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Simple Method For Determining Harmonic Sequences in a Machine, Transformer or Network

Claude Ziad Bayeh
EEE GROUP-R&D department
LEBANON
Email: c.bayeh@eee-group.net
c.bayeh@hotmail.com

ABSTRACT

Harmonics in power systems are very dangerous and may cause different problems for equipment, i.e. the third harmonic has the same amplitude and angle in each phase of a three phase system. It enters into the neutral and may damage or overheat it if the system is unbalanced. The fifth harmonic causes an inverse polarity in machines where the magnetic field rotates in the opposite direction than the normal magnetic field, this causes the rotor to reduce its speed. Harmonics can produce different distortions for all equipment, and may breakdown some of them and damage other sensitive equipment. They reduce the lifetime of super-capacitors and other equipment, etc. These harmonics are caused by non-linear loads or natural disasters. Thus, it is very important to analyze these harmonics and filter them. For many engineers, it is necessary to arrange harmonics into three sequences, direct sequence, inverse sequence and homopolar sequence. Therefore, to arrange these harmonics, they have introduced many methods, but unfortunately, till now the traditional methods are complicated and take a longer time to find the harmonic sequences. Moreover, students find difficulties in the traditional methods and need some faster and simpler method to help them determining the sequence of harmonics, whatever is their range. For this reason, this paper proposes a new method to determine harmonic sequences in the easiest and fastest way.

KEYWORDS

Determining Harmonics, Direct sequence, Inverse sequence, Homopolar sequence, Simple method, Transformer, Machine.

1 INTRODUCTION

In Electrical Engineering, it is very important to analyze unbalanced Three-Phase power systems and to know the sequence of harmonics and phases. The first paper about the symmetrical components was introduced by Charle L. Fortescue [1]. He demonstrated that any asymmetrical set of unbalanced M phasors could be decomposed into a linear combination of the same number M of a balanced symmetrical set of phasors. So the symmetrical components are a Direct sequence (also called Positive sequence), an inverse sequence (also called negative sequence), and a zero sequence (also called Homopolar sequence). These 3 sequences are the basis for analyzing power systems and their effect on equipment such as machines [2-7], transformers [8-11] and power systems [12-13].

The proposed method by Fortescue allows determining the harmonic sequences in order to filter them, because they cause problems on networks, transformers, and machines, and because they introduce heating, electrical pollution, inverse polarity in machines (i.e. Fifth Harmonic) and may affect the functionality of some sensitive loads. Thus, by knowing the sequence of harmonics, it permits to filter the unneeded harmonics. Hence, active and passive filters may be designed in a way to eliminate or reduce these harmonics.

For students, engineers and specialists, the time needed to find the harmonic sequences is very precious; the less time consumed to find harmonics, the better the achievement is for them.

This paper proposes a new and simple method to find Harmonic Sequences, just with one operation for each sequence, which makes the calculation much easier and faster. And it doesn't need even a calculator or any medium to calculate it. This can be also applied on programming part which finds the harmonic sequences, the program will be much shorter, more efficient and faster.

In the second Section, the Symmetrical Components are presented; the normal current is function of Harmonic Sequences. The Third Section discusses the most popular methods of determining the harmonic sequences. In the Fourth Section, this paper introduces a new formula called "Nth Fractional Part of Real Number", this new formula is used in Section Five to determine the harmonic sequences rapidly. The Fifth Section is the most important Section in this paper, where a new equation is defined to determine in one operation, whether the Harmonic has a direct sequence, an inverse sequence or a zero sequence. In the Sixth section, a discussion of the four methods is presented. And finally, a conclusion is developed in Section Seven.

2 SYMMETRICAL COMPONENTS

Fortescue [1] has decomposed any three phase vectors for an electrical system into three balanced systems which are (direct, inverse, and homopolar) in the following way:

$$V_a = V_{a0} + V_{a1} + V_{a2} \quad (1)$$
$$V_b = V_{b0} + V_{b1} + V_{b2} \quad (2)$$
$$V_c = V_{c0} + V_{c1} + V_{c2} \quad (3)$$

Where,
V_a, V_b, V_c are the three phases of the system,
V_{a0}, V_{b0}, V_{c0} are the homopolar components of the system,
V_{a1}, V_{b1}, V_{c1} are the direct components of the system,
V_{a2}, V_{b2}, V_{c2} are the inverse components of the system,

For the direct sequence
$V_{a1} = V_1, V_{b1} = a^2V_{a1} = a^2V_1, V_{c1} = aV_{c1} = aV_1$

For the inverse sequence
$V_{a2} = V_2, V_{b2} = aV_{a2} = aV_2, V_{c2} = a^2V_{c2} = a^2V_2$

For the homopolar sequence
$V_{a0} = V_0, V_{b0} = V_{a0} = V_0, V_{c0} = V_{c0} = V_0$

Where,
$a = 1\angle 120^0$,
$a^2 = 1\angle 240^0$,
$a^3 = 1\angle 360^0$,
$a^4 = a = 1\angle 120^0$.

The same concept can be applied to the current. Therefore, the Symmetrical components matrices of the voltages and currents are as following:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} \quad (4)$$
$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \cdot \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} \quad (5)$$

And

$$\begin{bmatrix} V_1 \\ V_2 \\ V_0 \end{bmatrix} = \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (6)$$
$$\begin{bmatrix} I_1 \\ I_2 \\ I_0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (7)$$

The current in each phase is:

$$I_a = \sum_{n=1}^{\infty}(I_{0n} + I_{1n} + I_{2n}) \quad (8)$$
$$I_b = \sum_{n=1}^{\infty}(I_{0n} + a^2I_{1n} + aI_{2n}) \quad (9)$$
$$I_c = \sum_{n=1}^{\infty}(I_{0n} + aI_{1n} + a^2I_{2n}) \quad (10)$$

The current of each sequence is:

$$I_0 = \sum_{n=1}^{\infty} I_{0n} \quad (11)$$
$$I_1 = \sum_{n=1}^{\infty} I_{1n} \quad (12)$$
$$I_2 = \sum_{n=1}^{\infty} I_{2n} \quad (13)$$

Where,
n is the harmonic number,
I_0 is the current of homopolar sequence,
I_1 is the current of direct sequence,
I_2 is the current of the inverse sequence,
I_a is the current of the phase a,
I_b is the current of the phase b,
I_c is the current of the phase c.

This method is the most used method to determine the harmonic sequences. An example of this method is presented in the following section.

3 TRADITIONAL METHODS FOR DETERMINING THE HARMONIC SEQUENCES

There are many developed methods to determine the sequence of harmonics, three of the most famous methods which are used by all engineers, researchers and students are discussed in this section:

3.1 Mathematical Method

The mathematical method based on Fortescue equations consists of determining the harmonic sequences using equations that calculate the vectors of currents, their magnitudes and their phase angles, this method is very long and it takes approximately 7 to 10 minutes to calculate the sequence of one harmonic, it is described as following:

3.1.1 First step-Finding the phase currents

This step consists of finding the phase currents for the three phase system I_a, I_b, and I_c, where,

$$I_a = I \cdot \cos(wt - 0^0) = I\angle 0^0 \quad (14)$$
$$I_b = I \cdot \cos(wt - 120^0) = I\angle -120^0 \quad (15)$$
$$I_c = I \cdot \cos(wt + 120^0) = I\angle 120^0 \quad (16)$$

And,

$$I_{a,h} = I \cdot \cos\big(h(wt - 0^0)\big) = I\angle(h \cdot 0^0) = I\angle 0^0 \quad (17)$$

$$I_{b,h} = I \cdot \cos\big(h(wt - 120^0)\big) = I\angle\big(h \cdot (-120^0)\big) \quad (18)$$

$$I_{c,h} = I \cdot \cos\big(h(wt + 120^0)\big) = I\angle\big(h \cdot (120^0)\big) \quad (19)$$

Where,
$I_{a,h}$, $I_{b,h}$, $I_{c,h}$ are the currents of the h harmonic in each phase.

3.1.2 Second step-Finding the direct, inverse and homopolar current sequences

After finding equations (17) to (19), the second step is to find the harmonic sequence using the matrix in equation (7) by replacing equations (17) to (19) into it, thus, the elements of this matrix for a harmonic h are determined as following:

The current of the homopolar sequence is:

$$I_{0,h} = \tfrac{1}{3}\left(I_{a,h} + I_{b,h} + I_{c,h}\right) \quad (20)$$

$$I_{0,h} = \frac{1}{3}\Big(I\angle 0^0 + I\angle\big(h \cdot (-120^0)\big) + I\angle\big(h \cdot (120^0)\big)\Big) \quad (21)$$

The current of the direct sequence is:

$$I_{1,h} = \tfrac{1}{3}\left(I_{a,h} + a \cdot I_{b,h} + a^2 \cdot I_{c,h}\right) \quad (22)$$

$$I_{1,h} = \frac{1}{3}\Big(I\angle 0^0 + a \cdot I\angle\big(h \cdot (-120^0)\big) + a^2 \cdot I\angle\big(h \cdot (120^0)\big)\Big)$$

$$I_{1,h} = \frac{1}{3}\Big(I\angle 0^0 + I\angle\big(120^0 + h \cdot (-120^0)\big) + I\angle\big(240^0 + h \cdot (120^0)\big)\Big) \quad (23)$$

The current of the inverse sequence is:

$$I_{2,h} = \tfrac{1}{3}\left(I_{a,h} + a^2 \cdot I_{b,h} + a \cdot I_{c,h}\right) \quad (24)$$

$$I_{2,h} = \frac{1}{3}\Big(I\angle 0^0 + a^2 \cdot I\angle\big(h \cdot (-120^0)\big) + a \cdot I\angle\big(h \cdot (120^0)\big)\Big)$$

$$I_{2,h} = \frac{1}{3}\Big(I\angle 0^0 + I\angle\big(240^0 + h\cdot(-120^0)\big) + I\angle\big(120^0 + h\cdot(120^0)\big)\Big) \quad (25)$$

By calculating equations (21), (23), and (25) we can determine the sequence of the harmonic, we should get two equations equal to zero and one equation different from zero, the equation different from zero gives the correct sequence of the harmonic.

3.1.3 Example

Calculate the sequence of the harmonic 29?

Firstly, one calculates $I_{a,h}$, $I_{b,h}$ and $I_{c,h}$ from equations (17) to (19),

$$I_{a,29} = I\cdot\cos\big(29(wt-0^0)\big) = I\angle(29\cdot 0^0) = I\angle 0^0$$

$$I_{b,29} = I\cdot\cos\big(29(wt-120^0)\big) = I\angle\big(29\cdot(-120^0)\big) = I\angle(-3480^0) = I\angle(-3480^0 + 9\cdot 360^0) = I\angle(-240^0)$$

Because 3480/360=9.666, so one needs $9\cdot 360^0$ to reduce the equation, and because $9\cdot 360^0 = 0^0$,

$$I_{c,29} = I\cdot\cos\big(29(wt+120^0)\big) = I\angle\big(29\cdot(120^0)\big) = I\angle(3480^0) = I\angle(3480^0 - 9\cdot 360^0) = I\angle(240^0)$$

Secondly, one calculates the homopolar, direct, and inverse current sequences $I_{0,29}$, $I_{1,29}$, and $I_{2,29}$ from equations (20) to (25),

Thirdly, one calculates the homopolar sequence:

$$I_{0,29} = \frac{1}{3}\big(I_{a,29} + I_{b,29} + I_{c,29}\big)$$

$$I_{0,29} = \frac{1}{3}\Big(I\angle 0^0 + I\angle\big(29\cdot(-120^0)\big) + I\angle\big(29\cdot(120^0)\big)\Big)$$

$$I_{0,29} = \frac{1}{3}\big(I\angle 0^0 + I\angle(-3480^0) + I\angle(3480^0)\big)$$

$$I_{0,29} = \frac{1}{3}\big(I\angle 0^0 + I\angle(-3480^0 + 9\cdot 360^0) + I\angle(3480^0 - 9\cdot 360^0)\big)$$

$$I_{0,29} = \frac{1}{3}\big(I\angle 0^0 + I\angle(-240^0) + I\angle(240^0)\big)$$

$$I_{0,29} = \frac{1}{3}\big(I\angle 0^0 + I\angle(120^0) + I\angle(-120^0)\big) = 0$$

Because the sum of three opposite vectors is equal to 0, as in figure 1.

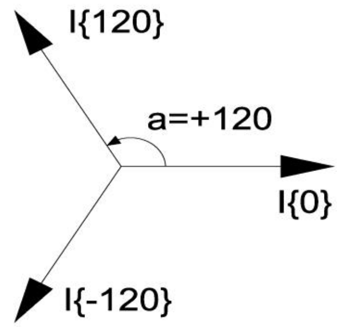

Figure 1: Three opposite vectors with difference in phase equal to 120 and -120 degrees, their sum is equal to zero.

Therefore, the homopolar sequence is equal to zero.

Fourthly, one calculates the direct sequence:

$$I_{1,29} = \frac{1}{3}\big(I_{a,29} + a\cdot I_{b,29} + a^2\cdot I_{c,29}\big)$$

$$I_{1,29} = \tfrac{1}{3}\Big(I\angle 0^0 + a\cdot I\angle\big(29\cdot(-120^0)\big) + a^2\cdot I\angle\big(29\cdot(120^0)\big)\Big)$$

$$I_{1,29} = \tfrac{1}{3}\Big(I\angle 0^0 + I\angle\big(120^0 + 29\cdot(-120^0)\big) + I\angle\big(240^0 + 29\cdot(120^0)\big)\Big)$$

$$I_{1,29} = \tfrac{1}{3}\big(I\angle 0^0 + I\angle(120^0 - 3480^0) + I\angle(240^0 + 3480^0)\big)$$

$$I_{1,29} = \frac{1}{3}\big(I\angle 0^0 + I\angle(-3360^0) + I\angle(3720^0)\big)$$

$$I_{1,29} = \tfrac{1}{3}\big(I\angle 0^0 + I\angle(-3360^0 + 9\cdot 360^0) + I\angle(3720^0 - 10\cdot 360^0)\big)$$

Because 3360/360=9.333, so one needs $9\cdot 360^0$ to reduce the equation, and because $9\cdot 360^0 = 0^0$, and 3720/360=10.333, so one needs $10\cdot 360^0$ to reduce the equation, and because $10\cdot 360^0 = 0^0$.

Thus,

$$I_{1,29} = \frac{1}{3}\left(I\angle 0^0 + I\angle(-120^0) + I\angle(120^0)\right) = 0$$

Therefore, the direct sequence is equal to zero.

Fifthly, one calculates the inverse sequence,

$$I_{2,29} = \frac{1}{3}(I_{a,29} + a^2 \cdot I_{b,29} + a \cdot I_{c,29})$$

$$I_{2,h} = \frac{1}{3}\Big(I\angle 0^0 + I\angle\big(240^0 + 29 \cdot (-120^0)\big) + I\angle\big(120^0 + 29 \cdot (120^0)\big)\Big)$$

$$I_{2,h} = \frac{1}{3}\big(I\angle 0^0 + I\angle(240^0 - 3480^0) + I\angle(120^0 + 3480^0)\big)$$

$$I_{2,h} = \frac{1}{3}\big(I\angle 0^0 + I\angle(-3240^0) + I\angle(3600^0)\big)$$

$$I_{2,h} = \frac{1}{3}\big(I\angle 0^0 + I\angle(-3240^0 + 9 \cdot 360^0) + I\angle(3600^0 - 9 \cdot 360^0)\big)$$

$$I_{2,h} = \frac{1}{3}(I\angle 0^0 + I\angle 0^0 + I\angle 0^0) = I$$

Therefore, the inverse sequence is different from zero, and one can conclude that the harmonic 29 has an inverse sequence because $I_{2,h} = I$.

3.1.4 Discussion

In order to complete this method for one harmonic one needs,

- Number of operations = 200
- Number of equal symbol "=" = 33
- Number of lines used to write equations = 24
- Time needed to find the harmonic sequence = 7.25 minutes (435 seconds)

3.2 Simplified Method

Because the Mathematical Method is too long, some engineers proposed a simplified method, it consists of calculating the harmonic sequences using simple arithmetic equations as following:

Direct Sequence (or Positive Sequence):

$$h = 3k + 1 \tag{26}$$

and

$$k = \frac{h-1}{3} \tag{27}$$

Where,
h is the harmonic sequence,
k represents an Integer Number.

Inverse Sequence (Negative Sequence):

$$h = 3k - 1 \tag{28}$$

and

$$k = \frac{h+1}{3} \tag{29}$$

Homopolar Sequence (Zero Sequence):

$$h = 3k \tag{30}$$

and

$$k = \frac{h}{3} \tag{31}$$

3.2.1 Example of determining the Harmonic Sequences using the Simplified Method

Consider the harmonic which is proposed in the previous example, h=29. Calculate the harmonic sequence.

For h=29, one uses equations (27), (29), and (31) until obtaining an integer number,

- For the Direct Sequence:

$$k = \frac{h-1}{3} = \frac{29-1}{3} = 9.3333$$

k is not an integer number, thus $h = 29$ is not a Direct Sequence.

- For the Inverse Sequence:

$$k = \frac{h+1}{3} = \frac{29+1}{3} = 10$$

k is an integer number, thus $h = 29$ is an Inverse Sequence.

- For the homopolar Sequence:

$$k = \frac{h}{3} = \frac{29}{3} = 9.666$$

k is not an integer number, thus $h = 29$ is not a homopolar Sequence.

3.2.2 Discussion

In order to complete this method for one harmonic, one needs,

- Number of operations = 10
- Number of equal symbol "=" = 9
- Number of lines used to write equations = 3
- Time needed to find the harmonic sequence = 80 seconds

This method is much practical than the Mathematical Method, but it can also be improved. For a large number of harmonics, this method takes a longer time to determine all harmonic sequences. So a simpler method should be developed to reduce the time of calculation. Practically the Simplified method needs improvement which will be proposed in this paper in section 5.

Currently, such calculation can be found using specific software or a table similar to Table 1. But even, the software can be more simplified in order to reduce the time of simulation and the consuming time to develop such software.

3.3 Determining the Harmonic Sequences using a table of harmonics

Table 1 can also be used to pick out the harmonic sequences, but the disadvantage of this table is that for high harmonics the table will be very large and not practical, and it needs to be always carried.

Table 1, A table presents the Harmonic number for each sequence [16].

	Sequence		
	Direct	Inverse	Homopolar
Harmonic Rang	1	2	3
	4	5	6
	7	8	9
	10	11	12
	13	14	15
	16	17	18
	19	20	21
	22	23	24
	…	…	…

3.3.1 Example of determining the Harmonic Sequences using the Table of harmonics

Consider the harmonic which is proposed in the previous examples, h=29, calculate the harmonic sequence?

The table 1 doesn't contain harmonics more than 24, so one has two choices, the first one is to do a larger table which contains at least the first 200 harmonics, or the second one is to interpret the harmonic sequences using a calculator and by adding +3 for every column until arriving to the desired sequence.

For our example:
22+3=25,
25+3=28,
Thus, 29=28+1, therefore this harmonic has an inverse sequence.

3.3.2 Discussion

In order to complete this method for one harmonic, one needs,

- Number of operations≥1 (it depends on the dimension of the given table), for h=29, one needs 3 operations,
- Number of equal symbol "=" ≥ 1 (it depends on the dimension of the given table), for h=29, one needs 3 equal symbols,
- Number of lines used to write equation≥1 (it depends on the dimension of the given table), for h=29, one needs 3 lines,
- Time needed to find the harmonic sequence≥20 seconds, (it depends on the dimension of the given table), for h=29, one needs 26 seconds to find the answer,

The disadvantage of this method is that for large harmonics such as 60, 90, 120, etc, it becomes more difficult to find the answer if the table is limited for a few number of harmonics, and the time needed to find the answer will become much higher than the Simplified method.

4 INTRODUCING THE NTH FRACTION PART OF A REAL NUMBER

Any Real Number is divided into two parts, the first part is called the "Characteristic", which contains the left part of a Real Number before the Decimal Point, and the second part is called the "Fractional Part" (also called Mantissa), which contains the right part of a Real Number after the Decimal Point [15].

For example: the Real Number "12.584" has two parts:

1- The first Part is "12" located before the Decimal Point, which is called "Characteristic", it is a pure integer number.
2- The second part is "584" which is located after the Decimal Point, and it is called the Fractional Part or Mantissa.

Thus,

$$R = C + F \tag{32}$$

Where,
• R is the Real Number,
• C represents the Characteristic,
• F designs the Fractional Part.

The digits after Decimal Point have names, the first digit is called the "tenths digit", in the above case it is "5", the second digit is called the "hundreds digit", in the above case it is "8", and so on.

4.1 Proposed Extractor of Nth digit

For instance, in Mathematics, and to the best of the author's knowledge, there is no formula that extracts a Fractional Digit. In this paper, it is vital to extract the first Fractional Digit which is the "tenths digit" that will be used in the following sections.

So, this paper proposes an original way to extract the Nth Digit of a Fractional Part, and it is denoted in equation (33)

$$\mathrm{FP}(n; A) \tag{33}$$

Where,
• FP is a Function which extracts the Nth Digit of the Fractional Part of a Real Number A.
• n designs the Nth Digit that will be extracted.
• A represents the Real Number.

Its programming is equivalent to

$$\mathrm{FP}(n; A) = \mathrm{mod}(\mathrm{Int}(A \cdot 10^{n}), 10) \tag{34}$$

This is the notation using Microsoft Excel.

Or it can be also written as

$$\mathrm{FP}(n; A) = \mathrm{Int}(A \cdot 10^{n}) \cdot \mathrm{mod}(10) \tag{35}$$

Where,
• $\mathrm{Int}(x)$ rounds a number (x) down to the nearest integer,
• $\mathrm{mod}(10)$ returns the remainder after a number $(\mathrm{Int}(A \cdot 10^{n}))$ is divided by advisor (10 in our case).

4.2 Example

Let's suppose $A = 13.4768901$

$\mathrm{FP}(1; A) = 4$ which is the tenths digit
$\mathrm{FP}(2; A) = 7$ which is the tenths digit
$\mathrm{FP}(3; A) = 6$ which is the tenths digit
$\mathrm{FP}(4; A) = 8$ which is the tenths digit
And so on.

5 PROPOSED METHOD FOR DETERMINING THE HARMONIC SEQUENCES

By using the proposed Infomath function developed in the paper [14], it will be much easier to determine the harmonic sequences with a very simple method comparing to the traditional one,

The equation is:

$$S = \underline{\mathrm{FP}\left(1; \frac{h}{3}\right); 3, 6; (D), (I)/(H)} \tag{36}$$

Where,
• S is the sequence of the Harmonic,
• $FP(n; A)$ is a Function which extracts the Nth Digit of the Fractional Part of a Real Number A. In our case, $n = 1$, and $A = h/3$, which means one is interested to extract the Tenths Digit of $h/3$,
• D designs the Direct Sequence of the Harmonic,
• I is the Inverse Sequence of the Harmonic,
• H represents the Homopolar Sequence of the Harmonic.

5.1 How does the InfoMath Function work?

This subsection gives an idea about how the Infomath Function works, but for more details, one can refer to the paper [14].

The form of the InfoMath function is as following:

$$\underline{\text{input}; \text{conditions}; \text{output}} \qquad (37)$$

It is divided into three parts;

1- The first part is the input, which can be a number, equation, or anything else.
2- The second part is the conditions which are applied to the input, if the conditions are verified; the output will give a certain value, expression, equation, or anything else.
3- The third part is the output, which can be a number, equation, or anything else.

Simple example:

$$S = \underline{3; \leq 1, \geq 6; (2), (5)/(8)}$$

In this example, the input is “3”, so the conditions must be applied to this input,

1- 3≤1?? If yes the output should be “2”
2- If not, is 3≥6?? If yes the output should be “5”
3- Else, the output should be “8”

In the above case, the input doesn’t verify the first two conditions, thus, the Infomath Function is replaced by the output “8”, which means

$$S = \underline{3; \leq 1, \geq 6; (2), (5)/(8)} = 8$$

If the first condition is verified, the output will be the first item “2”. If not, if the second condition is verified, the output will be the second item “5”. If not, Else, the output will be automatically the third item which is “8”.

Therefore, if the input is ≤ 1, the output is 2,

If the input is ≥ 6, the output is 5,

Else, the output is 8.

Now, returning to the equation (36),
The input is $FP\left(1; \frac{h}{3}\right)$,

The first condition is (=3), the second condition is (=6),

The outputs are: if the first condition is verified, the output is “D” which means Direct Sequence. If not, if the second condition is verified, the output is “I” which means Indirect Sequence. If not, Else, the output is “H”, which means Homopolar Sequence.

5.2 Application of the equation

Now, returning to the main subject, the main idea of the Infomath function is simple, and it is much easier to be applied. Let’s consider different values of harmonics. One has 3 harmonics on a transformer, a machine, or a network in which he wants to determine their sequences in order to see which one has a Direct Sequence, an Inverse Sequence, or a Homopolar Sequence. The three harmonics are:
$h_1 = 38, h_2 = 57, h_3 = 67$

By applying equation (36),
• $\frac{h_1}{3} = \frac{38}{3} = 12.6666$

The Tenths Digit is ≠3, but it is =6, thus, it is the Inverse Sequence, and
$S = \underline{6; 3, 6; (D), (I)/(H)} = I$

• $\frac{h_2}{3} = \frac{57}{3} = 19$

The Tenths Digit is ≠3 and ≠6, thus, it is the Homopolar Sequence, and

$S = \underline{0; 3, 6; (D), (I)/(H)} = H$

• $\frac{h_3}{3} = \frac{67}{3} = 22.3333$

The Tenths Digit is =3, thus, it is the Direct Sequence, and

$$S = \underline{3; 3, 6; (D), (I)/(H)} = D$$

In this way, the calculation is simplified to only 1 division, and one gets the answer rapidly without doing the whole traditional procedure which is described in section 3.

Now, if one has a large number of harmonics, it is much easier to determine what the sequence of each harmonic is.

5.3 Example

Considering the harmonic h=29 which is discussed in the previous sections. To calculate this harmonic using the proposed method, one uses the equation (36), therefore,

$$S = \underline{FP\left(1; \frac{h}{3}\right); 3, 6; (D), (I)/(H)}$$
$$= \underline{FP\left(1; \frac{29}{3}\right); 3, 6; (D), (I)/(H)}$$
$$= \underline{6; 3, 6; (D), (I)/(H)} = I$$

Therefore this harmonic has an inverse sequence.

5.4 Discussion

In order to complete this method for one harmonic, one needs,

- Number of operations = 1.
- Number of equal symbol "=" = 4,
- Number of lines used to write equation = 1,

Time needed to find the harmonic sequence = 28 seconds.

6 RESULTS AND DISCUSSIONS

In the previous sections, some examples are given and discussed briefly for each method. Therefore, it is necessary to make a table of comparison between these four methods. Table 2 presents a summary about the four used methods in this paper.

Table 2, A comparative table between all used methods in this paper.

Category	Mathematical Method	Simplified Method	Table Method	Proposed method
Needed time to obtain the answer	7.25 minutes	80 seconds	**≥20 seconds** In our example it took 36 seconds	**28 seconds**
Number of operations needed to obtain the answer	200	10	**≥0** In our example it took 3 operations	**1**
Number of lines used to obtain the answer	24	3	**≥1** In our example it took 3 lines	**1**
Programming difficulty	Very complex	medium	Medium	**simple**
Constraints	**Non**	**Non**	Limited by the size of the table	**Non**
Number of used equal symbols "="	33	9	≥1 For h=29 it takes 3 "="	**4**
Possibility of errors by hand calculation	Possible ≥ 0%	**0%**	**0%**	**0%**
Is the method practical	No	**Yes**	No	**Yes**
Complexity of the calculation	Very Complex	Easy	Easy	**Very Easy**
Mental Calculation can be used?	No	Yes but difficult	No	**Yes**

The advantages of each category are presented in bold red color, it is very clear that all advantages are presented in the proposed method in this paper.

The most important factor is the time needed to obtain an answer for one harmonic; hence, if one considers that the mathematical method is the standard method to do a comparison with, therefore, the reduced time ratio for each method is presented as below:

$$\text{Reduced Time Ratio}_{\text{Math method}} = \frac{435}{435} = 1$$

$$\text{Reduced Time Ratio}_{\text{Simplified method}} = \frac{435}{80} = 5.4375$$

$$\text{Reduced Time Ratio}_{\text{Table method}} \leq \frac{435}{20} = 21.75$$

$$\text{Reduced Time Ratio}_{\text{Proposed method}} = \frac{435}{28} = 15.537$$

7 CONCLUSION

In this paper, the harmonic sequences are defined, they are calculated using four methods, the first method is the mathematical method defined by Fortescue, it is considered as the basis of the harmonic decompositions. The second method is a simplified method, in which each sequence is defined by two simple equations; it is much simpler than the first one. The third method is presented in a table, where different harmonics are classified into three categories in the same table, which are the Direct, Inverse, and Homopolar sequences. The problem with this method is that the size of the table is limited to a certain number of harmonics, and it doesn't have a mathematical support. And finally, the fourth method which is proposed in this paper, it has many advantages over the traditional methods which are presented in table 2. The proposed method in this paper is recommended for students, and engineers who are willing to study the harmonics sequences of a certain system or equipment.

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