Nonlinear Composite Adaptive Control for Quadrotor

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ABSTRACT

A nonlinear composite adaptive control algorithm is done for a 6-DOF quadrotor system. The under-actuated system is divided to two subsystems using dynamic inversion. Sliding mode control is controlling the internal dynamics while the adaptive control is controlling the fully actuated subsystem. The plant parameters such as mass, system inertia, thrust and drag factors are considered as fully unknown and vary with time. The composite adaptive control is driven using the information from both the tracking error and the parameter error. The stability of the closed-loop system is shown in the flight region of interest. Also the performance of the proposed controller is illustrated to follow a desired position, velocity, acceleration and the heading angle of quadrotor despite the fully unknown parameters.

KEYWORDS

Nonlinear Quadrotor Control, Under-Actuated system, Composite Adaptive Control, Unknown Parameters

1 INTRODUCTION

Many different dynamic systems to be controlled have unknown or slowly varying parameters. Therefore, unless such parameter uncertainty reduced it may cause inaccuracy or instability for control system. For instance, firefighting air craft may experience considerable mass changes as they load or unload large quantities of water. One way to reduce the parameter uncertainty is to use parameter estimation technique where it could be done either online or offline. Offline estimation is more suited if the parameter is constant and there is time to estimate the parameter before controlling the system. Still, for time varying parameter online parameter estimation is more efficient to keep track of the parameter value.

Adaptive control is an admirable candidate for this type of systems because of its capability of tracking a desired output signal with the presence of parametric uncertainties. Adaptive control basic idea is to estimate the uncertainty plant parameters online using the inputs and the outputs signals of the system, and then use these parameters in the control input computation. There are two main approaches for constructing adaptive controllers. In this work, a so called model reference adaptive control (MRAC) method is used. The convergence of the parameters to the exact value depends on the richness of the input signal. Still the adaptive control will track the desired output signal despite it convergences to the exact parameter or not.

A lot of interest in developing a control algorithm for quadrotor has been grown lately; this is because of its low cost, the high ability for maneuver and vertically teak off and landing which make it very popular as a research platform especially for indoor applications.

For adaptive control scheme, there are different parameters can be estimated. Many proposed methods estimated only mass while the other estimated mass and inertia matrix. However, very few researches took other parameters into accounts. For estimating mass, a backstepping approach has been used in [1] while an improvement using adaptive integral backstepping has been done in [2]. An adaptive robust control has been proposed in [3] and a model reference adaptive control in [4]. However for mass and inertia estimating, a comparison between model reference adaptive control and model identification adaptive control has been done in [5]. A Lyapunov-based robust adaptive control has been used in [6], [7] and [8]. Also in [9], a composite adaptive controller has been shown. In addition, estimating extra parameters such as aerodynamic coefficients can be shown first in [10] using Lyapunov-based robust adaptive control also in [11] using adaptive sliding mode control and last in [12] using adaptive integral backstepping method.

Quadrotor is considered as under actuated subsystem; it has more degree of freedom than number of actuators. Hence, the introduced control method will divide the whole system to two sub systems. The first subsystem will control the internal dynamics using sliding mode control. However, the second subsystem, which is a full actuated system, will be controlled using an adaptive control; to remove the effects of the parameter uncertainty while tracking the desired output.

This research paper is organized as follows. In Section 2, the problem statement is described. The dynamic equation of the quadrotor model and the reference frames are introduced in the next section 3. The proposed adaptive control scheme is fully described in Section 4. The validation of the proposed control is done using Simulation in last section.

2 PROBLEM DEFINITION

Quadrotor, like any highly nonlinear system, suffers from parameters uncertainty; either totally unknown or vary with time. There are two main ways to control such systems; either by using robust controller or by using adaptive control. Adaptive control deals with parametric uncertainty without having any prior information about the parameter.

In this paper, an nonlinear control of a 6-DOF quadrotor to follow a desired position, velocity, acceleration and a heading angle despite the parameter uncertainty is aimed. In addition, plant parameters such as mass, system inertia, thrust and drag factors are considered as fully unknown and vary with time.

The quadrotor is considered as under-actuated system. Thus, to control it in 6DOF, a dynamic inversion technique to split the system into two sub systems is been done. Using four command signals, as input signals of quadrotor, the other two commands are driven.

3 QUADROTOR MODEL

3.1 Reference Frame:

Two reference frames has been defined to describe the motion of a 6 DOF rigid body; Earth inertial reference (E-frame) and Body-fixed reference (B-frame).

The earth-frame is an inertial right-hand reference denoted by (o_E, x_E, y_E, z_E). Using the E-frame, the linear position ($\boldsymbol{\Gamma^E}$ [m]) and the Euler angles ($\boldsymbol{\Theta^E}$ [rad]) of the quadrotor has been defined.

$$\boldsymbol{\Gamma^E} = [x \quad y \quad z]^T, \qquad \boldsymbol{\Theta^E} = [\phi \quad \theta \quad \psi]^T$$

Where x, y and z are the position of the center of the gravity of the quadrotor in E-frame and ϕ, θ and ψ are the Euler angles represent the roll, pitch and yaw in E-frame respectively.

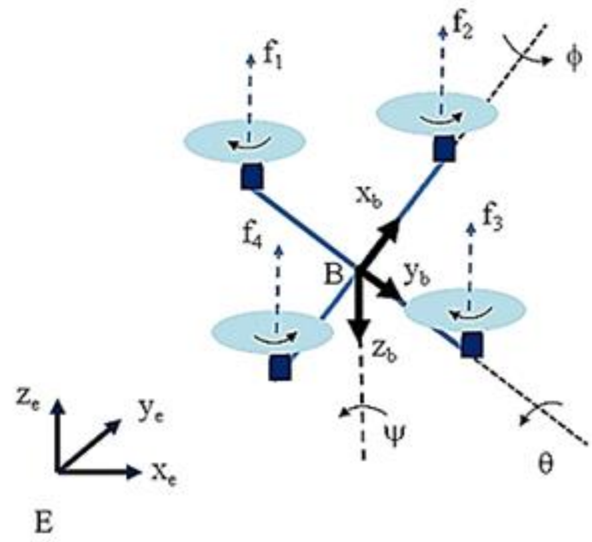

Figure 1: Reference Frame

The body-frame is a right-hand reference denoted by (o_B, x_B, y_B, z_B). This frame is attached to the body of the quadrotor. Using the B-frame, the torques (τ^B [Nm]) and the forces (F^B [N]) has been defined. See Figure 1.

A rotation matrix is needed to map the orientation of a vector from B-frame to E-frame. It is described as follows:

$$\boldsymbol{R_\theta} = \begin{bmatrix} c_\psi c_\theta & s_\psi c_\theta & -s_\theta \\ -s_\psi c_\phi + c_\psi s_\theta s_\phi & c_\psi c_\phi + s_\psi s_\theta s_\phi & c_\theta s_\phi \\ s_\psi s_\phi + c_\psi s_\theta c_\phi & -c_\psi s_\phi + s_\psi s_\theta c_\phi & c_\theta c_\phi \end{bmatrix}$$

Where c_x means $\cos(x)$ and s_x means $\sin(x)$.

3.2 Dynamic Equation:

Using the rotation matrix to map the forces and the torques from the body frame to the earth frame and by using Euler-Lagrange approach, the dynamic equation of the quadrotor is driven and described as following:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{k}{m} \begin{bmatrix} s_\psi s_\phi + c_\psi s_\theta c_\phi \\ -c_\psi s_\phi + s_\psi s_\theta c_\phi \\ c_\theta c_\phi \end{bmatrix} u - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \left(\frac{I_y - I_z}{I_x}\right) \dot{\theta}\dot{\psi} \\ \left(\frac{I_z - I_y}{I_y}\right) \dot{\phi}\dot{\psi} \\ \left(\frac{I_x - I_y}{I_z}\right) \dot{\phi}\dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{J_p}{I_x} \dot{\theta} \\ \frac{J_p}{I_x} \dot{\phi} \\ 0 \end{bmatrix} \Omega + \begin{bmatrix} \frac{k}{I_x} & 0 & 0 \\ 0 & \frac{k}{I_y} & 0 \\ 0 & 0 & \frac{d}{I_z} \end{bmatrix} \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \quad (2)$$

Where the inputs are defined as:

$$\begin{bmatrix} u \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -l & 0 & l & 0 \\ 0 & l & 0 & -l \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \quad (3)$$

$$f_i = w_i^2$$

$$\Omega = w_1 - w_2 + w_3 - w_4$$

where m is the quadrotor mass in [kg], I_x, I_y and I_z are the quadrotor inertia matrix around x y and z axis respectively in [N m s^2], w_i is the speed of the ith motor in [rad/s], Ω is the overall propellers' speed in [rad/s], J_p is the total rotational moment of inertia around the propeller axis in [N m s2], k is the thrust factor in [N.s^2], d is the drag factor in [N m s^2], l is the distance between the center of the quadrotor and the center of a propeller in [m] and $[u, \tau_\phi, \tau_\theta, \tau_\psi]^T$ are the inputs of the quadrotor representing the collective force, roll torque, pitch torque and yaw torque respectively.

4 CONTROL SCHEME

The quadrotor system is considered as under-actuated system; it has got 6-DOF and four actuators. To achieve tracking control for the desired command $[x_c, y_c, z_c, \psi_c]$, a dynamic inversion technique is been used to divide the system into two subsystems [13].

The first subsystem is the internal dynamics that have been defined using the feedback linearization on the system and it is given by:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \frac{k}{m} \begin{bmatrix} s_\psi s_\phi + c_\psi s_\theta c_\phi \\ -c_\psi s_\phi + s_\psi s_\theta c_\phi \end{bmatrix} u \quad (4)$$

A sliding mode control is been used to control the second subsystems and to generate the command $[\phi_c, \theta_c]$.

The second subsystem is considered as a full actuated system and it is defined as:

$$\begin{bmatrix} \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} -g \\ \left(\frac{I_y - I_z}{I_x}\right) \dot{\theta}\dot{\psi} \\ \left(\frac{I_z - I_x}{I_y}\right) \dot{\phi}\dot{\psi} \\ \left(\frac{I_x - I_y}{I_z}\right) \dot{\phi}\dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{k}{m} & 0 & 0 & 0 \\ 0 & \frac{k}{I_x} & 0 & 0 \\ 0 & 0 & \frac{k}{I_y} & 0 \\ 0 & 0 & 0 & \frac{d}{I_z} \end{bmatrix} \begin{bmatrix} u_z \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} \quad (5)$$

In the second subsystem, an adaptive control is been developed to overcome the unknown parameters. Furthermore, the adaptive control is designed to achieve attitude and the altitude control of the quadrotor $[z, \phi, \theta, \psi]$. Additionally, A composite adaptive controller is been introduce to improve the parameters estimation and the smoothness of the estimation. The below block diagram in Figure 2 shows the overall control scheme.

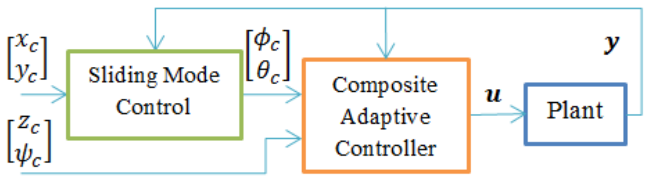

Figure 2: Control Scheme Block Diagram

4.1 Sliding Mode Control for Internal Dynamics:

Stabilizing the internal dynamics is essential to guarantee the stability of dynamic inversion technique. Thus, a proper command ϕ_c and θ_c for the roll and pitch respectively is been selected such that the tracking control for x_c and y_c is been achieved. As a result, the internal dynamics will be grantee to be stable. Figure 3 shows the block diagram of the sliding control.

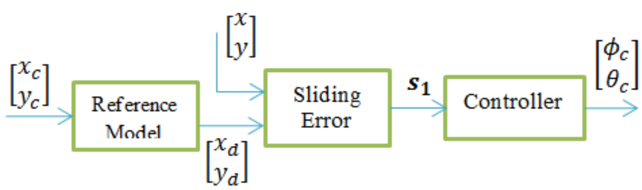

Figure 3:Sliding Control for the Internal Dynamics Block Diagram

Let us define the states of the first subsystem as:

$$\boldsymbol{q_1} = [x \quad y]^{\boldsymbol{T}}$$

Rewrite internal dynamics equation in (4) as:

$$\ddot{\boldsymbol{q}}_{\boldsymbol{1}} = \frac{\bar{k}}{m} u\, \boldsymbol{\Psi} \begin{bmatrix} s_\phi \\ s_\theta \end{bmatrix} \qquad (6)$$

$$\boldsymbol{\Psi} = \begin{bmatrix} s_\psi & c_\psi c_\phi \\ -c_\psi & s_\psi c_\phi \end{bmatrix}$$

Where the yaw angle (ψ) and the roll angle (ϕ) in the matrix $\boldsymbol{\Psi}$ are the current angles of the system.

4.1.1 Reference Model:

Let the desired trajectory for the first subsystem been defined using the following reference model in state space form:

$$\begin{bmatrix} \ddot{q}_{1_d} \\ \dot{q}_{1_d} \end{bmatrix} = \boldsymbol{A}_{m1} \begin{bmatrix} \dot{q}_{1_d} \\ q_{1_d} \end{bmatrix} + \boldsymbol{B}_{m1}\, \boldsymbol{q_{1_c}}$$

$$\boldsymbol{q_{1_c}} = [x_c, y_c]^T$$

where $\mathbf{A}_{m1}$ and $\boldsymbol{B}_{m1}$ represent the desired system performance. Also x_c and y_c represent the command inputs of the first subsystem.

4.1.2 Tracking and Sliding Mode Error:

Define the tracking error on XY-plane as:

$$\boldsymbol{e_1} = q_{1_d} - q_1 = \begin{bmatrix} e_x \\ e_y \end{bmatrix} = \begin{bmatrix} x_d - x \\ y_d - y \end{bmatrix}$$

The associated sliding mode error is defined as:

$$\boldsymbol{s_1} = \dot{\boldsymbol{e}}_{\boldsymbol{1}} + \boldsymbol{\Lambda_1 e_1}$$

$$\boldsymbol{s_1} = \begin{bmatrix} s_x \\ s_y \end{bmatrix} = \begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} + \begin{bmatrix} \lambda_x\, e_x \\ \lambda_y\, e_y \end{bmatrix}$$

$$\boldsymbol{\Lambda_1} = diag([\lambda_x \quad \lambda_y])$$

Where λ_x and λ_y are design positive constant and represent a stable Hurwitz polynomial.

4.1.3 Control Law:

Using the dynamic equation (1), a relation between the input force (u) and the gravitational acceleration (g) could be made to simplify the control law. Recall from the dynamic equation of the z-axis:

$$\ddot{z} = \frac{k}{m}\, u\, \cos(\phi) \cos(\theta) \;-\; g$$

At hovering state, the acceleration in z-axis equal to zero:

$$\ddot{z} = 0 \quad \rightarrow \quad \frac{\mathrm{k}}{m}\, u\, \cos(\phi)\cos(\theta) \;=\; g$$

$$\frac{k}{m}\, u = \frac{g}{\cos(\phi)\cos(\theta)} = G \qquad (7)$$

The control law is defined in the form of:

$$\begin{bmatrix}\phi_c \\ \theta_c\end{bmatrix} = sin^{-1}\left(\boldsymbol{\Psi}^{-1}.\left[\ddot{q}_{1_d} - \boldsymbol{\Lambda_1}\dot{\boldsymbol{e}}_1 - \boldsymbol{K_1 s_1}\right]/G\right) \quad (8)$$

$$\boldsymbol{K_1} = diag(k_x, k_y)$$

Where k_x and k_y are design positive constants and represent a stable Hurwitz polynomial. Substitute the control law in equation (6) yields:

$$\ddot{\boldsymbol{q}}_1 = \ddot{\boldsymbol{q}}_{1_d} - \boldsymbol{\Lambda_1}\dot{\boldsymbol{e}}_1 - \boldsymbol{K_1 s_1}$$

$$\dot{\boldsymbol{s}}_1 + \boldsymbol{K_1}\,\boldsymbol{s_1} = 0 \quad (9)$$

Therefore, gives exponential convergence for $\boldsymbol{s_1}$ which guarantees the convergence of XY-plane tracking error $(\boldsymbol{e_1})$.

4.2 Direct Adaptive Controller

The objective of using the adaptive control is to make the output asymptotically tracks the desired output $\boldsymbol{q}_{2_d}(t)$ despite the parameter uncertainty. The adaptive control is been applied for the second subsystem to guarantee the convergence of the attitude and altitude tracking $[z, \phi, \theta, \psi]$. See Figure 4.

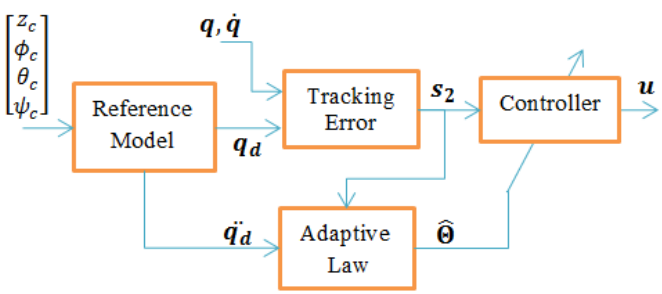

Figure 4: Adaptive Control Block Diagram

Adaptive control of nonlinear systems has been well studied in [16],[15] and [16].

4.2.1 Parameterization:

Define the states for the second subsystem as:

$$\boldsymbol{q_2} = [z \quad \phi \quad \theta \quad \psi]^{\boldsymbol{T}}$$

Rewrite (5) in linear-in-parameter form:

$$\boldsymbol{\Phi\Theta} = \boldsymbol{u} \quad (10)$$

where:

$$\boldsymbol{\Theta} = \begin{bmatrix}\boldsymbol{\Theta_1} \\ \boldsymbol{\Theta_2}\end{bmatrix}, \qquad \boldsymbol{\Theta_1} = \begin{bmatrix} m/\bar{k} \\ I_x/\bar{k} \\ I_y/\bar{k} \\ I_z/d \end{bmatrix}$$

$$\boldsymbol{\Theta_2} = \begin{bmatrix} I_z/\bar{k} \\ (I_x - I_y)/d \end{bmatrix}$$

$$\boldsymbol{u} = [u \quad \tau_\phi \quad \tau_\theta \quad \tau_\psi]^T$$

$$\boldsymbol{\Phi} = \boldsymbol{Q}(\ddot{\boldsymbol{q}}_2) + \boldsymbol{F}(f_\phi, f_\theta, f_\psi, g)$$

$$\boldsymbol{Q} = \begin{bmatrix} \ddot{z} & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddot{\phi} & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddot{\theta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddot{\psi} & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{F}(f_\phi, f_\theta, f_\psi, g) = \begin{bmatrix} g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -f_\phi & 0 & f_\phi & 0 \\ 0 & f_\theta & 0 & 0 & -f_\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & -f_\psi \end{bmatrix}$$

$$f_\phi = \dot{\theta}\dot{\psi}, \qquad f_\theta = \dot{\phi}\dot{\psi}, \qquad f_\psi = \dot{\phi}\dot{\theta}$$

4.2.2 Reference model

Similar to the first subsystem, let the desired trajectory for the second subsystem been defined using the following reference model in state space form:

$$\begin{bmatrix}\ddot{\boldsymbol{q}}_{2_d} \\ \dot{\boldsymbol{q}}_{2_d}\end{bmatrix} = \boldsymbol{A}_{m2}\begin{bmatrix}\dot{\boldsymbol{q}}_{2_d} \\ \boldsymbol{q}_{2_d}\end{bmatrix} + \boldsymbol{B}_{m2}\,\boldsymbol{q}_{2_c}$$

$$\boldsymbol{q}_{2_c} = [z_c, \phi_c, \theta_c, \psi_c]^T$$

where $\mathbf{A}_{m2}$ and $\boldsymbol{B}_{m2}$ represent the desired system performance Also $[z_c, \phi_c, \theta_c, \psi_c]$ represent the command inputs of the second subsystem.

4.2.3 Tracking and Sliding Mode Error:

Defining tracking error for adaptive system as:

$$\boldsymbol{e_2} = \boldsymbol{q_2} - \boldsymbol{q_{2_d}} = \begin{bmatrix} e_z \\ e_\phi \\ e_\theta \\ e_\psi \end{bmatrix} = \begin{bmatrix} z \\ \phi \\ \theta \\ \psi \end{bmatrix} - \begin{bmatrix} z_d \\ \phi_d \\ \theta_d \\ \psi_d \end{bmatrix}$$

And the associated sliding error as:

$$\boldsymbol{s_2} = \begin{bmatrix} s_z \\ s_\phi \\ s_\theta \\ s_\psi \end{bmatrix} = \begin{bmatrix} \dot{e}_z \\ \dot{e}_\phi \\ \dot{e}_\theta \\ \dot{e}_\psi \end{bmatrix} + \begin{bmatrix} \lambda_z e_z \\ \lambda_\phi e_\phi \\ \lambda_\theta e_\theta \\ \lambda_\psi e_\psi \end{bmatrix}$$

$$\boldsymbol{s_2} = \ \dot{\boldsymbol{e}}_2 + \boldsymbol{\Lambda_2 e_2}$$

$$\dot{\boldsymbol{s}}_2 = \ddot{\boldsymbol{e}}_2 + \boldsymbol{\Lambda_2} \dot{\boldsymbol{e}}_2 = \ddot{\boldsymbol{q}}_2 - \boldsymbol{q_r}$$

$$\boldsymbol{q_r} = \ddot{\boldsymbol{q}}_{2_d} - \boldsymbol{\Lambda_2}\, \dot{\boldsymbol{e}}_2$$

where $\boldsymbol{\Lambda_2}$ is positive definite diagonal matrix defined as:

$$\boldsymbol{\Lambda_2} = diag([\lambda_z \quad \lambda_\phi \quad \lambda_\theta \quad \lambda_\psi])$$

And $[\lambda_z \quad \lambda_\phi \quad \lambda_\theta \quad \lambda_\psi]$ are design positive constant and represent a stable Hurwitz polynomial.

4.2.4 Control Law:

Define the control law as:

$$\boldsymbol{u} = \ \bar{\boldsymbol{\Phi}}\hat{\boldsymbol{\Theta}} - \boldsymbol{K_2 s_2} \qquad (11)$$

where:

$$\bar{\boldsymbol{\Phi}} = \ \boldsymbol{Q}(\boldsymbol{q_r}) + \boldsymbol{F}(f_\phi, f_\theta, f_\psi, g)$$

$$\boldsymbol{K_2} = diag(k_z, k_\phi, k_\theta, k_\psi)$$

where k_z, k_ϕ, k_θ and k_ψ are design positive control gains. Substituting the control law (11) in equation (5) yields:

$$\boldsymbol{\Phi\Theta} = \ \bar{\boldsymbol{\Phi}}\hat{\boldsymbol{\Theta}} - \boldsymbol{K_2 s_2}$$

$$\boldsymbol{\Phi\Theta} = \ \bar{\boldsymbol{\Phi}}\hat{\boldsymbol{\Theta}} - \boldsymbol{K_2 s_2} \pm \bar{\boldsymbol{\Phi}}\boldsymbol{\Theta}$$

$$(\boldsymbol{\Phi} - \ \bar{\boldsymbol{\Phi}})\boldsymbol{\Theta} + \boldsymbol{K_2 s_2} = \bar{\boldsymbol{\Phi}}(\hat{\boldsymbol{\Theta}} - \ \boldsymbol{\Theta}) \qquad (12)$$

Using:

$$\boldsymbol{\Phi} - \ \bar{\boldsymbol{\Phi}} = \boldsymbol{Q}(\ddot{\boldsymbol{q}}) - \boldsymbol{Q}(\boldsymbol{q_r}) = \boldsymbol{Q}(\dot{\boldsymbol{s}}_2)$$

Rewrite (12) using the following simplification:

$$\boldsymbol{Q}(\dot{\boldsymbol{s}}_2)\boldsymbol{\Theta} = \ diag(\boldsymbol{\Theta_1})\dot{\boldsymbol{s}}_2$$

This yield:

$$diag(\Theta_1)\, \dot{\boldsymbol{s}}_2 \ + \boldsymbol{K_2}\, \boldsymbol{s_2} = \bar{\Phi}\tilde{\Theta} \qquad (13)$$

where $\tilde{\Theta}$ is the parameter error defined by:

$$\tilde{\boldsymbol{\Theta}} = \ \hat{\boldsymbol{\Theta}} - \ \boldsymbol{\Theta}$$

Therefore, $\boldsymbol{s_2}$ **c**onverge exponentially to the region defined by **($\tilde{\boldsymbol{\Theta}}$)**.

4.2.5 Adaptation Law:

Define the Lyapunov function as:

$$V\left(\boldsymbol{s_1}, \boldsymbol{s_2}, \tilde{\boldsymbol{\Theta}}\right) = \tfrac{1}{2}\left[\boldsymbol{s_1^T H s_1} + \boldsymbol{s_2^T} diag(\boldsymbol{\Theta_1})\boldsymbol{s_2} + \tilde{\boldsymbol{\Theta}}^T \boldsymbol{P}^{-1}\tilde{\boldsymbol{\Theta}}\right] \qquad (14)$$

where **H**, diag($\boldsymbol{\Theta_1}$) and **P** are symmetric positive definite matrix. The derivative of the Lyapunov function is calculated as:

$$\dot{V} = \boldsymbol{s_1}^{\boldsymbol{T}}\boldsymbol{H}\dot{\boldsymbol{s}}_1 + \ \boldsymbol{s_2}^{\boldsymbol{T}} diag(\boldsymbol{\Theta_1})\, \dot{\boldsymbol{s}}_2 \ + \dot{\tilde{\boldsymbol{\Theta}}}^{\,T}\, \boldsymbol{P}^{-1}\, \tilde{\boldsymbol{\Theta}}$$

Using the equation (9) and (13) yields:

$$\dot{V} = \ -\boldsymbol{s_1}^T\boldsymbol{K_1 s_1} - \boldsymbol{s_2}^T\boldsymbol{K_2 s_2} + \boldsymbol{s_2}^T\bar{\boldsymbol{\Phi}}\tilde{\boldsymbol{\Theta}} + \dot{\tilde{\boldsymbol{\Theta}}}^{\,T}\boldsymbol{P}^{-1}\tilde{\boldsymbol{\Theta}} \qquad (15)$$

Assuming constants or small time varying parameters ($\boldsymbol{\Theta}$):

$$\dot{\tilde{\boldsymbol{\Theta}}} \ = \dot{\hat{\boldsymbol{\Theta}}} - \dot{\boldsymbol{\Theta}} = \dot{\hat{\boldsymbol{\Theta}}}$$

Choosing the adaptation law as:

$$\left(\ \boldsymbol{s_2}^T\bar{\boldsymbol{\Phi}} + \ \dot{\hat{\boldsymbol{\Theta}}}^T\ \boldsymbol{P}^{-1}\right)\, \tilde{\boldsymbol{\Theta}} = 0$$

$$\dot{\hat{\boldsymbol{\Theta}}} = -\,\boldsymbol{P}\bar{\boldsymbol{\Phi}}^T\boldsymbol{s_2} \qquad (16)$$

Therefore, the derivative of the Lyapunov function (15) becomes:

$$\dot{V} = -\,\boldsymbol{s_1}^T\boldsymbol{K_1}\,\boldsymbol{s_1} - \boldsymbol{s_2}^T\boldsymbol{K_2}\,\boldsymbol{s_2} \qquad (17)$$

$$\dot{V} < 0$$

$$\boldsymbol{s_1}, \boldsymbol{s_2} \to 0 \Rightarrow e_1, e_2 \to 0 \; as \; t \to \infty$$

Since $\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are positive definite diagonal matrices, the above expression guaranteed global stability and global tracking convergence of both the sliding control and adaptive control systems.

4.3 Composite Adaptive Control

In the previous section, the information about the estimated parameters are been extract from the tracking error $(\boldsymbol{e_2})$. However, there is another source could be used to extract the information about the estimated parameters. Prediction error on the inputs also contains parameters information. Figure 5 shows the block diagram of the composite adaptive control [17].

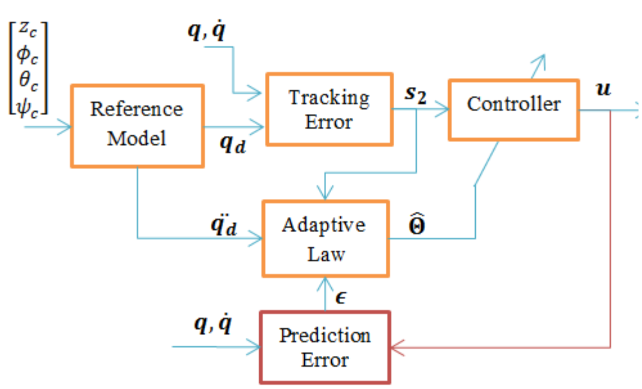

Figure 5: Composite adaptive control block diagram

4.3.1 Prediction Error:

The prediction error is defined as the difference between the actual input and the estimated control

$$\boldsymbol{\varepsilon} = \hat{\boldsymbol{u}} - \boldsymbol{u} = \boldsymbol{\Phi}\hat{\boldsymbol{\Theta}} - \boldsymbol{\Phi}\boldsymbol{\Theta} \qquad (18)$$

However, the presence of immeasurable acceleration in $\boldsymbol{\Phi}$ will prevent us from using this definition for the prediction error $(\boldsymbol{\varepsilon})$ directly for the parameter estimation. To avoid the appearance of the acceleration in this relation, let us use a proper first order filter as follows:

$$\left[\frac{\lambda_f}{s+\lambda_f}\right]\boldsymbol{\Phi}(\ddot{\boldsymbol{q}}, f_\phi, f_\theta, f_\psi, g) = \boldsymbol{W}(\dot{\boldsymbol{q}}, f_\phi, f_\theta, f_\psi, g)$$

$$\lambda_f\left[\boldsymbol{Q}(\dot{\boldsymbol{q}}) - \frac{\lambda_f}{s+\lambda_f}\boldsymbol{Q}(\dot{\boldsymbol{q}}) + \frac{1}{s+\lambda_f}\boldsymbol{F}\right] = \boldsymbol{W} \qquad (19)$$

where $\frac{\lambda_f}{s+\lambda_f}$ is stable filter with $\lambda_f > 0$.

Now, let us define a filtered prediction error $(\boldsymbol{\epsilon})$ as:

$$\boldsymbol{\epsilon} = \boldsymbol{W}\tilde{\boldsymbol{\Theta}} = \boldsymbol{W}\hat{\boldsymbol{\Theta}} - \boldsymbol{u}_1 \qquad (20)$$

where $\mathbf{u_1}$ is the filtered version of the actual input, defined as:

$$\boldsymbol{u_1} = \left[\frac{\lambda_f}{s+\lambda_f}\right]\boldsymbol{u} \qquad (21)$$

4.3.2 Adaptation Law:

The main idea of using the composite adaptive control, is to develop an adaptive law that is driven by the information from both errors, tracking error $(\mathbf{e_2})$ and prediction error $(\boldsymbol{\epsilon})$, without changing the control law.

Choosing the composite adaptation law as:

$$\dot{\hat{\boldsymbol{\Theta}}} = -\,\boldsymbol{P}[\bar{\boldsymbol{\Phi}}^T\boldsymbol{s_2} + \boldsymbol{W^T}\boldsymbol{R}(\boldsymbol{t})\boldsymbol{\epsilon}] \qquad (22)$$

where $\boldsymbol{R}(\boldsymbol{t})$ is a uniformly positive definite weighting matrix. This Matrix is used to indicate how much attention should the composite adaptation law pay for the parameter information that comes from the prediction error $(\boldsymbol{\epsilon})$. It is defined as:

$$\boldsymbol{R}(\boldsymbol{t}) = I_{6x6}\left(\boldsymbol{W^T}\boldsymbol{W} + I_{6x6}\right)^{-1} \qquad (23)$$

$$\boldsymbol{R}(\boldsymbol{t}) > 0$$

Substitute the new adaptation law in (17). The derivative of the Lyapunov function becomes:

$$\dot{V} = -\boldsymbol{s_1}^T\boldsymbol{K_1}\boldsymbol{s_1} - \boldsymbol{s_2}^T\boldsymbol{K_2}\boldsymbol{s_2} - \boldsymbol{\epsilon}^T\boldsymbol{R}(\boldsymbol{t})\boldsymbol{\epsilon} \qquad (24)$$

$$\dot{V} < 0$$

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$$\boldsymbol{s_1}, \boldsymbol{s_2}, \boldsymbol{\epsilon} \to 0 \Rightarrow e_1, e_2 \to 0 \; as \; t \to \infty$$

The above expression guaranteed global stability and global tracking convergence of both the sliding control and adaptive control systems.

5 SIMULATION RESULTS

Simulations are used to validate the proposed control algorithm and to illustrate the difference in behavior between the direct and the composite adaptive controller. Figure 6 shows the used simulink model.

In the simulation, time varying plant parameters are considered. The plant parameters are generated using square wave function which is defined as:

$$Squre(freq, Amplitude, \text{Offset}, DutyCycle\%)$$

The plant parameters have a common duty cycle of 50% and a common frequency of $5\text{x}10^{-4}$ Hz. However each parameter has got different amplitude and offset which are shown in Table 1.

Table 1: Plant Parameters

Time Varying Parameter	Frequency in Hz	Offset	Amplitude
m	$5\text{x}10^{-4}$	1.5	1
I_x	$5\text{x}10^{-4}$	0.01	0.005
I_y	$5\text{x}10^{-4}$	0.0125	0.005
I_z	$5\text{x}10^{-4}$	0.02	0.01
$\bar{k}$	$5\text{x}10^{-4}$	0.1	0.09
d	$5\text{x}10^{-4}$	0.002	0.001
Fixed Parameter	Value		
l	0.03		
J_p	$104\text{x}10^{-6}$		

Table 2 shows the command signals $[x_c, y_c, z_c, \psi_c]$ which are generated using square wave function with a common duty cycle of 50%, amplitude of one and zero offsets. However they have different frequencies.

Table 2: Command Signal

Command	Frequency in Hz	Offset
x_c	0.015	0
y_c	0.01	0
z_c	0.012	0
ψ_c	0.009	0

The control parameters have been selected to satisfy the desired performance and they are shown in Table 3.

Table 3: Control Parameter

Parameter	Value
$\boldsymbol{A}_{m1}$	$\begin{bmatrix} -2 & 0 & -2 & 0 \\ 0 & -2 & 0 & -2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
$\boldsymbol{B}_{m1}$	$\begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
$\boldsymbol{\Lambda}_1$	eye(2)
$\boldsymbol{K}_1$	2 eye(2)
$\boldsymbol{A}_{m2}$	$\begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{22} \\ \mathbf{I}_{4\text{x}4} & \mathbf{0}_{4\text{x}4} \end{bmatrix}$
$\boldsymbol{a}_{11}$	$diag([-2, -10, -10, -2])$
$\boldsymbol{a}_{22}$	$-diag([2,29,29,2])$
$\boldsymbol{B}_{m2}$	$\begin{bmatrix} \boldsymbol{a}_{22} \\ \mathbf{0}_{4\text{x}4} \end{bmatrix}$
$\boldsymbol{\Lambda}_2$	5 eye(4)
$\boldsymbol{K}_2$	6 eye(4)
$\mathbf{P}$	$diag(1,1,1,1,5,5)$
$\boldsymbol{\lambda}_\mathbf{f}$	20

Figure 7 and Figure 8 show the positions and the angles of the quadrotor respectively. The both figures show the difference in response between the Composite and direct adaptive control. The left column illustrate the responses at the beginning of the simulation where time is between $0 \le t < 100$. While the right column represent the responses at the time where the estimated parameters changed their values $1000 \le t < 1100$. It is easy to notice that the composite adaptive response is better and track the desired signal faster.

In Figure 9, the tracking error of the position and the angles of the quadrotor are shown.

Figure 10 shows the parameters estimation where the composite adaptive control is tracking the parameters exactly not like the direct adaptive control. Also Figure 11 illistrates the parameters error.

Table 4 shows the parameters estimation Mean Square Error (MSE). The composite adaptive control has got much lower MSE in all parameters.

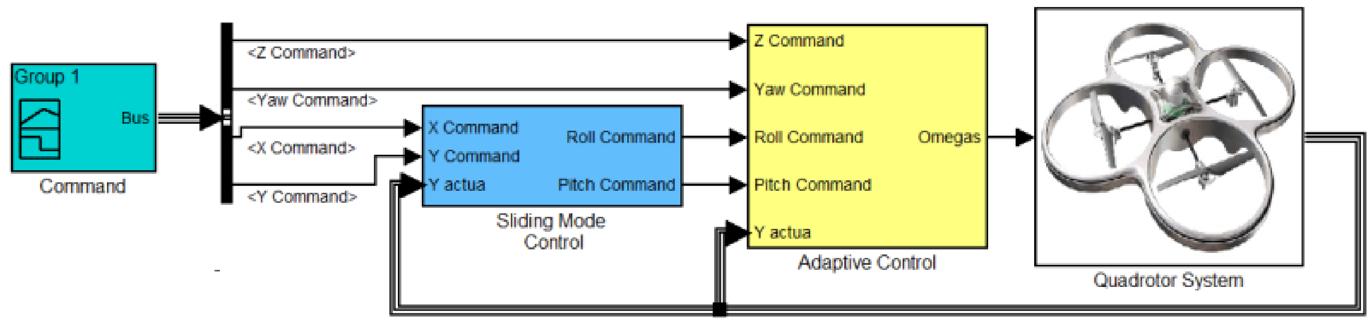

Figure 6: Simulink Block Diagram

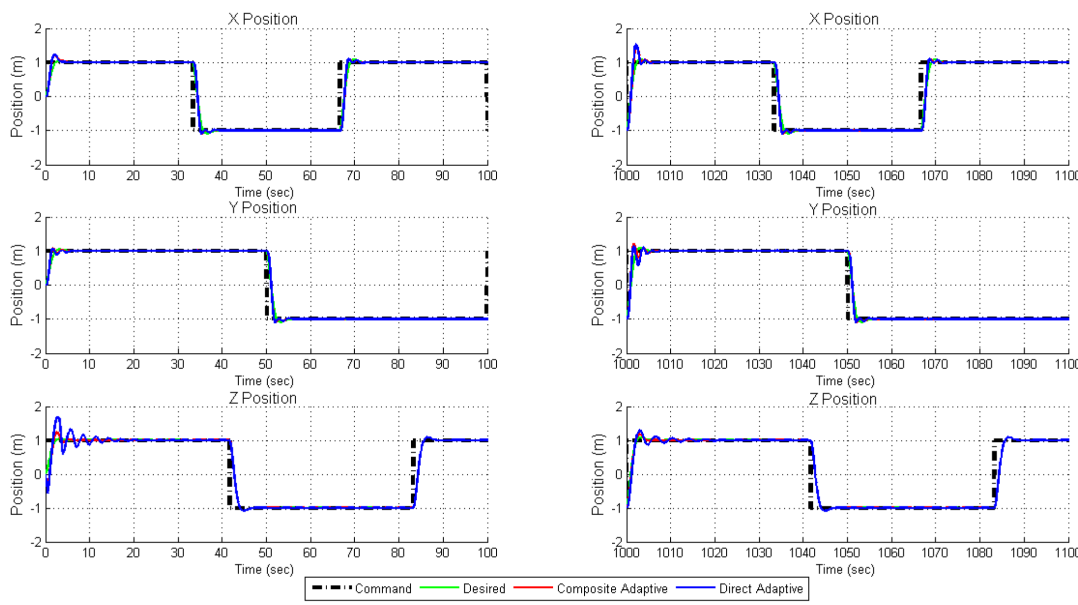

Figure 7: Command, Desired and Actual Positions

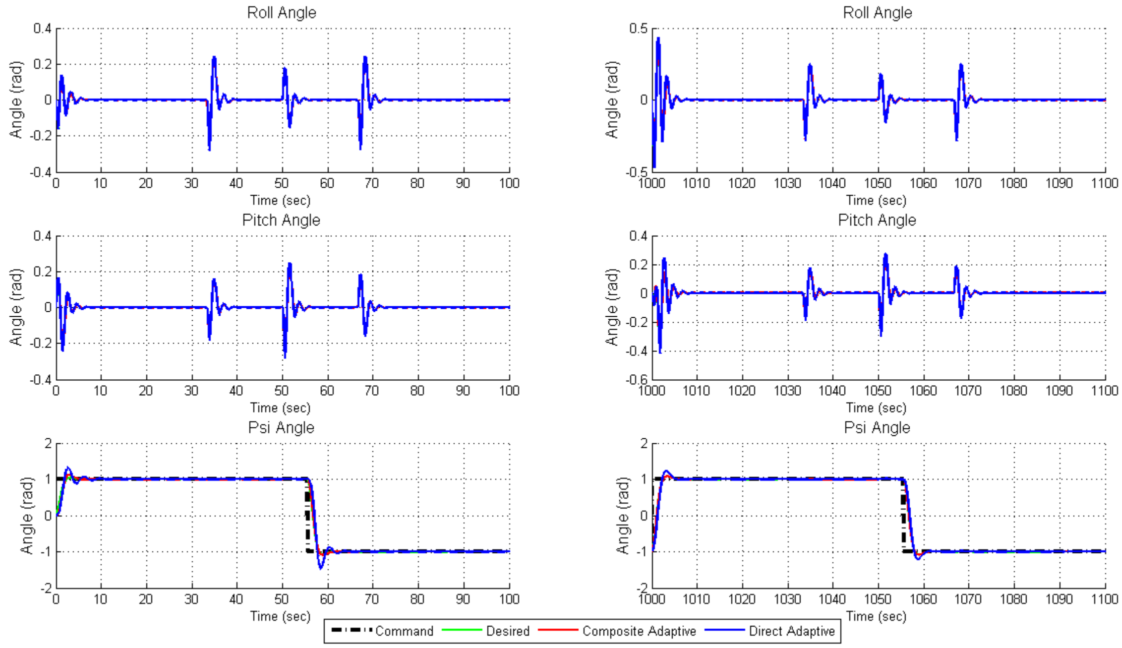

Figure 8: Command, Desired, and Actual Angles

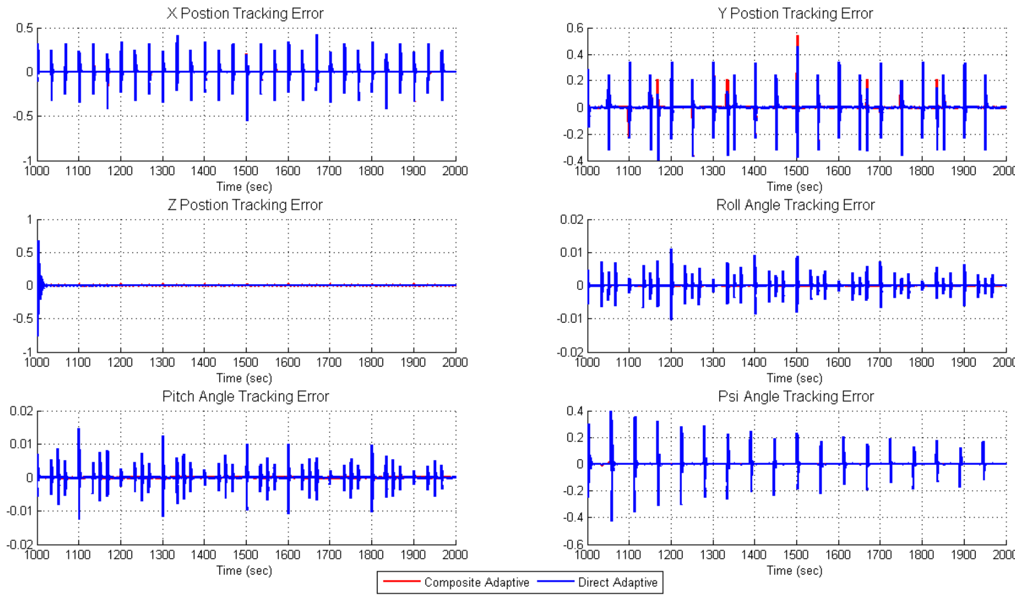

Figure 9: Tracking Error

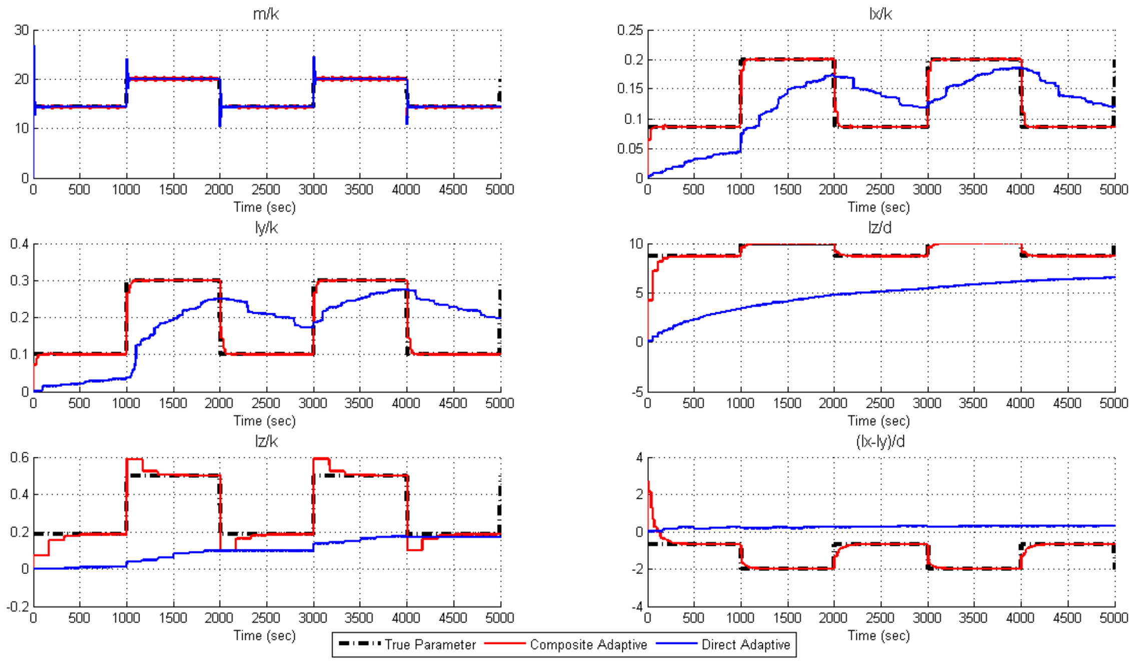

Figure 10: Parameters Estimation

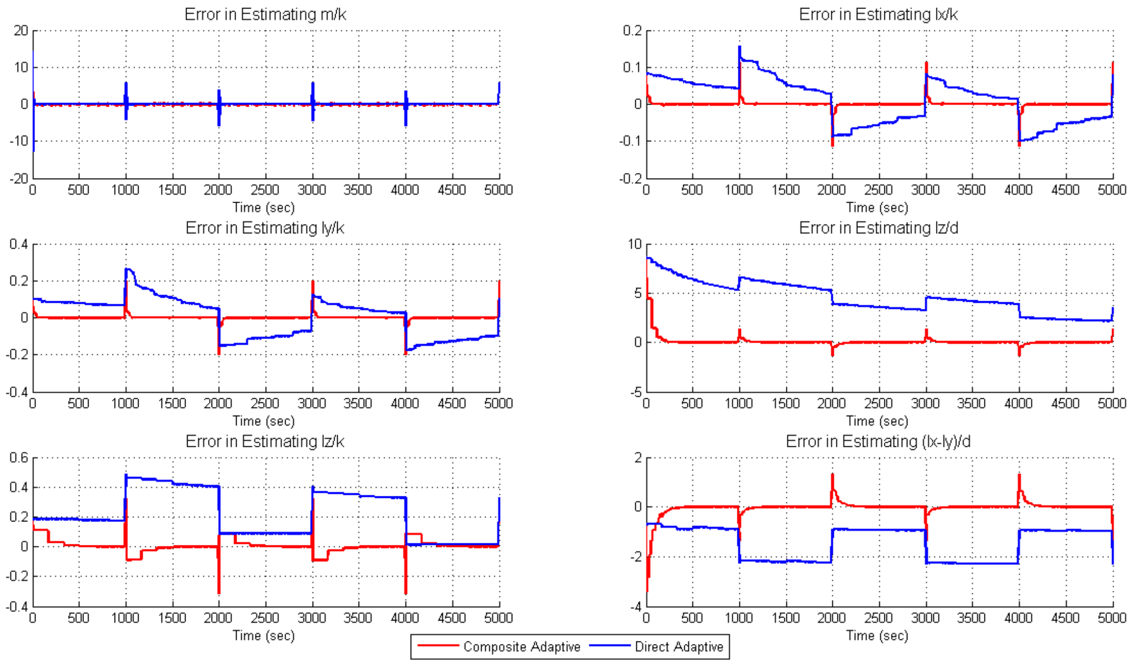

Figure 11: Parameters Error

Table 4: Parameter Estimation Mean Square Error (MSE)

Parameter	Method	
	Direct Adaptive	Composite Adaptive
$m/\bar{k}$	0.10981	0.036170
$I_x/\bar{k}$	0.00367	2.539e-05
$I_y/\bar{k}$	0.01143	8.021e-05
I_z/d	22.8098	0.271159
$I_z/\bar{k}$	0.06901	0.001622
$(I_x - I_y)/d$	2.51034	0.094554
$m/\bar{k}$	0.10981	0.036170

6 CONCLUSION

A nonlinear composite adaptive control algorithm is done for a 6-DOF quadrotor system. The proposed controller forced the quadrotor to follow a desired position, velocity, acceleration and the heading angle despite the parameter uncertainty. A Sliding mode control is used to control the internal dynamics while the adaptive control is controlling the fully actuated subsystem. All the parameters such as the mass, system inertia, thrust and drag factors are considered to be fully unknown and been estimated.

The direct adaptive controller has improved to become a composite adaptive controller. The composite adaptive controller is driven using the information from both the tracking errors and the parameter errors. The stability of the closed-loop system is shown in the flight region of interest. Also the comparisons between both sachems are illustrated .

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