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Two-Level Decomposition Method for Resource Allocation in Telecommunication Networks

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Abstract—**n this paper, we consider a two-level problem of resource allocation in a telecommunication network divided into zones. At the upper level the network manager distributes homogeneous resource shares among zones in order to maximize the total network profit, which takes into account the inner zonal payments from users and the implementation costs. This means that each zonal income calculation at a given resource share requires solution of the inner resource allocation problem. As a result, we obtain a two-level convex optimization problem involving non smooth functions whose values are calculated algorithmically. Unlike the usual convex non smooth optimization methods we suggest this problem to be solved by a Lagrangean duality method which enables us to reduce the initial problem to a sequence of hierarchical one-dimensional problems. Besides, we suggest new ways to adjust the basic problem to networks with moving nodes. We present results of computational experiments which confirm the applicability of the new method. n this paper, we consider a two-level problem of resource allocation in a telecommunication network divided into zones. At the upper level the network manager distributes homogeneous resource shares among zones in order to maximize the total network profit, which takes into account the inner zonal payments from users and the implementation costs. This means that each zonal income calculation at a given resource share requires solution of the inner resource allocation problem. As a result, we obtain a two-level convex optimization problem involving non smooth functions whose values are calculated algorithmically. Unlike the usual convex non smooth optimization methods we suggest this problem to be solved by a Lagrangean duality method which enables us to reduce the initial problem to a sequence of hierarchical one-dimensional problems. Besides, we suggest new ways to adjust the basic problem to networks with moving nodes. We present results of computational experiments which confirm the applicability of the new method. I**

Index Terms—**Resource allocation, telecommunication networks, non smooth optimization, Lagrangean duality method, decomposition.**

I. Introduction

Allocation of limited resources among competing entities according to predefined criteria is an optimization problem met in practice in numerous forms. In the quickly developing area of wireless communication it finds its application in such emerging technologies as mobile ad-hoc networks and sensor networks, which provide a new level of availability and quality of information and support a wide range of services. The current development of information technologies and telecommunications gives rise to new control problems related to efficient transmission of information and allocation of limited network resources (e.g., bandwidth, power capacity, etc); see e.g. [1], [2]. Usually, the decision making processes are based on solutions of the corresponding optimization problems. At the same time, experience of dealing with these very complicated and spatially distributed systems usually shows that these problems have to utilize a proper decomposition/clustering approach, which can be based on zonal, time, frequency and other attributes of nodes/units; see e.g. [3], [4]. In this paper, we consider one of such problems, i.e. optimal allocation of a homogeneous resource in telecommunication networks such that the income received from users payments is maximized and the implementation costs of the network operator are minimized. This means that each zonal income calculation at a given resource share requires solution of the inner resource allocation problem. The upper level (network manager) problem consists in optimal distribution of the resource shares among zones in order to maximize the total network profit. As a result, we obtain a two-level convex optimization problem involving non smooth functions whose values are calculated algorithmically. This problem can be solved by the usual subgradient optimization methods; see [5]. In this paper, we suggest this problem to be solved by a Lagrangean duality method which enables us to reduce the initial problem to a sequence of hierarchical one-dimensional problems. In such a way we develop a new dual decomposition approach for solution finding. Besides, we suggest new ways to adjust the basic problem to networks with moving nodes. We present results of computational experiments which confirm the applicability of the new method.

II. Notation and the problem statement

Let us consider a telecommunication network with nodes attributed to users (consumers) which is divided into zones (clusters). The problem of a manager of the network is to find the optimal allocation of a limited homogeneous network resource among the zones. That is, the optimal shares should maximize the value of the total profit containing the total income received from consumers' fees and negative resource implementation costs.

Let us use the following notation:

- n is the number of zones;
- I_k is the index set of nodes (currently) located in zone k ($k = 1, \ldots, n$);
- C is the total resource supply (the total bandwidth) for the system (network);
- a_k is an unknown quantity of the resource allotted to zone k and $h_k(a_k)$ is the cost of implementation of this quantity of the resource for zone k ($k = 1, \ldots, n$);
- x_j is the unknown resource amount allotted to node j and $U_j(x_j)$ is the fee (incentive) value paid by node j for the resource value x_j;
- $\alpha_j \geq 0$ and $\beta_j < +\infty$ are, correspondingly, the lower and upper bounds for x_j ($j \in I_k,\ k = 1, \ldots, n$).

Given the k-th resource share a_k, we denote by $f_k(a_k)$ the maximal income received from consumers' fees in this zone at a_k, thus defining the zonal income function. However, the implementation costs $h_k(a_k)$ for providing this quantity of the resource for the users in zone k must be taken into account. Then, the upper level problem of a network manager consists in maximizing the total network profit by means of the optimal resource allocation and is formulated as follows:

$$\max \rightarrow \sum_{k=1}^{n}(f_k(a_k) - h_k(a_k)) \tag{1}$$

subject to

$$\sum_{k=1}^{n} a_k \leq C, \tag{2}$$

$$a_k \geq 0,\ k = 1, \ldots, n. \tag{3}$$

Clearly, (1)–(3) can be rewritten equivalently as the minimization problem:

$$\min \rightarrow \sum_{k=1}^{n}(h_k(a_k) - f_k(a_k)) \tag{4}$$

subject to (2), (3).

Now we turn to the determination of the value $f_k(a_k)$. It is calculated as the optimal value of the optimization problem:

$$\max \rightarrow \sum_{j \in I_k} U_j(x_j) \tag{5}$$

subject to

$$\sum_{j \in I_k} x_j \leq a_k, \tag{6}$$

$$\alpha_j \leq x_j \leq \beta_j,\ j \in I_k. \tag{7}$$

Hence, $f_k(a_k)$ is determined algorithmically without any explicit formula, which imposes certain restriction on solution methods for the upper level problem (1)–(3).

Besides, we notice that both the above optimization problems involve separable functions, however, each function f_k is non-differentiable in general. For this reason, we should develop special solution methods in order take into account these peculiarities. Hence, we need in more detailed investigation of properties of the above optimization problems.

At the same time, we observe that the approach described enables us to take into account simultaneously income/expense values from two levels without utilizing usual complicated game-theoretical models described e.g. in [6], [7], see also the references therein.

III. Dual decomposition method

We now intend to suggest suitable solution methods for problem (1)–(3). We shall utilize the following natural assumptions. Suppose that

(a) each function U_j be continuous, concave and non-decreasing on $\mathbb{R}_+$;

(b)

$$\sum_{k=1}^{n}\sum_{j \in I_k} \alpha_j \leq C;$$

(c) each cost function h_k is convex on $\mathbb{R}_+$.

We intend to show that (1)–(3) (or (4), (2), (3)) becomes a convex optimization problem under these assumptions.

First we notice that problem (5)–(7) has always a solution due to (a) and (b), i.e., the function f_k is definite on $\mathbb{R}_+$. Moreover, we can now prove that f_k is concave on $\mathbb{R}_+$.

In fact, set $x_{(k)} = (x_j)_{j \in I_k}$, take arbitrary numbers $a'_k \geq 0$, $a''_k \geq 0$, let $x'_{(k)}$, $x''_{(k)}$ be solutions of problem (5)–(7) with $a_k = a'_k$ and $a_k = a''_k$, respectively. Take an arbitrary number $\lambda \in [0, 1]$, set $a_k(\lambda) = \lambda a'_k + (1-\lambda)a''_k$ and $x_{(k)}(\lambda) = \lambda x'_{(k)} + (1-\lambda)x''_{(k)}$. Then $x_{(k)}(\lambda)$ satisfies (6)–(7) with $a_k = a_k(\lambda)$, hence, by concavity of U_j, we have

$$\begin{aligned} f_k(x^*_{(k)}(\lambda)) &\geq f_k(x_{(k)}(\lambda)) = \sum_{j \in I_k} U_j(x_j(\lambda)) \\ &\geq \lambda \sum_{j \in I_k} U_j(x'_j) + (1-\lambda) \sum_{j \in I_k} U_j(x''_j) \\ &= \lambda f_k(x'_{(k)}) + (1-\lambda) f_k(x''_{(k)}), \end{aligned}$$

where $x^*_{(k)}(\lambda)$ denotes a solution of (5)–(7) with $a_k = a_k(\lambda)$. Therefore, f_k is concave, as desired. Clearly, this property together with assumption (c) implies that (1)–(3) (or (4), (2), (3)) is a convex optimization problem. However, the functions f_k have no explicit formulas and need not be differentiable. Due to these peculiarities we should carefully choose a solution method.

We take problem (4), (2), (3) as a basic one and, for brevity, rewrite it in a more general format:

$$\min \rightarrow \varphi(a) \quad \text{subject to (2) and (3),} \tag{8}$$

where $a = (a_1, \ldots, a_n)$ and

$$\varphi(a) = \sum_{k=1}^{n} \varphi_k(a_k),$$

with

$$\varphi_k(a_k) = (h_k(a_k) - f_k(a_k)).$$

In [5], it was proposed to solve this problem directly with suitable convex non-differentiable optimization methods or the heuristic Nelder-Mead method with utilization of the penalty functions, if necessary. However, those methods do not allow us to apply a decomposition techniques at the upper level. Now we intend to apply some other approach, namely, we suggest a dual iterative method which enables us to utilize its decomposable structure in full via sequential solution of families of only one-dimensional problems.

Let us write the Lagrange function for problem (8):

$$\begin{aligned} M(a,\lambda) &= \varphi(a) + \lambda\left(\sum_{k=1}^{n} a_k - C\}\right) \\ &= \sum_{k=1}^{n} (\varphi_k(a_k) + \lambda a_k) - \lambda C, \end{aligned}$$

defined on $W \times \mathbb{R}_+$, where W is the set of points satisfying (3), $\mathbb{R}_+ = \{\lambda \in \mathbb{R} \mid \lambda \geq 0\}$. Then we can define the dual problem:

$$\max_{\lambda \geq 0} \to \psi(\lambda), \tag{9}$$

where

$$\psi(\lambda) = \min_{a \in W} M(a,\lambda) = \sum_{k=1}^{n} \min_{a_k \geq 0} (\varphi_k(a_k) + \lambda a_k) - \lambda C. \tag{10}$$

Thus, (9) is an one-dimensional concave non-differentiable maximization problem, and the calculation of $\psi(\lambda)$ in (10) reduces to n one-dimensional convex non-differentiable minimization problems.

Now we indicate the way of calculation of values of the function $-f_k$ because the function h_k is supposed to be given explicitly.

Let us fix k and introduce the Lagrange function for the k-th zonal problem (5)–(7):

$$L(x_{(k)}, y_k) = -\sum_{j \in I_k} U_j(x_j) + y_k \left(\sum_{j \in I_k} x_j - a_k \right),$$

then, by duality,

$$-f_k(a_k) = \max_{y_k \geq 0} \min_{x_{(k)} \in \Omega_k} L(x_{(k)}, y_k), \tag{11}$$

where $\Omega_k = \prod_{j \in I_k} [\alpha_j,\ \beta_j],\ k = 1, \ldots, n$. It follows that

$$\begin{aligned} &-f_k(a_k) = \\ &\max_{y_k \geq 0} \left\{ -a_k y_k + \min_{x_{(k)} \in \Omega_k} \left\{ \sum_{j \in I_k} (y_k x_j - U_j(x_j)) \right\} \right\} \\ &= \max_{y_k \geq 0} \left\{ -a_k y_k + \sum_{j \in I_k} \min_{\alpha_j \leq x_j \leq \beta_j} (y_k x_j - U_j(x_j)) \right\}. \end{aligned}$$

So, in order to calculate the value of $-f_k(a_k)$, one can solve $n_k = |I_k|$ independent one-dimensional problems:

$$\min_{\alpha_j \leq x_j \leq \beta_j} (y_k x_j - U_j(x_j)), \quad j \in I_k; \tag{12}$$

for a fixed y_k and for each $k = 1, \ldots, n$. Observe that the cost function in (12) is again convex. Denote its solution by $x_{(k)}(y_k) = (x_j(y_k))_{j \in I_k}$ and set

$$\tau_k(y_k) = \sum_{j \in I_k} (y_k x_j(y_k) - U_j(x_j(y_k))).$$

Then τ_k is concave and we again have an one-dimensional problem instead of (11):

$$-f_k(a_k) = \max_{y_k \geq 0} \{-a_k y_k + \tau_k(y_k)\}.$$

Denote its solution by $y_k(a_k)$, then, clearly,

$$-f_k(a_k) = -a_k y_k(a_k) + \tau_k(y_k(a_k))$$

and $-y_k(a_k)$ is a subgradient of $-f_k$ at a_k. Thus, we can utilize this decomposition technique to implement the solution methods for the convex optimization problem (4), (2), (3). In order to solve the above one-dimensional optimization problems, we can utilize well-known iterative algorithms; see e.g. [8], [9].

Besides, we can calculate the value $f_k(a_k)$ easier for some special cases. For instance, suppose that the functions are linear (affine), i.e. $U_j(x_j) = c_j x_j$ with $c_j > 0$ for $j \in I_k$. In order to find a solution in this case we rearrange the prices c_j in the non-increasing order, i.e. $c_{j_1} \geq c_{j_2} \geq \ldots \geq c_{j_{n_k}}$ where n_k denotes the number of elements in I_k, and then assign the maximal feasible values x_{j_s} in this order. We give the explicit formula in case $\alpha_j = 0$ for $j \in I_k$:

$$x_{j_s} = \begin{cases} b_{j_s} & \text{if } s < t, \\ \min\{b_{j_s}, a_k - \sum_{l=1}^{s} b_{j_l}\} & \text{if } s = t, \\ 0 & \text{if } s > t; \end{cases}$$

for $s = 1, 2, \ldots, n_k$, where t is defined by

$$\sum_{l=1}^{t-1} b_{j_l} \leq a_k \quad \text{and} \quad \sum_{l=1}^{t} b_{j_l} > a_k.$$

Next, let us consider the case where all nodes of one zone have the same utility function $U_{(k)}$ and, consequently, the values x_j vary within the same interval $[\alpha_{(k)},\ \beta_{(k)}]$. Then we have

$$-f_k(a_k) = \max_{y_k \geq 0} \left\{ -a_k y_k + n_k \min_{\alpha_{(k)} \leq \xi \leq \beta_{(k)}} (y_k \xi - U_{(k)}(\xi)) \right\},$$

where $n_k = |I_k|$. So, in order to calculate the value of $-f_k(a_k)$, one can now solve only one one-dimensional problem:

$$\min_{\alpha_k \leq \xi \leq \beta_k} (y_k \xi - U_{(k)}(\xi)),$$

for a fixed y_k for each zone, which reduces essentially the computational expenses.

In order to solve problem (4), (2), (3) via (9) we can apply the *golden section method (GSM)* and find a suitable approximation of the optimal Lagrange multiplier λ^*. If the functions φ_k are strictly convex, then we can immediately obtain the optimal value a^* by solving the inner problems in (10) at λ^*. If this is not the case, the solutions of the inner problems in (10) can be infeasible with respect to the binding constraint in (2). We can overcome this drawback by applying regularization techniques, i.e. by adding strongly convex auxiliary terms to $\varphi_k(a_k)$ in order to provide the desired property. In particular, the application of the *proximal* *point method* (see e.g. [10]) consists in sequential calculation of an approximate solution a^t of the problem

$$\begin{array}{l}\min \to \left\{\varphi(a) + 0.5\tau\|a - a^{t-1}\|^2\right\} \\ \text{subject to (2) and (3) with } \tau > 0,\end{array} \tag{13}$$

which converges to a solution of (4), (2), (3) as $t \to \infty$, but now we can replace each function $\varphi_k(a_k)$ with $\tilde{\varphi}_k(a_k) = \varphi_k(a_k) + 0.5\tau(a_k - a_k^{t-1})^2$ which is strongly convex and maintain the decomposable structure of the initial problem. Hence, the above dual decomposition method should be applied to (13) and such a combined method enables us to obtain a solution of the primal problem.

Thus, we are able to replace the initial two-level optimization problem (4), (2), (3) with a sequence of embedded one-dimensional convex optimization problems. Another preference of this dual decomposition method over the usual subgradient methods consists in opportunity to obtain reaction values of each user independently of each other within each zone for a given local shadow resource price y_k. Similarly, the network manager obtains reaction values of each zone again independently of each other for a given total shadow resource price λ.

IV. Problem formulations for the case of moving nodes

In the above model it was assumed that users locations are known and unchangeable (within a given time slot). We now intend to suggest some adjustments of the above model to networks with more complex and non-stationary behavior of users (nodes), which is typical for various modern wireless systems; see e.g. [1], [2].

First of all we consider the situation when the network manager solve the resource allocation problem without preliminary knowledge about the distribution of users in zones. the above method enables one to receive the necessary information during the solution process. In fact, the network manager can report the current resource price λ to all the zones and they invite zonal users to participate in the resource distribution and send the data about their fee functions and resource requirements. After receiving this information and calculating the index set I_k, it is possible to determine the zonal utility function f_k and hence to find the optimal resource value a_k for a given λ. In conformity with the iterative scheme, all the interactions are carried out independently. Therefore, it can be easily adjusted to user data changes.

Next, in the case where the users choose a suitable zone, it is possible to provide a preliminary time slot, for preliminary calculations of prices and resource requirements. Afterwards, each user fixes the most suitable zone and participates in the resource allocation process, as above.

Now we turn to the problem of network profit evaluation for some future time slot. In this case we need some additional information about the behavior of users (nodes). It was suggested by I. Konnov (see e.g. [11]) to treat each node in such networks as a separate Markovian chain. However, our current models are essentially new in comparison with those in [11], [12], [5].

So, we determine a suitable grid $\mathcal{G}$ covering the domain of the network so that $\mathcal{G}_k$ denotes the index set of cells belonging to zone k. Next, we consider the discrete time model and suppose that, given a user (node) j we can determine the starting probability vector $\pi^{j,(0)}$, whose components $\pi^{j,(0)}_{\sigma}$ give its probabilities to be in cell $\sigma \in \mathcal{G}$ by stage 1, and the probability $\tilde{\pi}^{j}_{\sigma\tau}$ (for the sake of simplicity, it is supposed to be independent of time) of the one stage transition $\sigma \to \tau$ for each pair $\sigma, \tau \in \mathcal{G}$. Being based on these assumptions, we can create several network models.

Single slot planning. Suppose that we are interested in evaluation of resources at some separate slot t for $t = 1, \dots, T$. We shall use the following notation:

- I_{kt} is the index set of nodes located in zone k within stage t;
- a_{kt} is an unknown quantity of the resource allotted to zone k within stage t;
- x_{jkt} is an unknown resource amount allotted to node j in zone k within stage t;
- $\alpha_{jkt} \geq 0$ and $\beta_{jkt} < +\infty$ are, correspondingly, the lower and upper bounds for the resource amount allotted to node j in zone k within stage t;
- $U_{jkt}(x_{jkt})$ is the fee paid by node j for the resource amount x_{jkt} in zone k within stage t;
- $h_{kt}(a_{kt})$ is the cost of implementation of the resource quantity a_{kt} in zone k within stage t;
- C_t is the total resource supply for the system within stage t.

Knowing the starting and transition probability vectors for each node j, we can calculate its probability $\pi^{j,(t-1)}_{\sigma}$ to be in cell $\sigma \in \mathcal{G}$ by slot t via the standard Markovian chain technique (see e.g. [13]). Afterwards we calculate the value $\tilde{p}^{j,(t-1)}_{k} = \sum_{\sigma \in \mathcal{G}_k} \pi^{j,(t-1)}_{\sigma}$ for each zone k and assign node j to zone $\bar{k}$ where the probability $\tilde{p}^{j,(t-1)}_{\bar{k}}$ is maximal, i.e. then $j \in I_{\bar{k}t}$. Therefore, we replace the problem (4), (2), (3) with the following:

$$\min \to \sum_{k=1}^{n} \left(h_{kt}(a_{kt}) - f_{kt}(a_{kt})\right) \tag{14}$$

subject to

$$\sum_{k=1}^{n} a_{kt} \leq C_t, \tag{15}$$

$$a_{kt} \geq 0, \ k = 1, \dots, n; \tag{16}$$

where $f_{kt}(a_{kt})$ is the total fee paid by nodes of zone k for the resource amount a_{kt} within slot t, which is determined as the optimal value of the cost function in the problem

$$\max \to \sum_{j \in I_{kt}} U_{jkt}(x_{jkt}) \tag{17}$$

subject to

$$\sum_{j \in I_{kt}} x_{jkt} \leq a_{kt}, \tag{18}$$

$$\alpha_{jkt} \leq x_{jkt} \leq \beta_{jkt},\ j \in I_{kt}, \quad (19)$$

instead of (5)–(7). Clearly, we can apply the same dual iterative methods above to solve this problem.

Long-term planning. If we are interested in evaluation of resources during all the slots $t = 1, \ldots, T$, the above approach is still applicable. In fact, we can utilize the same way of calculation of elements of $I_{k,t}$ and the values of $f_{kt}(a_{kt})$ as solutions of problems (17)–(19) for all $k = 1, \ldots, n$ and $t = 1, \ldots, T$. Next, we replace the problem (14)–(16) with the following:

$$\min \rightarrow \sum_{t=1}^{T} \sum_{k=1}^{n} (h_{kt}(a_{kt}) - f_{kt}(a_{kt}))$$

subject to

$$\sum_{k=1}^{n} a_{kt} \leq C_t,\ t = 1, \ldots, T;$$
$$a_{kt} \geq 0,\ k = 1, \ldots, n;\ t = 1, \ldots, T.$$

It is not so difficult to see that we can solve this problem with the help of the dual iterative methods, whose implementation reduces to sequential solution of low-dimensional optimization problems. Thus, the problem is also decomposable essentially.

In addition we consider the situation when zones have nonempty intersection. Then the same user can in principle belong to several zones simultaneously. Clearly, this situation is rather usual for wireless networks. However, this case reduces to the previous ones after rather single modification. In fact, it is sufficient to assign a unique number to a fixed pair including a user and a zone, then the same user in different zones will be treated as different users. Notice that the same user need not have the same fee function and capacity bounds in different zones. Obviously, all the previous results remain valid for this modification. In case of moving nodes, user also can belong to several zones and he is included in the corresponding index sets $I_{\bar{k}t}$ if the probabilities $\tilde{p}_{\bar{k}}^{j,(t-1)}$ are near maximal, or greater than some predefined threshold value. Here again we should use a unique number for a fixed user/zone pair. Thus, in all the formulations, we maintain the decomposable structure of the problem and can apply either the same solution methods or their slight modifications.

V. Numerical experiments

In order to verify the method proposed we performed preliminary numerical experiments on special test problems, varying parameters related to dimensionality, capacity, and accuracy. The main goal of our numerical experience was to compare the proposed dual decomposition method with the previous streamlined non-differentiable optimization methods described in [5] for solving problem (1)–(3). Since the described above models for the case of of moving nodes are similar to the basic problem with stationary nodes, we investigated the methods only for the basic problem (1)–(3).

A. Implementation

We used the following quadratic and strongly concave fee functions:

$$U_j(x_j) = -(x_j - d_j)^2 + d_j^2,$$

where $d_j > 0,\ \forall j$, are given parameters, j is the node number. All the implementation expense functions were chosen to be linear, i.e.

$$h_k(a_k) = H_k a_k,$$

where $H_k > 0,\ \forall k$ are given parameters. We utilized the *golden section method (GSM)* with a predefined accuracy for solving all the one-dimensional optimization problems with the exception of the lowest level problems of form (12) since they were solved explicitly. In fact, given y_k, the solution is determined by the following formula:

$$x_j(y_k) = \begin{cases} \alpha_j & \text{if } \bar{x}_j < \alpha_j, \\ \bar{x}_j & \text{if } \alpha_j \leq \bar{x}_j \leq \beta_j, \\ \beta_j & \text{if } \bar{x}_j > \beta_j; \end{cases} \quad (20)$$

where $\bar{x}_j = d_j - \frac{y_k}{2}$ for each $j \in I_k$. Of course, in the general non quadratic case we can apply GSM to each problem (12). The GSM stops, when at some iteration i it finds the optimal localization segment $[A_i, B_i]$ with $B_i - A_i \leq \varepsilon$, where ε is a given accuracy. For short we denote by $\text{GSM}_1(\varepsilon_1)$ the GSM with the accuracy ε_1 applied to the upper level dual problem (9).

Implementation of $\text{GSM}_1(\varepsilon_1)$ is based on calculation of the function $\psi(\lambda)$ in (9) is defined *algorithmically*. Namely, in order to find its value at some fixed $\lambda = \bar{\lambda}$, we have to solve the inner minimization problem in (10) which is reduced to n independent one-dimensional problems of form

$$\min_{a_k \geq 0} (\varphi_k(a_k) + \lambda a_k)$$

for $k = 1, \ldots n$, i.e., we have to solve

$$\min_{a_k \geq 0} \{(\bar{\lambda} + H_k) a_k - f_k(a_k)\} \quad (21)$$

for $k = 1, \ldots n$. We again can apply GSM to these problems and denote it by $\text{GSM}_2(\varepsilon_2)$ where ε_2 is the unique accuracy for each k. Thus, each iteration of $\text{GSM}_1(\varepsilon_1)$ involve n $\text{GSM}_2(\varepsilon_2)$.

In turn, implementation of $\text{GSM}_2(\varepsilon_2)$ is based on calculation of the cost function in (21) which contains the term $f_k(a_k)$, hence we should calculate the values of the function f_k defined algorithmically. Recall that it is the optimal value function of problem (5)–(7), but due to (11) we have

$$f_k(a_k) = \min_{y_k \geq 0} \Big\{ a_k y_k - \sum_{j \in I_k} \min_{\alpha_j \leq x_j \leq \beta_j} (y_k x_j - U_j(x_j)) \Big\}. \quad (22)$$

for $k = 1, \ldots n$. We again can apply GSM to these problems and denote it by $\text{GSM}_3(\varepsilon_3)$ where ε_3 is the unique accuracy for each k. Thus, each iteration of $\text{GSM}_2(\varepsilon_2)$ involve $\text{GSM}_3(\varepsilon_3)$. Note that the inner function value in (22) is calculated by explicit formulas; see (20). So, we have the chain of embedded one-dimensional problems. Such a structure requires certain concordance of the accuracy values.

TABLE I
TEST PARAMETERS: n IS THE NUMBER OF ZONES, N IS THE TOTAL NUMBER OF NODES, AND C IS THE TOTAL RESOURCE OF THE SYSTEM.

	n	N	C
Test 1	2	10	210
Test 2	3	12	295
Test 3	6	20	474
Test 4	12	24	559

TABLE II
COMPUTATIONAL RESULTS.

	resource	profit	CPU-time (sec.)
Test 1, DM	210	5664	16
Test 1, PFM	210	4729	18
Test 2, DM	286	8965	12
Test 2, PFM	270	8912	0.8
Test 3, DM	262	14177	30
Test 3, PFM	259	14100	49
Test 4, DM	470	14737	58
Test 4, PFM	542	12904	134

B. Numerical results

The basic data of test problems are given in Table I. We compared the results of the dual decomposition method (DM) and one of the methods used in [5], namely, the combined penalty function and Nelder–Mead method (PFM); see [5] for more details. The results are presented in Table II. We note that high solution accuracy of the embedded problems at initial steps yielded additional time expenses, whereas the adaptive strategy provided significant preference. More precisely, we fixed the accuracy ε_1 for $\mathrm{GSM}_1(\varepsilon_1)$ and took ε_2 and ε_3 as $O(\frac{1}{J^2})$ and $O(\frac{1}{J^3})$, respectively, where J denotes the number of steps of $\mathrm{GSM}_1(\varepsilon_1)$.

From the results it follows that the dual decomposition method demonstrates as a rule better results both in goal function and time expenses, for large dimensional problems these difference becomes more essential. We should note that the calculation time for the dual decomposition method was less than 1 minute in all the cases.

VI. CONCLUSIONS

We considered a problem of optimal allocation of a homogeneous resource in telecommunication networks where the income received from users payments is maximized and the implementation costs of the network operator are minimized, i.e. one maximizes the total profit of the network. The problem is suggested to be formulated as a two-level convex optimization problem involving non smooth functions whose values are calculated algorithmically. In principle it can be solved by the usual subgradient optimization methods as in [5]. In this paper, we suggest a Lagrangean duality method which enables us to reduce the initial problem to a sequence of hierarchical (embedded) one-dimensional problems. That is some of embedded optimization problems are in fact multi-dimensional, but they can be decomposed into several one-dimensional problems solved in parallel. In such a way we develop a new dual decomposition approach for solution finding. Besides, we show that resource allocation problems for networks with moving nodes also fall into the basic problem format or represent its slight modifications. Hence, they can be solved by similar solution methods. We presented results of computational experiments which confirmed the applicability of the new method.

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