Evolution of Multimodulus Algorithm Blind Equalization Based on Recursive Least Square Algorithm

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ABSTRACT

Blind equalization is an important technique amongst equalization family. A Multimodulus algorithm based on blind equalization removes the undesirable effects of ISI and cater ups the phase issues, saving the cost of rotator at the receiver end. In this paper a new algorithm combination of recursive least square and Multimodulus algorithm named as RLSMMA is proposed by providing few assumption, fast convergence and minimum Mean Square Error (MSE) is achieved. Excellence of this technique is shown in the simulations presenting MSE plots and the resulting filter results.

KEYWORDS: Blind Equalizations, Constant Modulus Algorithm, Multimodulus Algorithm, Recursive Least square Algorithm, Quadrature Amplitude Modulation (QAM).

1 INTRODUCTION

Equalization is the basic building block of modern digital communication system. Equalization is defined as the process which helps to retrieve the transmitted signal free from the undesirable channel effects at the receiver end. Undesirable effect can be classified as (linear) channel distortion and additive noise commonly known as (ISI) that corrupts the transmitted signal making it cumbersome for the receiver end to recover the transmitted data directly. Equalization can be categorized into two major categories the Non-Blind equalization and the Blind equalization.

Non blind equalization is a technique which equalizes the received signal by the help of training bit to update the weights. Training bits are embedded with the transmitted signal and repeated every time, information about the training bits are pre known at the receiver so thereby the received training bits are analyzed and the channel response is calculated accordingly to match the equalizer output minimizing some criterion typically MSE(Mean Square Error). Non blind equalization gives a better performance but the main disadvantage is the wastage of bandwidth about 25% bandwidth consumption in GSM (global system for mobile communication) [1]. Blind Equalization or self-recovering algorithm defines an equalization without the use of training bits [3], [9]. Blind equalization has been a hot area of research from the last few decades. Constant Modulus Algorithm is considered as one of the famous algorithms of blind equalization proposed by Godard [1], [2], [3]. CMA utilizes method of steepest descent to equalize the signal. CMA due to its increased bandwidth efficiency which increases bits rate [1], and its simplicity like

LMS makes it very popular but its major weakness is that it has slow convergence seemingly its cost function is also dependent on the amplitude of the signal thus resulting lack of knowledge about the constellation, hence the overall performance suffers when using higher order QAM schemes [4], [5]. A replacement of CMA was proposed called as MMA [6], [7]. MMA despite of just minimizing the magnitude of equalizer's output y(n), it considers both the real $y_k(n)$ and $y_i(n)$ individually[8]. MMA achieves better convergence and the cost of rotator at the receiver end is also nullified.

In this research work a new MMA based algorithm has been proposed by having some enhancement to the Multimodulus algorithm results are quite considerable for the QAM constellations. In section 2 the existing algorithms are discussed which contains the brief description about CMA, RLSCMA and MMA. In section 3 new algorithm is proposed, followed by the simulation comparison included in section 4 and in section 5 the final work is concluded.

2 THE EXISTING BLIND ALGOTHMS

2.1 Constant Modulus Algorithm

Constant Modulus Algorithm was basically proposed by Godard [1], [2], [3]. CMA is also one of most famous algorithm amongst the Bussgang algorithm it utilizes memory less nonlinear function in filters output to obtain the results [1]. CMA algorithm is derived from the method of steepest descent when no training bits are utilized [1]. Cost function is formulated by:

$$J(n) = E[(|y_n|2 - R_p)2] \quad (1)$$

where y_n is the filter's output and Rp is the non-negative constant. Cost function is differentiated to get alike LMS algorithm, further it is only dependent on the modulus of filters output so the information about the carrier phase is lost. Equations of CMA algorithm is concluded as:

$$w_i(n+1)=w_i(n) + \mu x(n-i)y_n(Rp- |y_n|^2) \quad (2)$$

Step size is denoted by µ, $w_i(n)$ is the ith tap equation at a certain time n, similarly x(n) is the input of filter at time n, Rp is positive constant.

Error equation is calculated as:

$$e(n)=y_n(Rp-|y_n|^2) \quad (3)$$

where e(n) is the error term equation, y_n is the equalizer output.

Rp is calculated as:

$$Rp = \frac{E(|I_n|^4)}{E(|I_n|^2)} \quad (4)$$

where I_n is the transmitted source signal.

Constant Modulus Algorithm is preferred over other Godard algorithm due to its promising performance with respect to Mean Square Error (MSE), but due to slow convergence rate and lacking the knowledge about the carrier phase it leads to the research of other algorithms.

2.2 Recursive Least Square Constant Modulus Algorithm

Due to slow convergence and adaptation of CMA algorithm, Nassar Amin and Nahal Waleed introduced a modification of CMA algorithm and used Method of Least square over Method of steepest descent algorithm for CMA [11]. RLSCMA outperform CMA in term of MSE [11], [12].

Cost function of RLSCMA for the fast convergence is specified by:

$$J(w)=E[(|y(n)|^2 - 1)^2] \quad (5)$$

$$J(w)=\sum_{k=1}^{n} \lambda^{n-k}(|y(n)|^2-1)^2 \quad (6)$$

λ is the forgetting factor and valued between. $0 < \lambda < 1$. RLSCMA is derived from standard Recursive least square algorithm except it considers an input signal z(n) [11], rest of algorithm is concluded as:

$$z(n)=x(n)xH(n)w(n-1) \quad (7)$$

$$h(n)=p(n-1) * z(n) \quad (8)$$

p(n-1) is the inverse correlation matrix and Kalman gain is calculated as follow:

$$k(n)= \frac{h(n)}{\lambda+z^H(n)h(n)} \quad (9)$$

$$e(n)=1 - wH(n-1)z(n) \quad (10)$$

e(n) is the error term equation.

Weight update equation is as follow:

$$w(n)=w(n-1)+k(n)e^*(n) \quad (11)$$

P(n) inverse correlation matrix update is as follow:

$$p(n) = \frac{p(n-1)}{\lambda} - \frac{k(n)z^H(n)p(n-1)}{\lambda} \quad (12)$$

initial conditions are specified as w(0)=[1,1x(k-1)] ,$p(0)=\delta^{-1}I_{kxk}$ delta δ is assumed small positive constant like 10^{-3}.

2.3 Multimodulus Algorithm

Multimodulus algorithm (MMA) was introduced by Yang et al. [6], [7]. Carrier phase deficiency of CMA was catered in the this algorithm the cost function equation of MMA is as follow:

$$J_{MMA}(n)= J_R(n)+ J_I(n) \quad (13)$$

$$J_{MMA}(n)=E[(y^2_R(n) -R_{2,R})^2]+ E[(y^2_I(n) -R_{2,I})^2] \quad (14)$$

where $y_I(n)$ and $y_R(n)$ are the imaginary and real terms of equalizer's output. $R_{2,R}$ and $R_{2,I}$ can be calculated as per following equations:

$$R_{2,R}=\frac{E[s_R^4(n)]}{E[s_R^2(n)]} \quad (15)$$

$$R_{2,I}=\frac{E[s_R^4(n)]}{E[s_R^2(n)]} \quad (16)$$

where $s_I(n)$ and $s_R(n)$ are the imaginary and real part of the transmitted source signal. Weight update equation is given as:

$$w(n+1)= w(n) - \mu e(n).x^*(n) \quad (17)$$

similarly the error term is also the sum of real part of error $e_R(n)$ and the imaginary counterpart of error $e_I(n)$ given in following equations:

$$e_R(n)= y_R(n)(y_R^2(n)- R_{2,R}) \quad (18)$$

$$e_I(n)= y_I(n)(y_I^2(n)- R_{2,I}) \quad (19)$$

$$e(n)= e_R(n)+ e_I(n) \quad (20)$$

MMA can remove ISI and explicitly resolves the carrier phase issues due to its better phase tracking capability as stated above. However MMA does not performs well for the dense constellations, so to cater up this issue a new algorithm is proposed for fast convergence and providing better results for dense QAM.

3 NEW BLIND ALGORITHM

A new blind algorithm Recursive Least Square Multimodulus Algorithm (RLS-MMA) is proposed in this section. Combining the work done by S.Makino and Y.Kaneda for Recursive Least Square [13], [14], [15], [16]. Chen and et.al and for Recursive Least Square Constant Modulus Algorithm [17], [18] and various links for CMA and MMA convergence and MSE optimization a new algorithm is developed. Performance of the equalizers is analyzed by the MSE plots. Cost function of the algorithm is defined as:

$$J(w)=E[((y^2{}_R(k)-1)^2]+E[((y^2{}_I(k)-1)^2] \quad (21)$$

$$J(w)=\sum_{k=1}^{n} \lambda^{n-k}(y^2{}_R(k)-1)^2+\sum_{k=1}^{n} \lambda^{n-k}((y^2{}_I(k)-1)^2 \quad (22)$$

where $y_I(k)$ and $y_R(k)$ are the imaginary and real terms of equalizer's output and λ is the forgetting factor valued between. $0 < \lambda < 1$.

Algorithm of RLS-MMA is as follow, z(n) is the input of the equalizer:

$$z(n) = x(n)x^H(n)w(n-1) \quad (23)$$

$$h(n) = p(n-1)z(n) \quad (24)$$

p(n-1) is defined as inverse correlation matrix. Kalman gain is as follow:

$$k(n)= \frac{h(n)}{\lambda+z^H(n)h(n)} \quad (25)$$

$$e_R(n) = y_R(n)(\, y_R{}^2(n)- R_{2,R}) \quad (26)$$

$$e_I(n) = y_I(n)(\, y_I{}^2(n)- R_{2,I}) \quad (27)$$

$$e(n)= e_R(n)+ e_I(n) \quad (28)$$

$$R_{2,R}=\frac{E[s_R^4(n)]}{E[s_R^2(n)]} \quad (29)$$

$$R_{2,I}=\frac{E[s_R^4(n)]}{E[s_R^2(n)]} \quad (30)$$

where $s_I(n)$ and $s_R(n)$ are the imaginary and real part of the transmitted source signal.

Weight update equation is given as:

$$w(n)=w(n-1)+k(n)e^*(n) \quad (31)$$

P(n) inverse correlation matrix update is as follow:

$$p(n) = \frac{p(n-1)}{\lambda} - \frac{k(n)z^H(n)p(n-1)}{\lambda} \quad (32)$$

4 SIMULATION RESULTS

The following section evaluates the performance of the entire algorithm mentioned where the main parameter focused was lay on the Mean Square Error (MSE). Performance of CMA and RLSCMA was compared further MMA performance was compared with the proposed algorithm RLSMMA. In the simulations s(n) was the QAM signal, SNR is 30db. All the simulation considered a channel of a seventh order FIR filter similarly the equalizer is

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also FIR adaptive equalizer of seventh order is considered. CMA and MMA simulations used a stepsize μ of value 0.09 while forgetting factor λ is considered 0.99 for RLSCMA and RLSMMA.

Results are shown as follow:

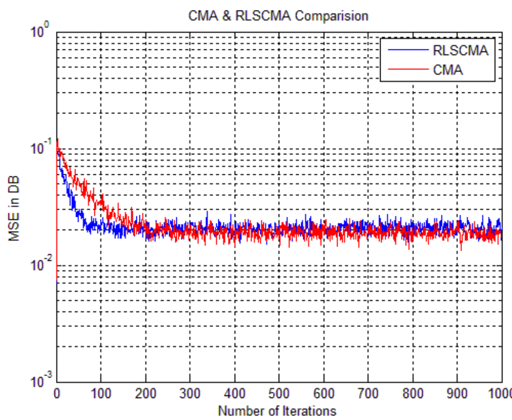

Figure1. Comparison of CMA and RLSCMA

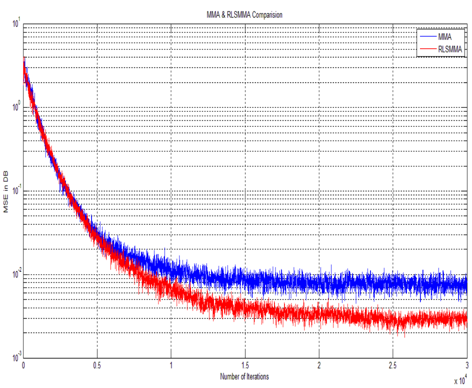

Figure 2. Comparison of MMA and RLSMMA

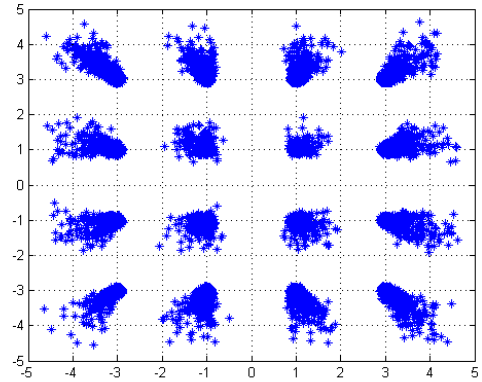

Figure 3. Equalized results of MMA

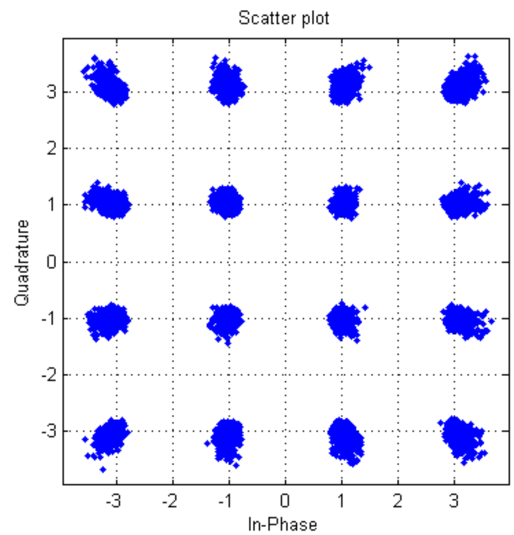

Figure 4. Equalized results of RLSMMA

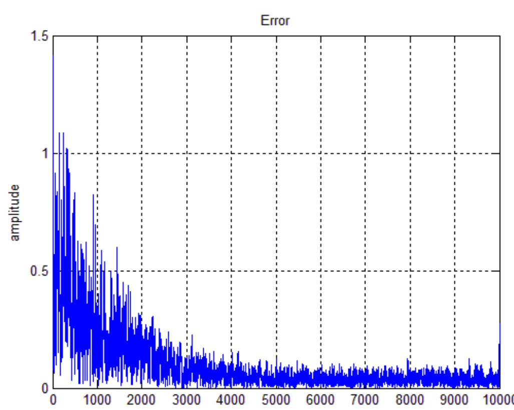

Figure 5. Error results of RLSMMA

Figure 1 shows the performance comparison of CMA and RLSCMA. RLSCMA outperforms CMA as it attains fast convergence and Mean Square Error is also lower then CMA.

Figure 2 shows the performance analysis of MMA and RLSMMA. It is clear that the performance of the newly proposed RLSMMA is much better than MMA for dense QAM as MSE is much lower in the RLSMMA case then MMA, clearly indicating the promising results as shown in figure 3.

Figure 5 shows the error results of the proposed new blind equalizer. The results are comparable and quite promising.

5 CONCLUSION

A new blind equalization technique RLSMMA was introduced in this paper. Simulations were carried on 16QAM symbol sequence. Results obtained for RLSMMA were quite promising as compared to the conventional blind equalization algorithms. The main advantages over other can be evaluated as follow, high rate of adaptation, minimum mean square error, less computational complexity, cost of rotator at receiver end saved.

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