes, M. C. Tzannes, J. Proakis, and P. N. Heller, "DMT systems, DWT systems and digital filter banks," in *Proc. IEEE Int. Conf. on Communications*, May 1994; 311-315.
- 7) H. Zhang, D. LeRuyet, D. Roviras, Y. Medjahdi, and H. Sun, "Spectral Efficiency Comparison of OFDM/FBMC for Uplink Cognitive Radio Networks," *EURASIP Journal on Advances in Signal Processing*, 2010; 14.
- 8) M. Shaat and F. Bader, "Computationally efficient power allocation algorithm in multicarrier-based cognitive radio networks: OFDM and FBMC systems," *EURASIP Journal on Advances in Signal Processing*, 2010; 13.
- 9) Y. Yang, T. Hidalgo Stitz, M. Rinne, and M. Renfors, "Mitigation of narrowband interference in single-carrier transmission with filter bank equalization," in *Proc. 2006 IEEE Asia Pacific Conference on Circuits and Systems*, Singapore, Dec. 2006; 749-752.
- 10) P. P. Vaidyanathan, "Filter Banks in Digital Communications" *IEEE Transactions on Circuits and Systems* 2001; 1(2): 4-25.
- 11) Y. Yang, T. Ihalainen, M. Rinne, and M. Renfors, "Frequency-domain equalization in single-carrier transmission: Filter bank approach," *EURASIP Journal on Advances in Signal Processing*, 2007; 16 pages.
- 12) T. Stitz, T. Ihalainen, A. Viholainen, and M. Renfors, "Pilot-Based Synchronization and Equalization in Filter Bank Multicarrier Communications," *EURASIP Journal on Advances in Signal Processing*, 2010; 18.



- 13) B. Hirosaki, "An orthogonally multiplexed QAM system using the discrete fouriertransform," *IEEE Transactions on Communications*, July 1981; 29(7): 982–989.
- 14) T. Karp and N. J. Fliege, "Computationally efficient realization of MDFT filterbanks," in *Proc. 8th European Signal Process. Conf.*, September 1996; 2: 1183–1186.
- 15) T. Karp and N. J. Fliege, "Modified DFT filter banks with perfect reconstruction," *IEEE Transactions on Circuits and Systems—Part II: Analog and Digital SignalProcessing*, Nov. 1999; 46(11): 1404–1414.