

Decision Making Principle from the Perspective of Possibilistic Theory

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Abstract—Mathematical programming plays a pivotal role in finding the solution for optimization problems in various practical, real-life applications. Conventionally, the modeling used in mathematical programming is based on numerical values. It is however complicated to accurately provide such rigid numerical values because uncertain elements do exist in the decision-making process. Furthermore, building a mathematical programming model with crisp and precise values can result in the production of an infeasible or improper solution. Hence, uncertain based decision making is exemplify in this paper by using possibilistic theory to capture human uncertain judgment to develop mathematical programming model which sufficiently able to find an acceptable solution. The implementation of the proposed method shows the significant capabilities to solve real application problem which retain the uncertainties in its problem model.

Keywords- *uncertain judgment; decision making; possibilistic theory; necessity measure*

1 INTRODUCTION

Decision-making theory has become one of the most important fields for real-world decision-making. Fundamentally, decision making involves imprecision and uncertainty when human knowledge and evaluation are considered in the decision-making process. In the real-world situation, the problems are much depending on the mathematical programming model which is used to explain the problem and to find the solution. Typically, a practical real-world problem is translated and developed into mathematical programming problem model with numerical values which neglects the uncertainties. However, providing precise values for mathematical problem models raises difficulties [1] because the nature of the decision-making process is inherently dependent upon the knowledge and professional experiences of decision makers (experts). Moreover, if the problem model's parameters are not appropriately determined as crisp values in the mathematical model, the formulated problem may yield an infeasible or improper solution [2]. In fact, the measurement and evaluation of imprecise values of decision criteria are difficult [3], and dealing with this imprecision is a challenging task in decision making.

In decision-making process, model setting and goal attainment are fundamental aspects of human decision-making. However, the information available to a decision

maker is often imprecise because of inaccurate attribute measurements and inconsistency in priorities. Until recently, the decision-making process still utilized subjective judgments when considering human evaluations for certain cases, such as resource planning problems. Therefore, a decision is often made on the basis of vague information or uncertain data. Because many evaluations depend on human judgment, which is usually based on intuition and experience, the expression of crisp values in mathematical models is a complicated problem. Moreover, extracting human judgment and personal subjectivity is difficult in the traditional decision-analysis models. Thus, certain approaches, such as probability distribution, fuzzy numbers, and different types of thresholds [4], have been used to model uncertainty and imprecision, in the distinct occurrence of the uncertainty. Yet, few studies discuss on the hybrid uncertainty in the decision-making problem model, even though it is important to consider such situation while modeling real-world decision-making problem.

As many evaluations depend on human judgment, which is usually based on intuition and experience, the expression of accurate values in mathematical models is a complicated problem. Given this imprecise situation, the uncertainties should be handled properly to ensure that the mathematical model developed for the problem takes the uncertainties in the evaluation into consideration. It is important to address uncertainty to obtain a proper solution, and to avoid the formulated problem model obtain misleading result. For this reason, fuzzy sets [5] are useful for representing uncertain and imprecise information in mathematical programming. It makes fuzzy mathematical programming is important for dealing with uncertainties for cases in which the mathematical programming model's parameters cannot be estimated precisely from the real situation in question.

Given this imprecise situation, the uncertainties should be handled properly to ensure that the mathematical model developed for the problem takes the uncertainties of evaluations into consideration. It is important to address uncertainty to obtain an optimal solution. For that reason, it is inspired to sufficiently explain the method which captures intuitive human judgment and preference to developed the mathematical programming model and solve the problem using possibilistic concept. The decision maker's aspiration is therefore reflected properly within the developed mathematical programming model, and the uncertainties are

retained to ensure that the model does not diverge from the problem. It is remarkable that the proposed method shows that a decision maker can realize the extent to which their target goal can be satisfied.

The remainder of this paper is organized as follows. Section 2 describes the background study based on possibilistic system. Section 3 explains the model's development of possibilistic decision making. Section 4 illustrates the model with a numerical example, and Section 5 concludes this paper with some additional remarks.

2 POSSIBILISTIC PROGRAMMING

Fuzzy mathematical programming models are classified into two categories [6]. One category addresses the fuzziness of the decision makers' aspirations with respect to goals and/or constraints (i.e., vagueness in fuzzy goals). The other category addresses the ambiguity of the coefficients of the objective functions and/or constraints. Possibilistic programming is the term used to describe the type of fuzzy mathematical programming produced if the vagueness in the decision maker's aspiration is modeled as an objective function using a fuzzy preference relation and the ambiguities in the coefficients are represented in terms of a possibility distribution [7]. Thus, possibilistic programming differs from fuzzy mathematical programming because the uncertainty in the former is incorporated into the coefficients of the goals and/or the constraints of the mathematical programming model, and these imprecise coefficients are restricted by possibilistic distributions. Studies ([2]; [7]; [8]) have shown that possibilistic programming provides advantages in addressing the ambiguity and vagueness contained in a decision-making model and, therefore, that its integration with other concepts and methods can improve the efficiency of such a model in solving problems.

Possibility theory ([9]; [10]) expresses an impression by means of a possibility distribution [9]. Within possibility theory, fuzzy parameters are associated with possibility distributions, just as random variables have traditionally been associated with probability distributions. Stochastic and possibilistic linear programming may be distinguished because the former considers uncertainty in model parameters due to randomness, whereas the latter considers the uncertainty in model parameters due to fuzziness. Since the 1980s, possibility theory has become more important in the decision-making field, and several methods have been developed to solve possibilistic programming problems. Additionally, possibilistic linear programming has also been applied to multi-objective programming problems in which all of the parameters are fuzzy. In multi-objective programming problems, parameters such as coefficients and the right-hand-side values of the constraints are conventionally assumed to be real numbers. However, in real world problems, we may face cases for which the expert knowledge is not sufficiently certain to specify that these

parameters as real numbers or cases in which parameters fluctuate in certain ranges.

Decision makers commonly face fuzziness, which can arise from factors such as the ambiguity of received information and the vagueness in a decision maker's goal [11]. When the mathematical model contains uncertain information, that is, the coefficients and goal are fuzzy or not exactly known, the problem should be modeled with an approach that addresses and incorporates these uncertainties into the solution of the mathematical model. Thus, the uncertainties that are included in decision making increase the complexities of problem modeling, and as a result, it is difficult to solve such models properly, as the uncertainties involved cannot be described precisely using numerical values. That is, in most real-world situations, it is reasonable to assume that the possible values of a model's attributes and its coefficients are uncertain. Hence, it is realistic to consider the estimated value of the coefficients as imprecise values rather than precise ones. A possible range for each coefficient can be represented by a fuzzy set, which is also regarded as a possibility distribution. Thus, the mathematical programming models for decision support must explicitly consider such issues, and correct treatment of the inherent uncertainty associated with the model coefficients is essential.

3 MODEL DEVELOPMENT

In possibilistic programming, the concepts of possibility and necessity measures [9] are introduced to deal with the vagueness and ambiguity included in the objective function and/or constraints. The interpretation of the problem plays an essential role in formulating the problem into mathematical programming model. From the perspective of possibility theory, the interpretation is developed based on the possibility measure and necessity measure.

3.1 Possibility and Necessity Measures

Let A and B be fuzzy sets of the universe X . A possibility measure $\Pi_A(B)$ and a necessity measure $\eta_A(B)$ are defined as follows:

$$\begin{aligned}\Pi_A(B) &= \sup_x \min\{\pi_A x, \mu_B x\}, \\ \eta_A(B) &= \inf_x \max\{1 - \pi_A x, \mu_B x\},\end{aligned}\tag{1}$$

where μ_A and μ_B are membership functions of fuzzy sets A and B . From (1), possibility measure $\Pi_A(B)$ evaluates to what extent it is possible that under the restrictions of the possibility distribution μ_A , the possibilistic variable α is in the fuzzy set B . Likewise, $\eta_A(B)$ evaluates to what extent it is certain that under the restrictions of the possibility distribution μ_A , the possibilistic variable α is in the fuzzy set B .

The following relations always hold:

$$\begin{aligned}\eta_A(B) &\leq \Pi_A(B), \\ \eta_A(B) &= 1 - \Pi_A(\bar{B}),\end{aligned}\quad (2)$$

where \bar{B} is the complement of B .

Let α be a possibilistic variable. Let $B = (-\infty, g]$ be a non-fuzzy set of real numbers which is not greater than g . The possibility and necessity measures defined by (1) are written as follows:

$$\begin{aligned}Poss(\alpha \leq g) &= \Pi_A((-\infty, g]), \\ &= \sup\{\mu_A(r) \mid r \leq g\} \\ Nec(\alpha \leq g) &= \eta_A((-\infty, g]), \\ &= 1 - \sup\{\mu_A(r) \mid r \leq g\}\end{aligned}\quad (3)$$

where $Poss(\alpha \leq g)$ and $Nec(\alpha \leq g)$ are the possibility and certainty degrees to what extent α is not greater than g , respectively.

3.2 Treating Uncertainties Through Necessity Measure

In this work, the expression of uncertainty adopted is based on fuzzy sets [5]. In this formulation, uncertain problem parameters are defined by fuzzy sets and characterized by membership functions. In general, the membership covers from zero to one. The definition of uncertainty by means of fuzzy sets enhances the ability to model real-world problems and gives a methodology for exploiting tolerance for imprecision or uncertainties [12].

A fuzzy number combines two ideas of confidence interval and membership degree or satisfaction level. Depending on the imprecise parameters, the constraints and the optimal solution constitute a class of alternatives whose boundaries are not well defined.

Let us consider the possibilistic linear programming problem (4) with constraints, as follows:

$$\begin{aligned}\max & \quad f(\mathbf{a}\mathbf{x}_j) \\ \text{subject to : } & \quad \bar{\mathbf{A}}\mathbf{x} \lesssim \bar{\mathbf{b}}; \mathbf{x} \geq 0.\end{aligned}\quad (4)$$

where x is an n -dimensional vector and \lesssim describes a fuzzy goal. \mathbf{a} , $\bar{\mathbf{A}}$ and $\bar{\mathbf{b}}$ are possibilistic variable vectors. $f_j = \mathbf{a}_j(\mathbf{x}_j)$ and $\bar{\mathbf{A}}\mathbf{x} \leq \bar{\mathbf{b}}$ denotes objective function and constraints, respectively.

To solve possibilistic programming problem (4), the constraints and objective function are treated using necessity and possibility measure. In this paper, we restrict ourselves to explain necessity measure in the treatment of constraints and objective function. Necessity measure evaluates to what extent the decision maker's aim can be achieved certainly.

3.3 Dealing the constraints

It is important to treat the constraint that is described in ambiguous coefficient in the mathematical model. The treatment is prepared by giving the interpretation to the

constraint so as the constraint in the model is closely translates the meaning of decision maker's desire. Using the necessity measure, the certainty of decision maker intention to the constraint is indicated.

Let $v^\eta \in [0,1]^m$ be a necessity aspiration degree that a decision maker is aspired to achieve certainly. The constraints $\bar{\mathbf{A}}\mathbf{x} \leq \bar{\mathbf{b}}$ can be treated as follows:

$$Nec\left(\left(\bar{\mathbf{A}}\mathbf{x} \leq \bar{\mathbf{b}}\right) \geq v^\eta\right) \quad (5)$$

Note that, this is the case where the decision maker feels that a certainty degree is not less than v^η . The symmetric

fuzzy number is written as $A = \left\langle \sum_{j=1}^n x_j a_j, \sum_{j=1}^n |x_j| d_j \right\rangle$.

From (5), let us assume that s is less than v^η to obtain the formulation as follows:

$$s = \sum_{j=1}^n x_j a_j + v^\eta \left(\sum_{j=1}^n |x_j| d_j \right) \quad (6)$$

Thus, expression (6) is the treated constraint which considers the certainty degree of decision maker's intention to the problem constraint.

3.4 Dealing the Objectives

In a fuzzy mathematical programming problem, each objective function value is not always a real number. The objective function value is frequently only restricted by a possibility distribution $\alpha(x)$. Therefore, the meaning of the objective should be interpreted.

Let us consider that the decision maker wants to maximize the certainty degree that the event is not smaller than g^η , and is modeled as $\max Nec(\alpha x \geq g^\eta)$.

Using additional variable h , the following model expresses the decision maker's intention.

$$\begin{aligned}\max & \quad h \\ \text{subject to : } & \quad Nec\left(\left(\alpha x \geq g^\eta\right) \geq h\right)\end{aligned}\quad (7)$$

Problem (7) is equivalent to the following.

$$\begin{aligned}\max & \quad h \\ \text{subject to : } & \quad \left(\frac{h}{\sum_{j=1}^n |x_j| d_j} \right) \geq h\end{aligned}\quad (8)$$

Problem (8) is rewritten by using the treated objective function and constraints as follows:

$$\begin{aligned} \max & \quad \left(\frac{\sum_{j=1}^n x_j \alpha_j}{\sum_{j=1}^n |x_j| d_j} \right) \\ \text{subject to} & \quad : \sum_{i=1}^m \sum_{j=1}^n x_j a_{ij} + v^\Pi \left(\sum_{j=1}^n |x_j| d_j \right), \quad (9) \\ & \quad x_i \geq 0. \end{aligned}$$

3.5 A Possibilistic Evaluation Scheme

The possibilistic evaluation scheme for decision making is simplified as follows:

- Problem description and modeling.**
Describe the problem and build the problem model. If there's a random situation exists then the problem model can be built by using fuzzy random regression method ([13]; [14]; [15]).
- Treating the constraints.**
Analyze the problem constraints to treat the ambiguity data as equation (5). Set the degree of certainty v^N and transform the constraints as expression (6).
- Treating the objective.**
Obtain the necessity aspiration level g^N of the objective function. Transform the objective function in an expression of $\max Nec(\mathbf{ax} \geq g^N)$, for the case of decision maker want to maximize the objective function.
- Modeling, solution and analysis.**
Develop a possibilistic programming model as Equation (9) which contain fuzzy random based coefficient. Solve problem model (9) to obtain the solution \mathbf{X} . Analyze the decision.

4 NUMERICAL EXAMPLE

Let us consider a production planning problems with two decision variables and one functional objective are investigated under four system constraints. The objective of the decision maker is to maximize the return of the profit, that are constrained with available resources; raw material, labor, mills capacity and capital. The coefficients values were estimated based on historical data. The vague target is derived from the aspiration of the decision maker to achieve some target, and fuzzy value in the data is considered as ambiguous data. Such situation results in possibilistic problem whereby the inherent uncertainties occur are characterized by fuzzy vague and ambiguous data.

The problem is modeled as follows:

$$\begin{aligned} \max \text{ profit} & \langle 1.86, 0.1 \rangle x_1 + \langle 3.10, 0.3 \rangle x_2. \quad (a) \\ \text{subject to} & : \left. \begin{aligned} \text{raw material} & : \langle 3.75, 0.06 \rangle x_1 + \langle 0.91, 0.08 \rangle x_2 \leq 87.75, \\ \text{labor} & : \langle 0.65, 0.55 \rangle x_1 + \langle 0.90, 0.09 \rangle x_2 \leq 14.42, \\ \text{mill capacity} & : \langle 17.35, 0.85 \rangle x_1 + \langle 2.16, 0.27 \rangle x_2 \leq 75.20, \\ \text{capital} & : \langle 0.87, 0.65 \rangle x_1 + \langle 0.98, 0.65 \rangle x_2 \leq 20.15, \end{aligned} \right\} (b) \end{aligned} \quad (10)$$

where all model coefficient is obtained by fuzzy random regression approach.

Based on Step 2 that is dealing with the constraints, let us assume that the decision maker decide that certainty degree not less than $v^N = 0.7$ is high enough for the system constraint (b) in the Problem (10). To satisfy the decision maker aim, analyze the constraints under expression (5). Use expression (5) to transform the constraints based on decision maker aim as follows:

$$\left. \begin{aligned} Nec(\langle \langle 3.75, 0.06 \rangle x_1 + \langle 0.91, 0.08 \rangle x_2 \leq 87.75 \rangle \geq v), \\ Nec(\langle \langle 0.65, 0.55 \rangle x_1 + \langle 0.90, 0.09 \rangle x_2 \leq 14.42 \rangle \geq v), \\ Nec(\langle \langle 17.35, 0.85 \rangle x_1 + \langle 2.16, 0.27 \rangle x_2 \leq 75.20 \rangle \geq v), \\ Nec(\langle \langle 0.87, 0.65 \rangle x_1 + \langle 0.98, 0.65 \rangle x_2 \leq 20.15 \rangle \geq v), \end{aligned} \right\} (b) \quad (11)$$

For the objective part, the decision maker aims to maximize the certainty degree of profit is larger than 5.0 million dollars. Based on (7), the decision maker target is modeled as follows:

$$\begin{aligned} \max & \quad h \\ \text{subject to} & : Nec(\langle 1.86 x_1 + 3.10 x_2 \geq 12.30 \rangle \geq h) \end{aligned} \quad (12)$$

The problem (5.22) is rewritten as follows by applying model (5.8):

$$\begin{aligned} \max & \quad \frac{1.86 x_1 + 3.1 x_2 - 12.30}{0.1 x_1 + 3.1 x_2} \\ \text{subject to} & : \begin{aligned} 3.79 x_1 + 0.95 x_2 & \leq 87.75, \\ 1.03 x_1 + 0.96 x_2 & \leq 14.42, \\ 17.94 x_1 + 2.34 x_2 & \leq 75.20, \\ 1.32 x_1 + 1.43 x_2 & \leq 20.15, \\ x_i & \geq 0. \end{aligned} \end{aligned} \quad (13)$$

The equivalent linear programming for problem (9) is as follows:

$$\begin{aligned} \max & \quad 1.86 z_1 + 3.1 z_2 - 12.30 t \\ \text{subject to} & : \begin{aligned} 3.79 z_1 + 0.95 z_2 - 87.75 t & \leq 0, \\ 1.03 z_1 + 0.96 z_2 - 14.42 t & \leq 0, \\ 17.94 z_1 + 2.34 z_2 - 75.20 t & \leq 0, \\ 1.32 z_1 + 1.43 z_2 - 20.15 t & \leq 0, \\ 0.1 x_1 + 0.3 x_2 & = 1, \\ t, z_i & \geq 0. \end{aligned} \end{aligned} \quad (14)$$

The optimal solution of the fractional programming is $(x_1, x_2) \approx (2.6, 11.6)$. The solution of problem (10) makes the certainty degree of the event that the profit is not smaller than 12.30 million dollars. It means that, the solution (14)

confirms the decision maker that the profit is not smaller than 12.30 million dollars are certain.

The solution of problem (10) makes the certainty degree of the event that the profit as higher as 0.7. It means that, the solution (10) confirms the decision maker that the profit rate is as larger as 0.7 are certain.

5 CONCLUDING REMARKS

In this paper, the decision making process is explain to include the ambiguous and vague data using possibilistic theory. The necessity measure is used to express the decision maker's aims to achieve certainly the objective function value and the ambiguous coefficients. From the result, it is shown that possibilistic programming is efficient to deal with the uncertainty. The proposed method can be repeated iteratively and various solutions can be obtained depending on the decision maker aim. It is remarkable that the proposed method shows that a decision maker can realize the extent to which their target goal can be satisfied. In the above models, the difficult issues to address include determining and transmitting the decision maker's objectives, preferences, and intentions as well as developing the initial stage of an improved mathematical programming model.

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