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Population Mean Estimate for Adaptive Modulation under Large Phase Error in Single Beamforming Sensor Array

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ABSTRACT

Beamforming is a signal processing technique to focus the transmitted energy so that maximum energy is radiated in the intended destination and communication range is enhanced. Data rate improvement in Transmit beamforming can be achieved with adaptive modulation. Though modulation adaptation is possible under zero-mean phase error, it is difficult to adapt it under non-zero mean Gaussian distributed phase error conditions. Phase errors occur due to channel estimation inaccuracies, delay in estimation, sensor drift, quantized feedback etc resulting in increased outage probability and Bit error rate. Preprocessing of beamforming weights adjusted by Sample Mean Estimate (SME) solves the problem of adaptive modulation. However, under large phase error variation, the SME method fails. Hence, in this paper, Population Mean Estimate (PME) approach is proposed to resolve these drawbacks for a Rayleigh flat fading channel with White Gaussian Noise. To correct the population mean error if any, Least Mean Square correction algorithm is proposed and is tested up to 80% error in PME and the corrected error fall within 10% error. Simulation results for a distributed beamforming sensor array indicate that the proposed method performs better than the SME based existing methods under worst-case phase error distribution.

KEYWORDS

Wireless sensor array, Transmit Beamforming, Modulation Adaptation, Phase error correction, Population Mean approach.

1 INTRODUCTION

Distributed Wireless Sensor Networks are popularly used to sense physical quantities and transmit the collected data to the controlling stations. The challenges of these sensor networks are scalability, power consumption, locality issues, security etc [1]. The hardware design requires robustness and lower power consumption. One of the methods of power conservation in Wireless Sensor Networks (WSN) is Cooperative Beamforming technique. For an N element array, the Signal to Noise Ratio (SNR) in the intended direction due to beamforming is N^2 and in the other directions, it is N [2]. The increase in gain in the intended direction is caused by coherence of the participating array signals and this requires phase synchronization of each participating node. The performance characteristic of uniform and Gaussian distributed cluster of sensor nodes was studied in [3] and [4]. Since sensors are typically randomly distributed in the field, phase errors are caused due to positional variations of the sensors from the ideal positions [5, 6]. Though the position error does not affect the beam direction [7], it reduces the SNR because the position error can be transformed to phase error and the correlated phase noise has effect on the beamforming [8]. The outage probability increases and the Bit Error Rate (BER) increases as the SNR reduces. Phase errors can also occur due to channel estimation inaccuracies, delay in estimation, sensor drift, quantized feedback etc

As Collaborate beamforming of WSN can be used to enhance the communication range and transfer message [9, 10], it can complement sensor routing and it finds wide application in sensor and other wireless communication applications. However, with phase error in beamforming, there is the possibility of SNR reduction and increase in BER.
The average BER of sensor distributed beamforming including phase error is analyzed for

Binary Phase Shift Keying (BPSK) modulation with 10 and 20 nodes array for Static, Rayleigh and Rician Channel [11] and it indicates a increase in BER with increasing phase error. The Average Packet Error Rate (PER) was analyzed in [12] for Distributed beamforming with phase errors caused by delay in estimation, estimation inaccuracies etc for BPSK modulation and the phase errors was modeled as Gaussian distributed. The analysis results indicate that PER increases with introduction of phase errors. The Phase State Information (PSI) required for phase synchronization can be estimated using either Mean Square Error, Signal to Noise Ratio (SNR), Maximum Likelihood (ML) and Minimum Variance Distortion reduced (MVDR) as in [13]; however each method has its error in estimates.

Adaptive modulation has been proven to provide data rate improvement compared to fixed communication [14,15]. The adaptive communication for Multi Input Multi Output system with coded and uncoded Quadrature Amplitude modulation (QAM) has been analyzed in terms of power allocation, BER etc. with partial channel state information [16] and using Sample Mean Feedback [17]. The adaptive modulation based on Packet Error probability has been analyzed in [18]. Adaptive modulation combined with two-dimensional Eigen beamforming using Sample Mean Feedback has been proposed in [19].

However, the Sample Mean of phase error is difficult to estimate incase of Multi Input Single Output (MISO) systems and it needs calibration. One of the methods of estimating Sample Mean is as given in [20] and the second technique is using orthogonal vectors to parallelize the N channel paths as in [21]. However, both methods yields approximated result, fail under large mean, and phase variation. Hence, in this proposed work, Population Mean estimate (PME) based approach is used for adaptive modulation in distributed sensor MISO beamforming under phase error for Rayleigh flat fading channel with Additive White Gaussian noise (AWGN). The phase error is modeled as Gaussian variable as in [12] but with Nonzero-Mean. Least Mean Square (LMS) correction algorithm is also proposed to correct error in PME.

This paper is organized as follows. Section 1 gives the introduction and existing works. Section 2 explains the required model and the proposed work. Section 3 presents the simulation results and discussion. Section 4 outlines the conclusions. The boldface letters in equation represent vector or matrix quantity.

2 PROBLEM FORMULATION AND PROPOSED SOLUTION

This section explains the modeling of MISO beamforming array and adaptive communication in MISO beamforming under phase error. The problem in using Sample Mean Estimates (SME) is described followed by the proposed method using Population Mean Estimate.

Large number of factors like estimation error, quantization error, delayed feedback, finite precision effect; incremental channel variation, detector device tolerance, position error etc contribute to phase error. As the WSN is energy conserving [1], the preprocessing of beamforming weights by population mean to equalize the effect of phase error is analyzed and finally the Adaptive Modulation under phase error in Transmit beamforming algorithm is presented.

2.1 Modeling of Phase Error

Let the random variable Y represent the process phase error. Let x1 represents the errors due to device aging, x2 represents quantization effect, x3 represents feedback delay effect, x4 represents the temperature effect, x5 represents estimation and positional variation errors, x6 represents power variations, x7 represents startup delays, x8 represents crystal error, x9 is channel variation, x10 is detector error etc. Then, the variable Y is sum of its contributing parts. By central limit theorem (CLT), the variable Y approaches Gaussian distribution as given in equation (1) with (population) mean θ_μ and standard deviation θ_σ.

$$Y = \sum_{i=1}^{10} X_i \qquad (1)$$

where each X_i is the random variable, representing the contributing process as defined above. The distribution of Y follows Gaussian with mean (θ_μ) and standard deviation (SD) (θ_σ) is shown in Fig 1. The modeling is assumed to be stationary .i.e. the mean and SD does not vary with time. $\mathbf{Y} \in \mathbf{N}(\theta_\mu, \theta_\sigma)$

The instantaneous samples of Y from the N nodes is represented as

$$\boldsymbol{\theta}_{err} = (\theta_{err,1}, \theta_{err,2}, \theta_{err,3},,, \theta_{err,N}) \in \mathbf{N}(\theta_\mu, \theta_\sigma) \qquad (2)$$

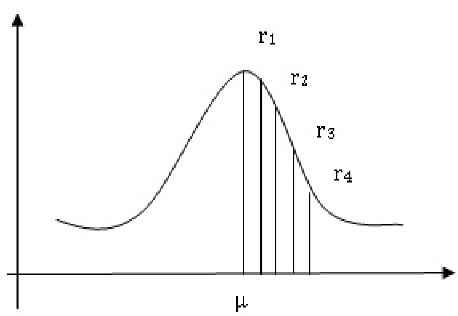


Figure 1. Gaussian Distributed Phase Error with Non zero-Mean

2.2. Modeling of MISO Transmit Sensor Beamforming with phase error

Beamforming is the one of the important techniques of reducing transmission power and this is achieved by phase synchronization of signals from all the array nodes so that the signal gets maximum energy in the intended direction whereas less energy is radiated in the unintended direction. The average beam formed from N nodes with the phase constant θi from each node to destination and beamforming weights θ_{wi} is given by equation (3) as in [22].

$$AF = \frac{1}{N} \sum_{i=1}^{N} a_i e^{j(\theta_i - \theta_{wi})} \qquad (3)$$

where a_i is the attenuation constant and it is considered as Rayleigh distributed. Consider a cluster of sensor nodes transmitting signal to destinations as shown in Fig 2

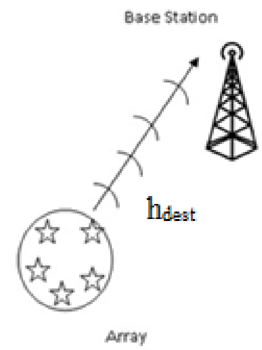


Figure 2. Cluster of Sensor Array transmitting messages to destination

The received signal at the destination with transmit beamforming in the presence of AWGN noise n(t) and channel h_{dest} is given by equation (4) in vector form

$$y_{dest} = \mathbf{w}^{-1} \mathbf{h}_{\mathbf{dest}} \mathbf{x} + n \qquad (4)$$

where

$$\mathbf{x} = [x_1, x_2, x_3, x_4,,,,, x_N]^T$$

$$\mathbf{h}_{\mathbf{dest}} = [r_{dest,1} e^{-j\theta_{dest,1}}, r_{dest,2} e^{-j\theta_{dest,2}}, r_{dest,3}\, e^{-j\theta_{dest,3}},,,, r_{dest,N}\, e^{-j\theta_{dest,N}}]$$

$r_{dest,i}$ are path attenuation and θ_{desti} are the phase constants, i = 1 to N, N is number of nodes in the array.

In case of transmit beamforming without phase error, the estimated weight is w = h_{dest}. Suppose, the estimated weight has error h_{err} it is given by equation (5)

$$\mathbf{w} = \mathbf{h}_{dest} + \mathbf{h}_{err} \qquad (5)$$

where

$$\mathbf{w} = [r_{dest,1} e^{-j\theta_{dest,1}}, r_{dest,2} e^{-j\theta_{dest,2}}, r_{dest,3}\, e^{-j\theta_{edst,3}},,,, r_{dest,N}\, e^{-j\theta_{dest,N}}]^T$$

$$\mathbf{h}_{err} = [r_{err,1} e^{-j\theta_{err1}}, r_{err,2} e^{-j\theta_{err,2}}, r_{err,3}\, e^{-j\theta_{err,3}},,,, r_{err,N}\, e^{-j\theta_{err,N}}]^T$$

The attenuation variable $r_{err,i}$ is assumed to be distributed as Rayleigh variable in accordance with channel model and $\theta_{err,i}$ is the error in estimated values. Substituting beamforming weight w as given by equation (5) in equation (4) and using matrix inverse identity given by equation (6), we get the following equation (7)

$$(\mathbf{A} + \mathbf{B})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}(\mathbf{A}^{-1} + \mathbf{B}^{-1})^{-1}\mathbf{A}^{-1} \qquad (6)$$

$$y_{dest} = (\mathbf{h}_{dest}^{-1} - \mathbf{h}_{dest}^{-1}(\mathbf{h}_{dest}^{-1} + \mathbf{h}_{err}^{-1})^{-1}\mathbf{h}_{dest}^{-1})\mathbf{h}_{dest}\mathbf{x} + n \quad (7)$$

On simplification, the following equation is obtained.

$$y_{dest} = (1 - \mathbf{h}_{dest}^{-1}(\mathbf{h}_{dest}^{-1} + \mathbf{h}_{err}^{-1})^{-1}\mathbf{x} + n \quad (8)$$

The channel error $\mathbf{h}_{err\text{-}final}$ is defined as

$$\mathbf{h}_{err-final} = 1 - \mathbf{h}_{dest}^{-1}(\mathbf{h}_{dest}^{-1} + \mathbf{h}_{err}^{-1})^{-1} \quad (9)$$

The response of sensor array with phase error can be rewritten as follows

$$y_{dest} = \frac{1}{N}\mathbf{h}_{err-final}\mathbf{x} + n \quad (10)$$

Therefore, the effect of error in estimate is to reduce the Received Signal Strength (RSS). This in turn reduces the SNR, increases the BER and outage probability increases.

2.3 Adaptive Modulation under phase error

Adaptive modulation is a method of achieving higher data rate [23] through which the symbol error probability can be reduced. In general, adaptive modulation problem can be defined as follows [16]

$$M = \underset{M\in\{Mi\}}{\operatorname{argmax}}(M), \mathrm{BER(M)} \le \mathrm{BER}_{target} \quad (11)$$

where M is modulation index. With phase error, the received signal strength reduces. Assuming $\sigma_{ph\text{-}err}^2$ is variance due to phase error, the following equation represents SNR as in [12].

$$SNR = \frac{P_r}{\sigma_n^2 + \sigma_{ph-err}^2} \quad (12)$$

It can be also defined using signal level as follows

$$SNR = \frac{|\sum_{i=1}^{n1} r_i e^{j(\theta_i)}|^2}{\sigma_n^2 + |\sum_{k=1}^{n2} r_j e^{j(\theta_k)}|^2} \quad (13)$$

where n1 is the number of nodes whose phase error is within the modulation threshold, n2 is number of nodes whose phase error is above modulation threshold as given in Table 1 and n1+ n2 = N is the total number of nodes.

Table 1. Phase Error Threshold value and region of MPSK modulation

Sl. No	MPSK	Region	Phase error threshold values
1	BPSK	r_4	π/2-π
2	QPSK	r_3	π/4-π/2
3	8PSK	r_2	π/8-π/4
4	16PSK	r_1	0-π/8

With this definition of SNR, the BER becomes a function of Phase Error and Noise

$$BER = BER(\boldsymbol{\theta}_{err}, n) \quad (14)$$

where the phase error vector is defined as

$$\boldsymbol{\theta}_{err} = ang(\mathbf{h}_{err-final}) \quad (15)$$

Under this condition, the selection of modulation schemes (M-PSK) is based on the phase error as can be seen in Fig.1. In a Rayleigh fading channel, QAM modulation suffers from high BER than MPSK. Hence, MPSK is selected in this work. The phase error distribution is divided into the four regions as shown in Fig 1. In low phase error region, higher order modulation has to be used to improve data rate as well as to keep BER under control. Hence, in the region r1, 16PSK, in r2, 8PSK, in r3, QPSK and in r4, BPSK are to be used.
The average data rate over the SNR range with inclusion of phase error is defined as follows in equation (16)

$$\mathrm{R} = (\sum_{i=1}^{n}\log_2(M_i)\int_{r_i} p(\theta_{err})d\theta_{err}) * (\log_2(1+SNR)) \quad (16)$$

The lower bound for data rate is obtained when BPSK is used.
The Table-1 is applicable for the situation where the phase error distribution has Zero-Mean distributed. However, in this paper the phase error is assumed to be distributed as Non Zero-Mean with mean of π. Therefore, phase error estimation is required. The use of phase error occurring on each path would give better result than using mean of error over all N paths. However, considering the feedback path nature and to reduce the number of feedbacks, the mean of phase error is assumed in this paper.

2.4 Adaptive Modulation in MISO Transmit Beamforming under Phase error

The phase error estimate is required to select the appropriate modulation order. However, the estimation of phase error in each path from node to destination is difficult in case of MISO system. In addition, the estimation of phase average poses the problem of sending it back to transmitter. There may be delay in feedback, quantization effect, noise influence etc. The existing methods using sample mean phase error estimates [20, 21] work up to certain limit of phase errors. The first technique [20] uses the measurement of received signal phase and the pilot phase and subtraction of it. This gives the mean phase of received signal. The second method [21] uses orthogonal vector to get the individual channel phase error and mean is found from it.

The third method [24] uses Kalman filter technique for phase estimation for individual participating sensor. Using Kalman filter technique the phase mean is estimated by suitably modeling the given problem.

All these estimates are approximated values only, but have higher complexity due to requirement of pilot symbol or orthogonal vector generation, etc. The proposed PME has advantage of reduced feedback and less computational complexity at the source using low power sensor devices. Nevertheless, the proposed PME also yields the approximated value. Hence, the error in PME has to be taken into consideration. The error in PME is assumed uniform distributed as follows.

$$\theta_{\mu e} = U(\theta_\mu - \%err * \theta_\mu, \theta_\mu + \%err * \theta_\mu) \tag{17}$$

where $\theta_{\mu e}$ is error in population mean and %errs is error in percentage. To correct this error, Least Mean Square (LMS) algorithm to minimize the mean square error (MSE) is used. Assuming that θ_μ and $\theta_{\mu e}$ are actual mean and mean with error and the error is assumed to fall within upper and lower limit as given by equation (17), the population mean can be corrected like closed loop feedback control algorithm [25]. The mean square error is given by

$$\min E(\theta_\mu - \theta_{\mu_e})^2 \tag{18}$$

Pseudo code for Population Mean Correction Algorithm

1. Estimate the BER moving average for last 10 samples
2. Calculate the BER gradient from the present BER value and BER moving average value
3. Estimate the $\theta_{\mu,new} = \theta_{\mu,old} + \theta_\Delta$ if the BER gradient is negative else $\theta_{\mu,new} = \theta_{\mu,old} - \theta_\Delta$
4. $\theta_{\mu,old} = \theta_{\mu,new}$
5. Repeat the above steps for 100 Iterations.

The flow chart for the algorithm is shown in fig. 3.

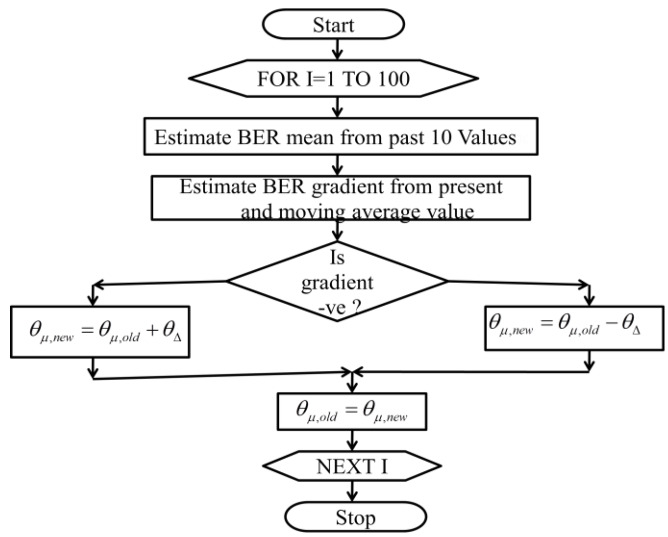


Figure 3. Flow chart for the population mean correction algorithm

The advantage of population mean correction algorithms are as follows. It uses only the past values of BER to correct the mean and it does not require any prediction, therefore prediction related errors are avoided. The calibrations of measured values to actual values are not required because BER is directly measurable. Only 1 bit of information for sign change is used. However, with Sample mean measurement the current values of mean is required and this requires more number of bits. In addition, this algorithm need not be executed every time as the process parameter is assumed to be non-varying over time. The populations mean correction algorithm can also be used to correct mean from some assumed value as initial population mean. In this paper, the error up to 80% is assigned to initial population mean value and calibration is carried out.

2.5 Preprocessing of beam forming weights by Population Mean approach

In this section, the estimation of phase error distribution with and without PME is discussed.
If $f_{w,peadjust}(w)$ denotes the probability density function of w with phase error and $f_{w,nophadjust}(w)$ with no phase error, then.

$$f_{w,\text{no}\, phadjust}(w) = \mathbf{N}(\theta_\mu, \theta_\sigma{}^2) \tag{19}$$

Subtracting the population mean (θ_μ) from each $\theta_{err,\ i}$, the resultant distribution is modeled with Zero-Mean as follows.

$$\boldsymbol{\theta}_{\mathbf{err}} - \theta_\mu \in \mathbf{N}(0, \theta_\sigma) \tag{20}$$

and

$$f_{w,,phadjust}(w) = \mathbf{N}(0, \theta_\sigma{}^2) \tag{21}$$

Now the preprocessing of the h_{err} by h_μ, gives the following equation (22).

$$y_{dest} = (\mathbf{h}_{err} - \mathbf{h}_\mu)\mathbf{x} + n \tag{22}$$

Therefore, beamforming vector adjustment has made the phase error to be Zero-Mean distributed at the receiver. Without phase error adjustment, the phase error at the receiver follows the same distribution as the source and hence it is difficult to maintain the given BER. Using equations (19) and (21), and estimating the expectation, we get the following expression.

$$E[\boldsymbol{\theta}_{\text{err}}] = \theta_\mu \tag{23}$$

$$E[\boldsymbol{\theta}_{err} - \theta_\mu] = 0 \tag{24}$$

Therefore, the phase error effect is reduced by preprocessing the beamforming vector by the Population Mean Estimate (PME). From equation (24), it is proved that the result of using Population Mean Estimate with Nonzero-Mean distributed phase error is same as Zero-Mean distributed phase error. The received signal after beamforming vector preprocessing using population mean is given by the equation (25) for the destination and the system model is shown in Fig. 3

$$y_{dest}(t) = \text{Re}(\frac{1}{N}\sum_{i=1}^{N} r_{err,i}\, e^{j(\omega t + \theta_{\text{err,i}} - \theta_\mu)} x(t) + n(t)) \tag{25}$$

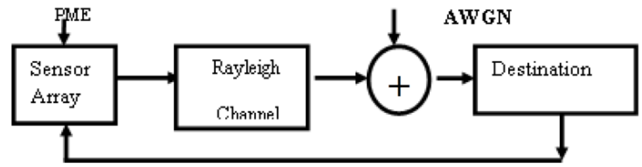


Figure 4. System Model using PME

From the system model in Fig 4, it is understood that the PME method requires only the predicted SNR value for modulation adaptation whereas the SME method requires SNR and as well as sample mean estimate for transmission.
The modulation adaptation is carried out to maintain the BER at the destination. It is assumed that the beamforming vector estimates W_{est} is available and h_μ is the mean of phase error estimated from PME

Pseudo code for Adaptive Modulation Algorithm using Population Mean Estimates

1) Estimated Population Mean ($\theta_{\mu,Y}$) from Distribution Variable Y.
2 Estimate the Population Mean Correction (θ_{err}) and add it with $\theta_{\mu,Y}$. i.e. $\theta_\mu = \theta_{\mu,Y} + \theta_{err}$
3) Adjust the already estimated Beamforming Vector (W_{est}) with Population Mean $\mathbf{W}_{adj} = \mathbf{W}_{est} - \mathbf{h}_\mu$
4) Based on the RSS received, select the modulation M if BER (M) $\leq BER_{target}$, Otherwise, Select next lower modulation M and repeat it until all M-PSK have been checked.
5) Transmit message after modulating using M-PSK
6) Repeat from step 4 until all message have been transmitted.
The flowchart is shown in Fig. 5.

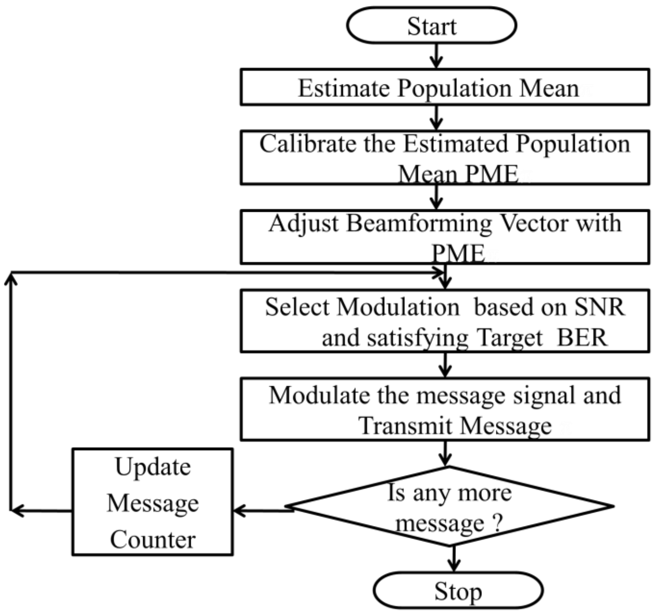


Figure 5. Flow chart of the Adaptive modulation using PME algortihm

3 SIMULATION RESULTS AND DISCUSSION

For this work, it is assumed that sensors are spread on a plane surface and no movement of sensors is allowed. Each sensor has a single isotropic antenna. The frequency synchronization is done already. Table 2 gives the parameters used for simulation. The phase error is assumed to be independent of each other. The SNR is fedback to transmitter through feedback path. The simulation is done in MATLAB software using equation (25) and using the system model explained. The results are obtained for a 10 nodes array. The target BER is taken as 10^{-2}. The population mean can be corrected up to 10% and therefore the population mean is assumed to have 10% uniformly distributed error for PME.

Table 2. Simulation Parameters

Sl.No	Parameter	Value
1	No. of Array Sensors	10
2	Iterations	1000
3	Data Bits Sequence	1200 Bits Randomly selected
4	Modulation Schemes	M-PSK -2,4,8 and 16
5	Channel Type	Rayleigh Flat Fading
6	Channel Noise	AWGN, N(0,1)
7	SNR	0 -30dB
8	Phase Error Distribution	Gaussian with Mean = π radians Sigma 0.9 radians
9	Pilot Sequence	One symbol

3.1 Adaptive Modulation without Phase Error

Fig 6 shows the BER performance assuming no phase error. There are six graphs in Fig 4. The graphs, Blue for BPSK, Red for QPSK Black for 8PSK and yellow for 16PSK modulation indicate BER average and green curve is the target BER. The Merun color indicates the adaptive modulation. Though phase error is not considered, the Rayleigh attenuation is considered along with AWGN.

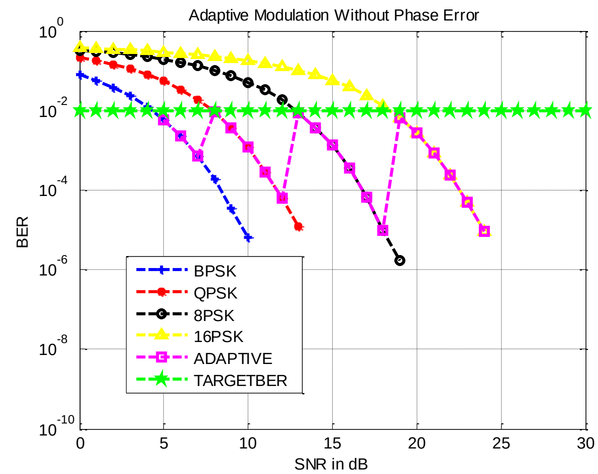


Figure 6. Modulation Adaptation without any phase error.

From the Fig. 6, for the given target BER, the adaptive modulation is able to achieve higher data rate. It is found to be 2.5484 approximately using 10 nodes array for the SNR range of 0 to 30dB. This similar result is obtained for similar trials. Therefere, this becomes the upper bound obtainable without phase error effect and it depends on the SNR range.

3.2 Adaptive Modulation with Zero-Mean phase error with phase variation of 90^0 from mean

Fig 7 shows the BER performance with phase error for 90^0 from zero mean. There are five graphs in the figure, and same color convention is followed. The experiments have been simulated for 10 nodes array for SNR range of 0 to 30 dB

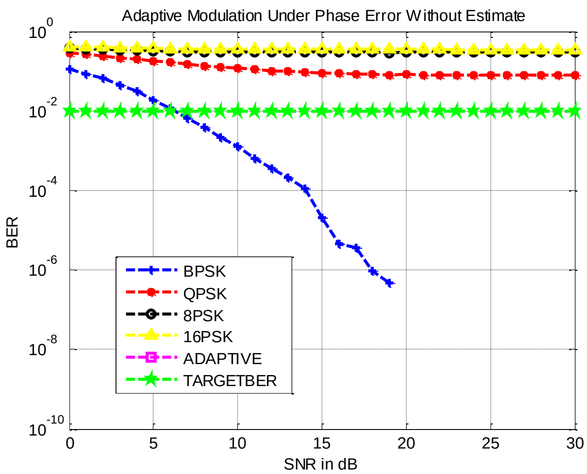


Figure 7. Zero-Mean phase error with 90^0 variation

From Fig. 7, the BER graph follows conventional model and it is easy to adapt modulation under this condition. In the above Fig 7, the modulation adaptation is possible even without estimates. The outage occurs up to 5dB. Table 3 shows the average data rate over SNR in bits per cycles with different trials for M-PSK modulation. The modulation adaptation is better when the phase error variation is less as expected. Therefore, the above results show that the adaptive modulation is possible under Zero-Mean phase error distribution without any estimates.

Table 3. Average Data Rate under Zero-Mean Phase Error Distribution with 30^0,60^0 and 90^0 variation

Phase Variation	30^0	60^0	90^0
Average Data Rate in Bits/Cycle	1.9032	1.3548	0.7742

3.3 Adaptive Modulation under phase error for Mean 180^0 and without any estimates

Fig. 8 shows the simulation results for adaptive modulation under phase error for mean 180 degree with phase error variation of 30degree. There are five graphs in Fig. 8 and the same color convention is used as in Fig. 7. To analyze the possibility of adaptive modulation under Nonzero-Mean phase without estimation, the following result is simulated.

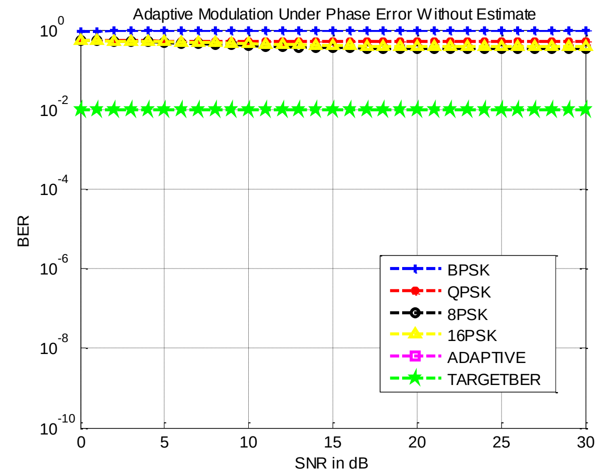
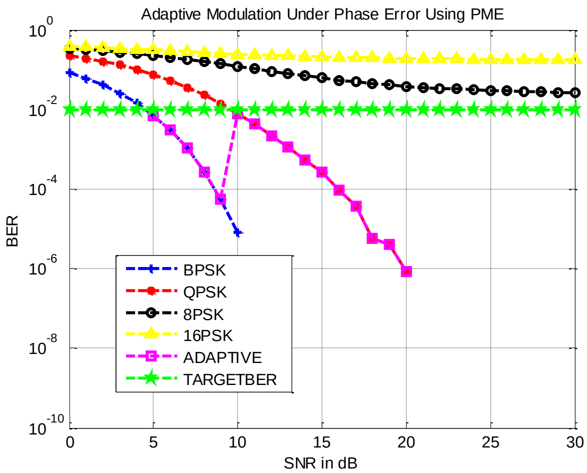


Figure 8. Adaptive Modulatio under mean phase error of 180^0 and maximum variation of 30^0 and without any estimates

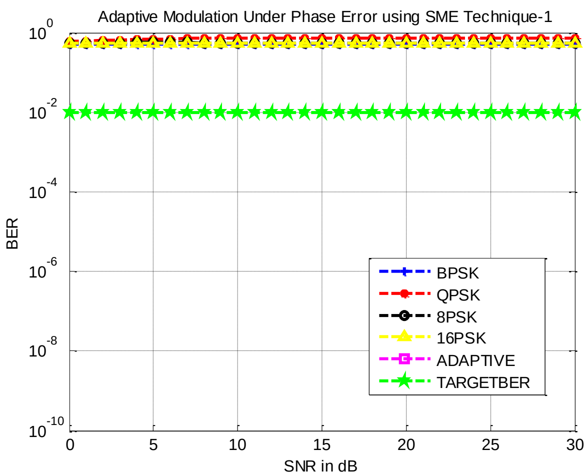
From the above Fig. 8, it is understood that the modulation adaptation is not possible with non zero- mean phase error without using phase error estimates even under phase variation of 30^0. The BER graph is almost flat. It is justified that the mean of phase error (180^0) is π radian and from Table 1 it is equal to BPSK phase error threshold. Though the mean has higher probability of occurrence, it is at the edge of the region even for BPSK. Therefore, it is possible for every bit to be in error even in case of BPSK, which is supposed to have the least BER. Even after 30dB, reception is not possible, i.e. outage occurs at the given BER. This enforces the use of at least the mean estimate for adaptive modulation to be implemented under the above condition.

3.4 Comparison of Adaptive Modulation using PME with existing works

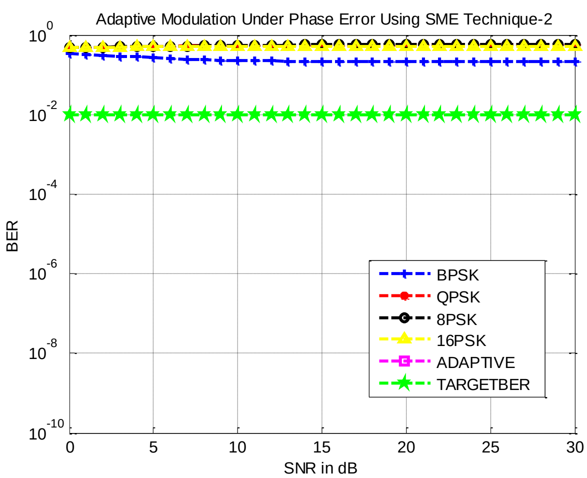
Comparison of the population mean estimator with existing works [20, 21, 24] has been simulated for maximum variation of 30^0 from mean 180^0. There are six graphs in Fig 9(a) and (c) Blue is average BER for BPSK, red for QPSK, Black for 8PSK, yellow for 16PSK and magenta for Adaptive modulation. The target BER is shown in green. The graph 9(a) is for proposed PME; 9(b) is for Sample Mean Estimate (SME) [20], 9(c) for SME [21] and 9(d) for SME [24].



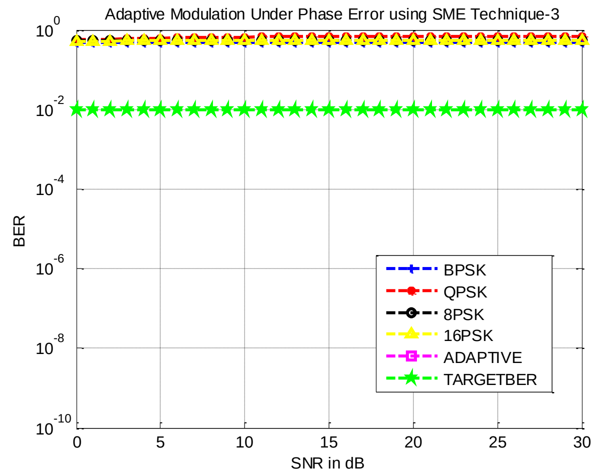
(a)



(b)



(c)



(d)

Figure 9. Comparison with existing work for maximum variaion of 30^0 from mean 180^0 (a) proposed PME (b) existing work [20] (c) existing work [21] and (d) exiting work [24]

From the above graph 9(a) using the PME, we are able to adapt modulation at the BER of 10^{-2} under phase error. The modulation is able to adapt between BPSK and QPSK as compared to Fig 6. In addition, the outage occurs up to 5dB in both Fig 6 and 9(a). Therefore, its performance is similar to the Zero-Mean phase error distribution. . However, from figures 9(b), 9(c) and 9(d), it is inferred that the adaptive modulation is not possible with existing works [20, 21 and 24] as the measurement fails at the mean phase error of 180^0. Table 4 shows comparative Performance of Average Data rate in Bits per cycle over given SNR range of 0 to 30 dB using Sample Mean Estimator (SME) [20], [21], [24] and Population Mean Estimator (PME) using M-PSK Modulation for the phase error variation of 30^0,60^0 and 90^0 from mean.

Table 4. Comparative Average Data Rate for SME and PME

Phase Error Variation (in Degree)	Average Data Rates in Bits/Cycles			
	SME [20]	SME [21]	SME [24]	PME
30	0	0	0	1.5161
60	0	0	0	1.3871
90	0	0	0	0.8065

From the Table 3 and 4 above, the achievable data rate using PME under Nonzero-Mean phase error with mean of 180^0 matches with Zero-Mean Phase Error Scenario.

The estimation capability of SME is presented in the following Tables 5, 6 and 7. The Table 5 shows the Sample Mean Estimate [20] and Table 6 is for SME [21] and Table 7 for SME [24] for the SNR range of 0-30 dB for the maximum phase error variation of 10%, 20% and 50% from phase angle -180 to 180.

Table 5. Estimation of Sample Mean Estimate using SME[20] for the maximum variation of 10% 20% and 50% variation from different phase angle ranging from -180^0 to 180^0

Phase Angle (Degree)	10%	20%	50%
-180	115.483	131.585	127.852
-150	-135.59	-128.71	82.7999
-120	-121.45	-125.68	-135.417
-90	-92.965	-97.3749	-109.79
-60	-62.395	-65.4166	-74.1195
-30	-31.262	-32.8615	-37.3357
0	3.5129	3.3445	3.7103
30	31.3586	32.7699	37.0321
60	62.3229	65.1781	74.2587
90	93.1610	97.4355	110.1988
120	121.516	125.8760	135.2541
150	135.112	128.6423	-79.6403
180	-114.53	-132.131	-127.549

Table 6. Estimation of Sample Mean Estimate using SME [21] for the maximum variation of 10% 20% and 50% variation from different phase angle ranging from -180^0 to 180^0

Phase Angle (Degree)	10%	20%	50%
-180	102.94	119.66	116.04
-150	-134.430	-114.03	26.1935
-120	-121.733	-125.977	-121.823
-90	-93.1486	-97.6052	-109.103
-60	-62.2409	-65.2834	-73.8956
-30	-31.2674	-32.9112	-37.2278
0	3.4316	3.3619	3.5942
30	31.4512	32.9190	37.2095
60	62.3584	65.5053	74.1196
90	93.3592	97.3584	108.3897
120	121.118	125.827	121.035
150	134.582	115.697	-26.7407
180	-101.86	-118.77	-114.56

Table 7. Estimation of Sample Mean Estimate using SME [24] for the maximum variation of 10% 20% and 50% variation from different phase angle ranging from -180^0 to 180^0

Phase Angle (Degree)	10%	20%	50%
-180	170.231	157.221	132.28
-150	-156.546	-161.597	171.16
-120	-124.691	-129.927	-142.879
-90	-94.7930	-100.99	-111.904
-60	-63.2484	-68.1629	-73.83
-30	-31.3794	-32.6717	-38.23
0	-0.058	-0.001	0.052
30	1.3488	33.344	37.140
60	63.4666	64.967	77.845
90	94.2281	100.90	114.72
120	125.683	133.904	156.064
150	156.950	168.283	-172.44
180	-172.03	-157.59	-147.354

From the above Table 5-7, SME [20] [21] and [24] give approximated values with 10% error in estimates for the phase angle range from 120 to -120. These results are average of SNR over 0-30 dB and for 1000 iterations. The estimates at 150^0 and 180^0 are wrong for both SME [20] [21] and [24] and for even minimum variation of 10%. The BER simulation were carried out with 50% variation in Table 5 and 7 at 180^0 level. However, at this level, the SME Technique fails to estimate correctly. Therefore, the adaptive modulation assuming phase error mean of 180^0 does not give desired results using SME [20] [21] and [24]. However, using PME, the adaptive modulation is achieved.

The performance of PME has been simulated with $60^0$and 90^0 There are six graphs in Fig 10 and 11. Blue is average BER graph for BPSK, red for QPSK, Black for, 8PSK, yellow for 16PSK and magenta for Adaptive modulation. The target BER is shown in green. The Figure 10 is for 60^0 variations and Figure 11 is for 90^0 variations.

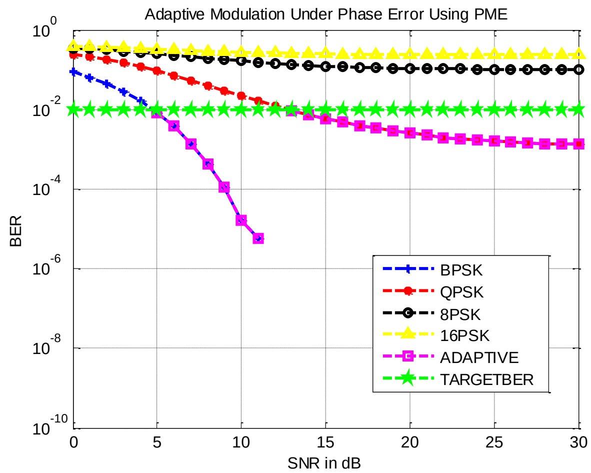


Figure 10. Adaptive Modulation using PME for maximum variaion of 60^0 from mean 180^0

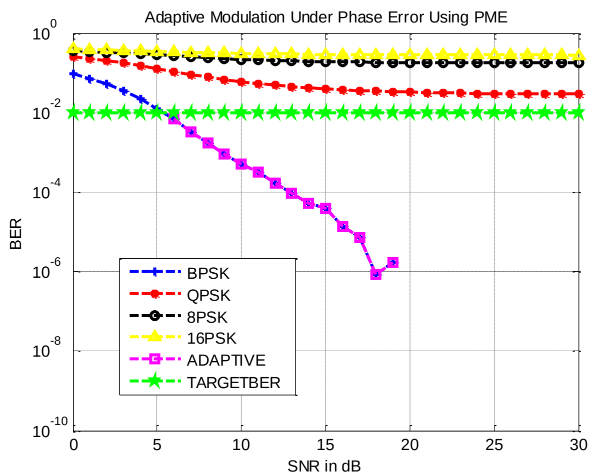


Figure 11. Adaptive Modulation using PME for maximum variaion of 90^0 from mean 180^0

From the above figures 10 and 11 using the PME, we are able to adapt modulation at the BER of 10^{-2} under phase error. Nevertheless, using SME technique [20, 21, 24], the adaptive modulation is not possible as the methods fails at the mean phase error of 180^0 as given in Tables 5, 6 and 7.

3.4 Comparison of Phase error tracking with existing works for maximum variation of 30^0 from mean of 180^0

Fig 12 shows the comparison of tracking capability of Sample Mean Estimate (SME) [20], proposed PME and actual sample mean with the 30^0 variation from mean of 180^0. There are 3 graphs in each Fig 12. The Yellow color represents actual sample mean variation, Blue is for PME and Red is for sample mean estimate technique [20].

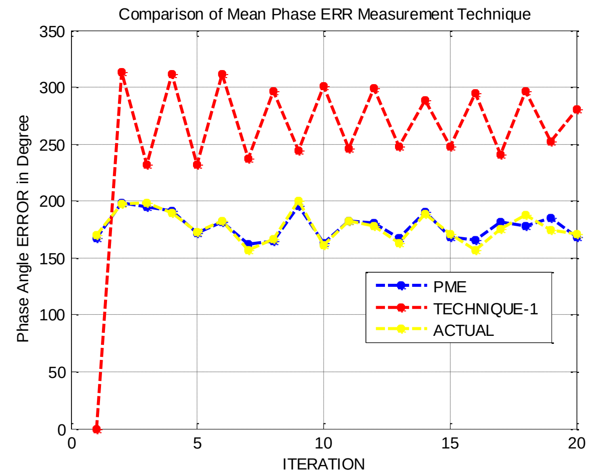


Figure 12. Comparison of Mean Phase Error Tracking using SME [20] and proposed PME for 30^0 variation from mean of 180^0 for 10 nodes array

From the above Fig 12, it is understood that the SME [20] is unable to track the least variation of 30^0 from mean of 180^0, whereas the PME is able to track better. The PME is assumed to have 10 % in estimation always. The maximum 10% error in PME is referred from the Table 7 given below. If the 10% error in PME were not assumed, the adaptive modulation results would be better.

To show the capability of population mean correction algorithm, the simulation of population mean correction algorithm was executed and the results are given in the Table 8 with the initial error of +/-80%. The results are represented in degrees. The final value attained should be $\theta_\mu=\pi=180^0$. From the Table 7, the final value falls within the 10% of Population Mean and it is possible to correct positive as well as negative variations.

Table 8. Result of Population Mean Correction Algorithm

Node	Initial error (%)	Initial Value (degree)	Final Value (FV) (degree)	Error % = (180-FV) *100 / FV
10	- 80	36	177.8335	1.2036%
10	+80	324	182.1931	1.2183%

To compare the tracking capability of SME [21], the following Fig 13 has been plotted with the variation of 30^0 from mean of 180^0.

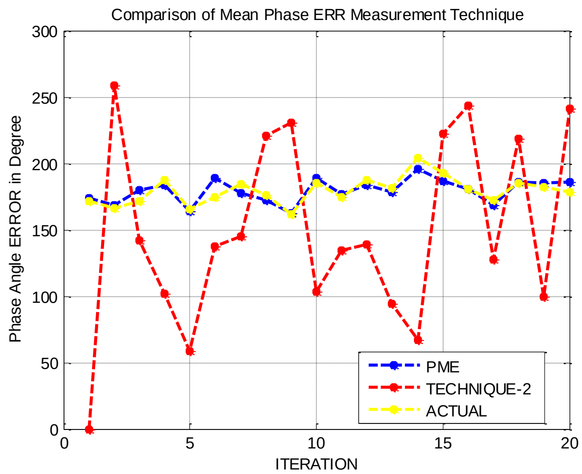


Figure 13. Comparison of Mean Phase Error Tracking using SME [21] and PME for 30^0 variations from mean of 180^0 for 10 nodes array

Similar to the previous results, from the above Fig. 13, it is understood that the SME [21] is unable to track the least variation of 30^0 from mean of 180^0. Similar results are observed for SME [24] with the variation of 30^0 from mean of 180^0 as shown in Fig.14. On the other hand, the PME is able to track better than the existing works.

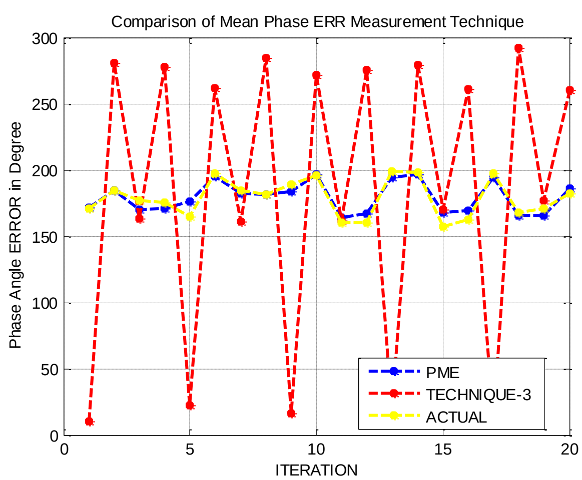


Figure 14. Comparison of Mean Phase Error Tracking using SME [24] and PME for 30^0 variation from mean of 180^0 for 10 nodes array

Table 9 shows the maximum data rate achievable over given power level for the 30^0,60^0 and 90^0 variation of phase error mean 180^0 using PME. As the phase error variation reduces the maximum data rate also, increases with simultaneous reduction in transmit power level.

Table 9. Maximum Data Rate in Bits/cycle

Error (in Degree)	Maximum Data Rate in Bits/Cycle	SNR in dB
30	2	10
60	2	12
90	1	5

4 CONCLUSION

Modulation adaptation in Transmit Beamforming for distributed sensor array under non-zero mean Gaussian Distributed Phase Error has been proposed using Population Mean Estimates (PME) for a Rayleigh flat fading channel. It is observed that without estimates, the adaptive modulation is not possible under given conditions, and Sample Mean Estimates (SME) can be used. However, the existing SME techniques fail under large phase errors for the case of MISO system. To resolve the problem, the Population Mean Estimate approach is proposed as an alternative estimator. At the transmitter side, adjusting the beamforming weights by simple preprocessing results in reduction of computational complexity for the participating sensor and improves the achievable data rates. To correct for error in PME, the Population Means correction algorithms has been proposed and is able to correct both positive and negative deviation from 80% to within 10%. It is also demonstrated from the results that using the proposed method, the average data rate achievable is similar to the case of phase error with zero mean. Thus, it is concluded that maximum data rate achievable increases under reduced phase error variation and the SNR required also reduces under similar conditions with the proposed method.

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