Modeling Performance of VoIP Traffic over 802.11 Wireless Mesh Network Under Correlated Inter-arrival Times

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ABSTRACT

Wireless Mesh Networks (WMN) are an attractive communication paradigm for VOIP traffic due to their low cost and relative ease of deployment. In a recent study, the end to end delay of VOIP packets in a WMN were modeled using an M/D/1 queue based model. In this model, the arrival and service patterns is assumed to follow Poisson and deterministic distributions respectively. However, the long held paradigm in the communication and performance studies that voice and data traffic follow Poisson distribution and the service time deterministic is inaccurate and inefficient. In most cases the arrivals are correlated and the sizes of requests are not similar. In this study, we propose to overcome the above challenge by modeling queuing delay and average queue length of VoIP packets in WMN to capture correlated arrivals, general size distribution and multiple channels using the $\text{MMPP}/G/m$ queue model. The numerical results obtained from the derived models show that increasing the number of mesh routers, arrival rate and load leads to increase in queuing delay and queue length. In addition, it is observed that correlated arrivals exhibits a higher queuing delay and queue length as compared to Poisson arrivals especially at high values of mesh routers, arrival rates and load.

KEYWORDS

Correlated arrivals, general service time, Wireless Mesh Networks

1 INTRODUCTION

Voice over Internet Protocol (VoIP) is a technology that transports voice packets across packet switched networks using the Internet Protocol [1]. VoIP involves digitization of voice streams and transmitting the digital voice as packets over conventional IP-based packet networks like the Internet.

Wireless Mesh Networks (WMNs) often consist of mesh clients, mesh routers and gateways. The mesh clients may be laptops, cell phones and other wireless devices while the mesh routers forward traffic to and from the gateways which may, but need not, connect to the Internet [2].

WMNs are an attractive communication model because of the low cost and relative ease of deployment [3]. The capability of being self-healing, self-organized, and auto-configurable makes WMNs very reliable and resilient [3]. The Wireless Mesh Network backbone provides alternative paths between each pair of end points which increases reliability and eliminates the single point of failure within the mesh [4]. When one node fails, the rest of the nodes can still communicate with each other, directly or through one or more intermediate nodes.

The quality of a VoIP call in a WMN is affected by many parameters such as delay, packet loss, jitter, delay, etc. The common metric used to measure the performance of Wireless Mesh Networks is delay[5]. Delay is defined as the time taken by the VoIP packet to travel from a mobile node to gateway in a Wireless Mesh Network.
Delay can be measured in either one-way or round-trip delay. A typical VoIP traffic can tolerate up to 150ms end to end delay in a single direction before the quality of the call becomes unacceptable [6].

There are various challenges facing VoIP in WMN. A number of efforts have been carried out to investigate the performance of VoIP over Wireless Mesh Network [7, 8]. Chhabra et al. [5] modeled average delay for VoIP by varying number of mesh points in WMNs. The authors developed an M/D/1 queue based model to estimate the average delay incurred by the VOIP traffic. In an M/D/1 model, M stands for Markovian arrival with exponentially distributed inter arrival times, D stands for deterministic service times and 1(one) server.

The expressions for the average queuing delay incurred by VOIP packets from source mesh client to destination (Internet gateway) in a wireless mesh network having \( n \) mesh routers can be expressed as [5]:

\[
Q_{nd} = s + n\lambda s^2/(2(1-n\lambda s)) \tag{1}
\]

where \( Q_{nd} \) is the average queuing delay taken by VOIP packets from the source client to the destination via \( n \) mesh routers in a network. \( s \) is the service rate of each VOIP packet.

However, the long held paradigm that voice traffic and data traffic are adequately characterized by certain Markovian models (e.g., Poisson) has been proven to be inaccurate, inefficient and greatly affects the accuracy of the results since real system workloads exhibit significant correlation patterns [9].

Correlation in traffic arrivals has been observed in compressed digital video streams [10], Ethernet traffic [11] and Web traffic between browsers and servers [12]. In addition, the study of social patterns in online social networks taking place on the World Wide Web shows that the inter-arrival times between subsequent tagging events can only be explained by taking into account correlation in users’ behaviors [13].

Similarly, correlation of word inter-arrival times have been detected in texts [13]. Furthermore, the assumption of deterministic service times is unrealistic because the voice packets have varying packet sizes.

In this paper, we investigate the performance of VoIP over WMN using an MMPP/G/m model that captures the correlated arrivals, general service times and multiple channels (servers).

The contribution of this paper is two fold. Firstly, this study proposes an analytical model for evaluating the performance of VoIP traffic in WMN in terms of queuing delay and queue length under correlated inter arrival times using MMPP/G/m queue system.

Secondly, this study evaluated the performance of VoIP traffic in WMN in terms of queuing delay and queue length under correlated arrivals using MMPP/G/m queue system. In this case, the number of mesh routers, average arrival rate, and load in the system are varied to determine the effect of these parameters on the queuing delay and queue length.

2 SYSTEM MODELS

We consider an MMPP/G/m queue system to model the correlations between consecutive inter arrival times. In this case, the arrivals of requests are modeled using Markov Modulated Poisson Process (MMPP). (MMPP) is one of the most used models [14, 15, 16] to capture the characteristics of the incoming traffic such as correlated traffic, burstiness behavior, and long range dependency, and is simply a Poisson process whose mean value changes according to the evolution of a Markov Chain [17].

\( G \) stands for general service time distribution, meaning the service time can take on any distribution, e.g exponential or Bounded Pareto, and \( m \) denotes the number of servers.

MMPP is normally used for modeling bursty traffic owing to its ability to capture the time-varying arrival rate and the correlation between inter-arrival times while still maintaining
analytical tractability. MMPPs are most frequently observed in queuing theory, however it has also been applied to other applications, such as analysis of Web surfing behavior [18], and for telephone network fraud detection [19]. When there is correlation between inter arrival times as shown in Figure 1, M/M/1 queue system becomes inaccurate, therefore MMPP/G/m queue system is an optimal model to capture the correlation between the inter arrival times.

Figure 1. Correlated Inter arrival times (Batch arrivals). Adopted [20]

We model the Wireless Mesh Network using the MMPP/G/m queue. G denotes general service time, and m the number of servers. The arrival process is assumed to follow a Markovian Modulated Poisson Process (MMPP). The service time is assumed to follow a general distribution. In this case the exponential and Bounded Pareto distributions are used as special cases of the general distribution. The system is assumed to consist of multiple servers, m. Two servers are used as a representation of multiple servers. Each mesh router is assumed to be a station in the queuing network representation.

The derived models are used to investigate the effect of the number of mesh routers, average arrival rate, and load in the system on the queuing delay and average queue length of VoIP packets in a WMN.

In the next section, we derive the expressions for average queuing delay and queue length for an MMPP/G/m queue. We approximate the behavior of an MMPP/G/m queue analytically. The method employed consists in approximating the MMPP/G/m queue system using a weighted superposition of different M/G/m queues.

3 EXPRESSIONS FOR THE PERFORMANCE METRICS

Consider an MMPP/G/m queue, and let the MMPP that models the incoming data traffic be composed by H states (S1...SH).

We use the notation Mi/G/m to refer to an M/G/m queue whose average arrival rate is λi observed in state Si and the service rate is µ and is constant among all the Si states. The analytical approximations is based on the observation that if the MMPP stays in state Si long enough without transiting to another state, the average waiting time at time t reach the same steady state observed for the corresponding Mi/G/m queue. The values are pinned on the same steady state value of Mi/G/m as long as the MMPP does not change its state from Si. Similar approximations have been used to approximate mean queue length and average response time for an MMPP/M/1 queue system [21].

The average packet queuing delay is an average evaluated over the number of incoming requests which are not distributed equally over time (during the state Si the arrival rate of requests is λi, while during the state Sj the arrival rate of requests is a different amount λj). Therefore, the average packet queuing delay of the MMPP/G/m is a weighted sum of the average packet queuing delays of the Mi/G/m queues, expressed as:

\[ Q = \sum_{i=1}^{H} Q_i r_i \tag{2} \]

The weights (ri) are not simply composed by the asymptotic probabilities of the MMPP (as in the case of the average queue length) but it is scaled to keep into account the different arrival rate per each state. Hence;
\[ r_i = \frac{p_i \lambda_i}{\sum_{j=1}^{m} p_j \lambda_j}. \]

### 3.1 Expression for average packet queuing delay under MMPP/M/m

The expression for average packet queuing delay for M/M/m, which is a special case of M/G/m, is derived as follows: When there are more than one server, a new arriving customer may enter any of the free servers. Let \( \lambda \) and \( \mu \) be the rate of the Poisson process for the arrivals and the parameter of the exponential distribution for the service times respectively.

The balance equations are:

\[
\lambda P_{i-1} = \begin{cases} 
  i \mu P_i, & \text{for } i \leq m \\
  m \mu P_i, & \text{for } i > m 
\end{cases}
\]  

\[ P_i = \begin{cases} 
  \frac{P_o \lambda^i}{i!}, & \text{for } i \leq m \\
  \frac{P_o \lambda^m}{m!}, & \text{for } i > m 
\end{cases}
\]

where \( \rho = \frac{\lambda}{m \mu} \). The probability that an arriving customer is forced to join the queue is given by

\[
P_Q = \sum_{i=m}^{\infty} P_i = \sum_{i=m}^{\infty} \frac{P_o \lambda^m}{m!} \rho^i = \frac{P_o \lambda^m \rho^m}{m!} \sum_{i=m}^{\infty} \rho^i = \frac{P_o \lambda^m \rho^m}{m!} \frac{\rho}{1-\rho}
\]

The expression for the average packet queuing delay for two states for MMPP/M/m is given as:

\[
Q_d = \sum_{i=1}^{2} r_i \left( \frac{P_o (\rho)^m}{m!} \cdot \frac{\rho}{\lambda (1-\rho)^2} \right)
\]

The expression for average packet queuing delay for two states for MMPP/M/m is given as:

\[
Q_d = \sum_{i=1}^{2} r_i \left( \frac{P_o (\rho)^m}{m!} \cdot \frac{\rho}{\lambda (1-\rho)^2} \right)
\]

where \( r_i = \frac{p_i \lambda_i}{\sum_{j=1}^{m} p_j \lambda_j} \) and \( P_0 = \left[ \sum_{i=0}^{m-1} \frac{(\rho)^i}{i!} + \frac{(\rho)^m}{m! (1-\rho)} \right]^{-1} \).

The expression for total average packet queuing delay for two states for MMPP/M/m is given as:

\[
Q_d = E[x] + \sum_{i=1}^{2} r_i \left( \frac{P_o (\rho)^m}{m!} \cdot \frac{\rho}{n \lambda (1-\rho)^2} \right)
\]

where \( \rho = \frac{n \lambda}{\mu} \), \( n \) is the number of mesh clients and \( E[x] \) the service time for each packet.

### 3.2 Expression for average packet queuing delay under MMPP/BP/m

The expression for average packet queuing delay for MMPP/BP/m can be derived from the general expression for MMPP/G/m. However, the average packet delay for MMPP/G/m queue system can be got by approximating the MMPP/G/m queue system using a weighted superposition of different M/G/m queues. There are numerous approximations for the average delay a job experiences under the M/G/m queue system [25, 23, 24].

A naturally refined heavy-traffic approximation exploiting the exact M/M/m results is given in [28] as:

\[
E[W^{M/G/m}] = \frac{C^2 + 1}{2} E[W^{M/M/m}]
\]

where \( C^2 \) is the coefficient of variation of the service time distribution, \( E[W^{M/G/m}] \) is the mean delay under M/G/m queue system, and \( E[W^{M/M/m}] \) is the mean delay under M/M/m queue system.

The expression for the average packet delay under MMPP/G/m can be deduced as
follows: From equation 6, the average packet delay under M/G/m is given as:

$$Q_d = \frac{1}{2} \cdot \frac{P_o (m \rho)^m}{m!} \cdot \rho \cdot \frac{1}{\lambda (1 - \rho)^2} \tag{10}$$

The expression for average packet queuing delay for two states for MMPP/G/m is given as:

$$Q_d = \sum_{i=1}^{2} r_i \cdot \frac{1}{2} \cdot \frac{P_o (m \rho)^m}{m!} \cdot \rho \cdot \frac{1}{\lambda (1 - \rho)^2} \tag{11}$$

where $$r_i = \frac{p_i \lambda_i}{\sum_{j=1}^{m} p_j \lambda_j}$$ and $$P_o = \left[ \sum_{i=0}^{m-1} \frac{(m \rho)^i}{i!} \right] + \frac{(m \rho)^m}{m! (1 - \rho)} \cdot \frac{1}{1 - \rho}$$.

The expression for total average packet queuing delay for two states for MMPP/G/m is given as:

$$Q_d = E[x] + \sum_{i=1}^{2} r_i \cdot \frac{1}{2} \cdot \frac{P_o (m \rho)^m}{m!} \cdot \rho \cdot \frac{1}{\lambda (1 - \rho)^2}$$

where $$\rho = \frac{\lambda}{\mu}$$, $$\lambda$$ is the number of mesh clients and $$E[x]$$ the service time for each packet.

In the next section, we derive the expressions for average packet queue length for MMPP/M/m

### 3.3 Expression for average packet queue length under MMPP/M/m

The evolution of the average packet queue length value of the MMPP/G/1 can be described as follows: each time a state transition occurs, there is a transient phase ($$t_{12}$$ for a transition from $$S_1$$ to $$S_2$$ and $$t_{21}$$ for a transition from $$S_2$$ to $$S_1$$) after which the mean queue length of the MMPP reaches the steady state of the corresponding Mi/G/1. Denote;

(i) $$Q_i$$ to be the steady state queue length of Mi/G/1 queue.

(ii) $$p_i$$ to be the asymptotic probability for the MMPP to stay in state $$S_i$$.

The mean queue length of the MMPP/G/1 queue can be approximated as:

$$Q = \sum_{i=1}^{2} p_i Q_i \tag{13}$$

The expression for the average queue length under MMPP/G/1 can be expressed as:

$$E[N_q] = \sum_{i=1}^{2} p_i \frac{\lambda_i^2 x^2}{2(1 - \rho_i)} \tag{14}$$

The expression for the average queue length of MMPP/M/m can then be derived as follows:

The average queue length for an M/M/m queue system is given in equation 5 as:

$$N_q = \frac{P_o (m \rho)^m}{m!} \cdot \frac{\rho}{(1 - \rho)^2} \tag{15}$$

Therefore, the expression for average packet queue length for two states for MMPP/M/m is given as:

$$N_q = \sum_{i=1}^{2} p_i \left( \frac{P_o (m \rho)^m}{m!} \cdot \frac{\rho}{(1 - \rho)^2} \right) \tag{16}$$

where $$P_o = \left[ \sum_{i=0}^{m-1} \frac{(m \rho)^i}{i!} \right] + \frac{(m \rho)^m}{m! (1 - \rho)} \cdot \frac{1}{1 - \rho}$$.

### 3.4 Expression for average packet queue length under MMPP/BP/m

The expression for average packet queue length for MMPP/BP/m can be derived from the general expression for average queue length of MMPP/G/m. Since queuing delay and queue length change directly proportionately, we can derive the expression for average queue length using a similar approach as in equation 9. Hence, the average queue length under M/G/m can be expressed as:

$$E[N_{M/G/m}] = \frac{C^2 + 1}{2} \cdot E[N_{M/M/m}] \tag{17}$$

where $$C^2$$ is the coefficient of variation of the service time distribution. Hence the expression for the average queue length under M/G/m can be deduced as follows:

$$N_q = \frac{(C^2 + 1) P_o (m \rho)^m}{2m!} \cdot \frac{\rho}{(1 - \rho)^2} \tag{18}$$
Therefore, the expression for average packet queue length for two states for $MMPP/G/m$ is given as:

$$N_q = \sum_{i=1}^{2} p_i \frac{(C^2 + 1) P_o(m \rho)^m}{2} \frac{\rho^m}{m!} \frac{\rho}{(1 - \rho)^2}$$

where $P_o = [\sum_{i=0}^{m-1} \frac{(m \rho)^i}{i!} + \frac{(m \rho)^m}{m(1-\rho)^m}]^{-1}$.

### 4 PERFORMANCE EVALUATION

In this section, we evaluate the performance of $M/D/1$ and $MMPP/G/1$ using MATLAB and present numerical results. The analysis is extended to the comparison of $M/D/m$, $MMPP/M/m$ and $MMPP/BP/m$. The results nicely illustrate the impact of correlated arrivals as compared to Poisson arrivals.

#### 4.1 Job Size Distributions under Consideration

Two job size distributions are considered, that is, the exponential and Bounded Pareto distribution. Exponential distribution is used to model the service time. Exponential distributed has a low variability since its coefficient of variation is 1. Exponential distribution models service time of requests with similar sizes. The PDF (probability density function) of an exponential distribution is given as:

$$f(x) = \mu e^{-\mu x}, \quad x \geq 0, \quad \mu \geq 0,$$

where $\mu$ is the mean service rate. Bounded Pareto (BP) distribution is commonly used in analysis because it has a high Coefficient of Variation and therefore it can exhibit the high variability property as observed in the Internet traffic and also because the maximum job size can be set to mimic the largest observed Internet flow sizes [22]. Denote the Bounded Pareto distribution by $BP(k, P, \alpha)$ where $k$ and $P$ are the minimum and the maximum job sizes and $\alpha$ is the exponent of the power law. The probability density function of the Pareto is given in [22] as

$$f(x) = \frac{\alpha k^\alpha}{1 - (k/P)^\alpha} x^{-\alpha - 1}.$$

$k \leq x \leq P, \quad 0 \leq \alpha \leq 2$.

#### 4.2 Model Parameters

There are many codecs available for digitizing speech. The three commonly used codec in Internet telephony are G.711, G.723.1 and G.729 [26]. We consider parameters used for G.711 codec because G.711 codec gives the best voice quality, introduces the least delay, and is less sensitive than other codecs to packet loss. G.711 codec uses service time of 0.01 seconds. In addition G.711 uses a 100ms sample period corresponding to a maximum arrival rate of 10 packets per second. The $MMPP$ parameters are set on the basis of the results reported in [27], which has shown, via real traces analysis, the feasibility to model incoming traffic to a GRID server. According to the data reported in [27], the incoming data traffic of the analyzed GRID server can be modeled by a 2-state MMPP model. The transition probability $p_1$ from state $S_1$ to state $S_2$, is 0.17, while the reciprocal transition probability $p_2$, from state $S_2$ to state $S_1$, is 0.08.

The hypothetical parameters used in the analysis is consistent with parameters used in literature [5, 28, 26, 27]. Number of mesh routers used is 10, service time is 0.1 seconds, transition states is two, arrival rate varies from 5 to 10 packets/second, $p_1 = 0.17, p_2 = 0.08$, $\lambda_1 = 22.10$ requests/second, $\lambda_2 = 7.16$ requests/second. Squared coefficient of variation for BP distribution, $C^2 = 0.53$ [28]. Mean for Bounded Pareto distribution, $E[X] = 2.56$ [22].

#### 4.3 Evaluation of queuing delay for $M/D/1$ and $MMPP/M/1$

We compare the performance of $M/D/1$ against $MMPP/M/1$ in terms of queuing delay. The results nicely illustrate the impact of correlated arrivals as compared to Poisson arrivals in terms of queuing delay as the performance metric.
Figure 2. Queuing delay as a function of number of mesh routers

Figure 2 shows a graph of queuing delay as a function of number of mesh routers. We used equations 1 and 8 to plot the graph. It can be observed from figure 2 that queuing delay increases with increase in the number of mesh routers. The increase in queuing delay as the number of mesh routers increase can be explained by the fact that at each mesh router packets experience queuing delay and this accumulates as the number of mesh routers increase. We further observe that for low number of mesh routers, the queuing delay for MMPP/M/1 queue system and M/D/1 queue system are almost the same, however as the number of mesh routers increase the queuing delay for MMPP/M/1 queue system is higher than for M/D/1 queue system. The difference in queuing delay between MMPP/M/1 queue system and M/D/1 queue system is more pronounced for higher numbers of mesh routers.

Figure 3. Queuing delay as a function of arrival rate of VoIP packets

Figure 3 shows a graph of queuing delay as a function of arrival rate of VoIP packets. We used equations 1 and 8 to plot the graph in figure 3. It can be observed from figure 3 that queuing delay increases with increase in arrival rate of VoIP packets regardless of the queue system. The increase in queuing delay as the arrival rate of VoIP packets increase can be explained by the fact that increase in arrival rate leads to increase in the number of packets at each mesh router hence increasing the queuing delay. We also observe that queuing delay for MMPP/M/1 queue system is higher than the queuing delay for M/D/1 queue system for all considered arrival rates. The difference in queuing delay between MMPP/M/1 queue system and M/D/1 queue system is observed to be higher for higher arrival rates of VoIP packets.

Figure 4. Queuing delay as a function of load due to VoIP packets

Figure 4 shows a graph of queuing delay as a function of load due to VoIP packets. We used equations 1 and 8 to plot the graph in figure 4. It can be observed from figure 4 that queuing delay generally increases with increase in load due to VOIP packets regardless of the queue system. Increase in load leads to increase in the number of packets in the system which in turn leads to increased queuing delay. We also observe that queuing delay for MMPP/M/1 queue system is slightly lower than the queuing delay for M/D/1 queue system for load values less than 0.05, however for load values higher than...
0.05, the queuing delay for MMPP/M/1 queue system is higher than for M/D/1 queue system. In addition, the difference in queuing delay between MMPP/M/1 queue system and M/D/1 queue system is observed to be higher for higher load values of VOIP packets.

5 EVALUATION OF QUEUING DELAY FOR MMPP/M/1 AND MMPP/M/2

In this section, we compare the performance of MMPP/M/1 against MMPP/M/2 in terms of queuing delay. The results nicely illustrate the impact of increasing the number of servers on the performance of VOIP packets under correlated arrivals in terms of queuing delay as a performance metric.

Figure 5 shows a graph of queuing delay as a function of number of mesh routers. We used equation 8 to plot graph 5. We observe from figure 5 that queuing delay generally increases with increase in number of mesh routers regardless of the number of servers. We also observe that initially queuing delay for MMPP/M/1 and MMPP/M/2 queue systems are the same, however as the number of mesh routers increase, the queuing delay for MMPP/M/1 is higher than for MMPP/M/2. Queuing delay for MMPP/M/1 is higher than for MMPP/M/2 due to the fact that MMPP/M/2 has higher number of servers as compared to MMPP/M/1. The more the number of servers, the higher the processing power and hence the lower the queuing delay. The difference in queuing delay for MMPP/M/1 and MMPP/M/2 queue systems is more pronounced as the number of mesh routers increase.

Figure 6 shows a graph of queuing delay as a function of arrival rate of VOIP packets. We used equation 8 to plot graph 6. It can be observed from figure 6 that queuing delay generally increases with increase in arrival rate irrespective of the number of servers. We also observe that queuing delay for MMPP/M/1 queue system is higher than for MMPP/M/2 queue system for all considered arrival rates. Queuing delay for MMPP/M/1 is higher than for MMPP/M/2 due to the fact that MMPP/M/2 has higher number of servers as compared to MMPP/M/1. The more the number of servers, the higher the processing power and hence the lower the queuing delay. The difference in queuing delay for MMPP/M/1 and MMPP/M/2 queue systems increase as the arrival rate increases.

Figure 7 shows a graph of queuing delay as a function of load for MMPP/M/m queue system. We used equation 8 to plot graph 7. It can be observed from figure 7 that queuing delay generally increases with increase in load due to VOIP packets regardless of the number of servers. We also observe that for low load values the queuing delay for MMPP/M/1 and MMPP/M/2 queue systems are the same.
however as the load increases, queuing delay for MMPP/M/1 queue system is higher than for MMPP/M/2 queue system. There is a marked difference in queuing delay for MMPP/M/1 and MMPP/M/2 queue systems as the load increases. Queuing delay for MMPP/M/1 is still higher than for MMPP/M/2 due to the fact that MMPP/M/2 queue system has higher number of servers as compared to MMPP/M/1 and hence higher processing power thereby lowering the queuing delay.

6 EVALUATION OF QUEUING DELAY FOR M/D/1 AND MMPP/BP/1

In this section, we compare the performance of M/D/1 against MMPP/BP/1 which is a special case of MMPP/G/1 in terms of queuing delay. The results illustrate well the impact of correlated arrivals as compared to Poisson arrivals in terms of queuing delay.

Figure 8 shows a graph of queuing delay as a function of load due to VOIP packets for MMPP/BP/1 and M/D/1 queue systems. We used equations 1 and 12 to plot graph 8. It can be observed that queuing delay generally increases with increase in number of mesh routers regardless of the queue system. We also observe that for low values of number of mesh routers, the queuing delay for MMPP/M/1 and M/D/1 queue systems are much closer, however as the number of mesh routers increases, queuing delay for MMPP/M/1 is higher than for M/D/1 queue system. Furthermore, we observe that there is a marked difference in queuing delay for MMPP/M/1 and M/D/1 queue systems as the number of mesh routers increase.

Figure 9 shows a graph of queuing delay as a function of arrival rate of VOIP packets for MMPP/BP/1 and M/D/1 queue systems. We used equations 1 and 12 to plot the graph in figure 9. It can be observed from figure 9 that queuing delay generally increases with increase in arrival rate of packets regardless of the queue system. We also observe that for low arrival rates, the queuing delay for MMPP/BP/1 and M/D/1 queue systems are much closer, however as the arrival rate increases, queuing delay for MMPP/BP/1 is higher than for M/D/1 queue system. Furthermore, we observe that there is a marked difference in queuing delay for MMPP/BP/1 and
M/D/1 queue systems as the arrival rate increase.

Figure 10 shows a graph of queuing delay as a function of load due to packets for MMPP/BP/1 and M/D/1 queue systems. We used equations 1 and 12 to plot the graph in figure 10. We observe from figure 10 that queuing delay increases with increase in load without regard to the queue system. We also observe that for low load values, the queuing delay for MMPP/BP/1 is lower than the queuing delay for M/D/1 queue system, however after the load of approximately 0.065 the queuing delay for M/D/1 queue system is lower than for MMPP/BP/1 queue system. It can further be noted that the difference in queuing delay between MMPP/BP/1 and M/D/1 queue systems are much higher as the load increases.

Next, we evaluate the average queuing delay for MMPP/BP/1 and MMMPP/BP/2 queue systems.

7 EVALUATION OF QUEUE DELAY FOR MMPP/BP/1 AND MMMPP/BP/2

We evaluate average queuing delay for MMPP/BP/1 and MMMPP/BP/2 queue systems to investigate the effect of increasing number of servers on the MMPP/BP/m, queue system.

Figure 11 shows a graph of queuing delay as a function of number of mesh routers for MMPP/BP/m queue system. We used equation 12 to plot graph 11. We observe from figure 11 that queuing delay generally increases with increase in number of mesh routers irrespective of the queue system. We also observe that for low number of mesh routers the queuing delay for MMPP/BP/1 and MMPP/BP/2 queue systems are closer, however as the number of mesh routers increase, queuing delay for MMPP/BP/1 queue system is higher than for MMPP/BP/2 queue system.

There is a marked difference in queuing delay between MMPP/BP/1 and MMPP/BP/2 queue systems as the number of mesh routers increase. Queuing delay for MMPP/BP/1 is higher than for MMPP/BP/2 due to the fact that MMPP/BP/2 has higher number of servers as compared to MMPP/BP/1 and hence higher processing power thereby lowering the queuing delay.

Figure 12. Queuing delay for MMPP/BP/m as a function of arrival rate of VoIP packets
Figure 12 shows a graph of queuing delay as a function of arrival rate of VoIP packets for MMPP/M/m queue system. We used equation 12 to plot graph 12. We observe from figure 12 that queuing delay increases with increase in arrival rate of packets into the system. We also observe that for low arrival rate values, the queuing delay for MMPP/BP/1 and MMPP/BP/2 queue systems are closer, however as the arrival rate increases, queuing delay for MMPP/BP/1 queue system is higher than for MMPP/BP/2 queue system. This trend is due to the fact that MMPP/BP/2 has a higher number of servers than MMPP/BP/1 and therefore higher processing rate that results in lower queuing delay. We further observe that as the arrival rate increases, the difference in queuing delay between MMPP/M/1 and MMPP/M/2 increases.

Figure 13. Queuing delay for MMPP/BP/m as a function of load due to VoIP packets

In the next section, we evaluate the average queue length for M/D/1 and MMPP/M/1 queue systems.

8 EVALUATION OF AVERAGE QUEUE LENGTH FOR M/D/1 AND MMPP/M/1

In this section, we compare the performance of M/D/1 against MMPP/M/1 in terms of queue length. Queue length is one of the performance metrics that are commonly used to evaluate the performance of systems.

Figure 14. Average queue length as a function of number of mesh routers

Figure 14 shows a graph of queue length as a function of number of mesh routers. We used equations 1 and 19 to plot the graph. It can be observed from figure 14 that average queue length increases with increase in the number of mesh routers regardless of the queue system. The increase in queue length as the number of mesh routers increase can be explained by the fact that at each mesh router packets queue hence increasing the queue length as the number of mesh routers increase. We further observe that for low number of mesh routers, the average queue length for MMPP/M/1 queue system and M/D/1 queue system are almost the same, however as the number of mesh routers increase the average queue length for MMPP/M/1 queue system is higher than for M/D/1 queue system. The difference in average queue length between MMPP/M/1 queue system and M/D/1 queue system is more pronounced as the number of mesh routers increase.
Figure 15. Average queue length as a function of arrival rate of VoIP packets

Figure 15 shows a graph of queue length as a function of arrival rates for VOIP packets. We used equations 1 and 19 to plot the graph. We observe from figure 15 that average queue length generally increases with increase in arrival rate of VOIP packets irrespective of the queue system. The increase in queue length as the arrival rate of VOIP packets increase can be explained by the fact that at each mesh router packets queue hence increasing the queue length as the number of mesh routers increase. We further observe that the average queue length for MMPP/M/1 is higher than for M/D/1 queue system regardless of the average arrival rate. The difference in average queue length between MMPP/M/1 queue system and M/D/1 queue system is more pronounced as the average arrival rate increases.

Figure 16. Average queue length as a function of load due to VoIP packets

Figure 16 shows a graph of queue length as a function of load due to VOIP packets. We used equations 1 and 19 to plot the graph. We observe from figure 16 that average queue length generally increases with increase in load due to VoIP packets regardless of the queue system. The increase in queue length as the load due to VOIP packets increase can be explained by the fact that if load increases more time is taken processing packets and this in turn leads to an increase in the queue length. We further observe that for low load values the queue length for MMPP/M/1 and M/D/1 queue systems are the same, however as the load increases, the queue length for MMPP/M/1 queue system is higher than for M/D/1 queue system. There is an observed marked difference in queue length for MMPP/M/1 and MMPP/M/2 queue systems as the load increases.

9 EVALUATION OF AVERAGE QUEUE LENGTH FOR MMPP/M/1 AND MMPP/M/2

In this section, we compare the performance of MMPP/M/1 against MMPP/M/2 in terms of queue length to investigate the effect of increasing number of servers on the average queue length.

Figure 17. Average queue length as a function of number of mesh routers for MMPP/M/1 and MMPP/M/2

Figure 17 shows a graph of queue length as a function of number of mesh routers for MMPP/M/1 and MMPP/M/2 queue systems. We used equation 19 to plot the graph. We observe from figure 17 that average queue length generally increases with increase in number of
mesh routers irrespective of the queue system. The increase in queue length as the number of mesh routers increase can be explained by the fact that at each mesh router packets queue hence increasing the queue length as the number of mesh routers increase. We also observe that for low number of mesh routers the queue length for MMPP/M/1 and MMPP/M/2 queue systems are much closer, however as the number of mesh routers increase, the queue length for MMPP/M/1 queue system is higher than for MMPP/M/2 queue system. The difference in queue length between MMPP/M/1 and MMPP/M/2 is even higher as the arrival rate increases.

![Figure 18. Average queue length as a function of arrival rate of VoIP packets for MMPP/M/1 and MMPP/M/2](image)

Figure 18 shows a graph of queue length as a function of arrival rate for MMPP/M/1 and MMPP/M/2 queue systems. We used equation 19 to plot the graph. We observe from figure 18 that average queue length generally increases with increase in load due to VoIP packets. At low load values, the average queue length for MMPP/M/1 and MMPP/M/2 are the same, however as the load increases, the average queue length of MMPP/M/1 is higher than average queue length for MMPP/M/2. The difference in queue length between MMPP/M/1 and MMPP/M/2 increases with increase in load after attaining a load of 0.04.

In the next section, we evaluate the average queue length for M/D/1 and MMPP/BP/1.

![Figure 19. Average queue length as a function of load due to VoIP packets for MMPP/M/1 and MMPP/M/2](image)

Figure 19 shows a graph of queue length as a function of load due to VoIP packets for MMPP/M/1 and MMPP/M/2 queue systems. We used equation 19 to plot the graph. We observe from figure 19 that average queue length generally increases with increase in load due to VoIP packets. At low load values, the average queue length for MMPP/M/1 and MMPP/M/2 are the same, however as the load increases, the average queue length of MMPP/M/1 is higher than average queue length for MMPP/M/2. The difference in queue length between MMPP/M/1 and MMPP/M/2 increases with increase in load after attaining a load of 0.04.

In the next section, we evaluate the average queue length for M/D/1 and MMPP/BP/1.

### 10 Evaluation of Average Queue Length for M/D/1 and MMPP/BP/1

In this section, we compare the performance of M/D/1 against MMPP/BP/1 in terms of queue length. Figure 20 shows a graph of queue length as a function of number of mesh routers. We used equations 1 and 12 to plot the graph in figure 20. We observe from figure 20 that average queue length generally increases with increase in number of mesh routers. We further
observe that for lower number of mesh routers, the average queue length for MMPP/BP/1 and M/D/1 are closer, however as the number of mesh routers increase, the average queue length of MMPP/BP/1 is higher than average queue length for M/D/1.

Figure 21 shows a graph of queue length as a function of arrival rate of VoIP packets. We used equations 1 and 12 to plot the graph in figure 21. We observe from figure 21 that average queue length generally increases with increase in arrival rate of VoIP packets. We further observe that at lower arrival rates of packets into the system, the average queue length for MMPP/BP/1 and M/D/1 are the same, however as the arrival rate increases, the average queue length of MMPP/BP/1 is higher than average queue length for M/D/1.

**11 EVALUATION OF AVERAGE QUEUE LENGTH FOR MMPP/BP/1 AND MMPP/BP/2**

In this section, we compare the performance of MMPP/BP/1 against MMPP/BP/2 in terms of queue length to investigate the effect of increasing number of servers on the average queue length.
Figure 23 shows a graph of queue length as a function of number of mesh routers. We used equation 12 to plot graph 23. We observe from figure 23 that average queue length generally increases with increase in number of mesh routers. We further observe that for lower number of mesh routers, the average queue length for MMPP/BP/1 and M/D/1 are closer, however as the number of mesh routers increase, the average queue length of MMPP/BP/1 is higher than average queue length for M/D/1.

Figure 24 shows a graph of queue length as a function of arrival rate of VoIP packets for MMPP/BP/1 and MMPP/BP/2. We used equation 12 to plot graph 24. We observe from figure 24 and that average queue length generally increases with increase in arrival rate of VoIP packets. We further observe that for lower number of mesh routers, the average queue length for MMPP/BP/1 and MMPP/BP/2 are closer, however as the arrival rate increases, the average queue length of MMPP/BP/1 is higher than average queue length for MMPP/BP/2.

Figure 25 shows a graph of queue length as a function of load due to VoIP packets for MMPP/BP/1 and MMPP/BP/2. We observed from figure 25 that average queue length increases with increase in load due to VoIP packets. We further observe that for lower load values, the average queue length for MMPP/BP/1 and MMPP/BP/2 are the same, however as the load increases, the average queue length of MMPP/BP/1 is higher than average queue length for MMPP/BP/2.

12 DISCUSSIONS

In a recent study, the end to end delay of VoIP packets in a WMN were modeled using an M/D/1 queue based model. In this model, the arrival pattern is assumed to follow Poisson distribution. However, the long held paradigm in the communication and performance communities that voice traffic and data follow Poisson distribution is inaccurate and inefficient. To overcome the above challenge, we modeled queuing delay and queue length of VoIP packets in WMN using MMPP/G/m queue system with arrivals that follow Markov Modulated Poisson Process, and general service time. The numerical results obtained from the derived models show that increasing the number of mesh routers, arrival rate and load leads to increase in queuing delay and queue length. Increase in queuing delay and queue length as a result of increasing the number of mesh routers can be explained by the fact that at each mesh router packets experience delay implying the more the mesh routers, the more the delay. Furthermore, increase in load and arrival rate lead to increase in the number of packets being processed and this in turn leads to increased queuing delay and queue length. In addition, it is observed that correlated
arrivals modeled by MMPP/M/m and MMPP/BP/m queue systems exhibit a higher queuing delay and queue length as compared to Poisson arrivals modeled by M/D/1 queue system especially at high values of mesh routers, arrival rates and load.

13 CONCLUSION

We presented an analytical model for evaluating the performance of VoIP traffic in WMN under correlated inter arrival times and general service time using MMPP/G/m queue system. The model is used to evaluate the performance of VoIP traffic in WMN in terms of queuing delay and queue length. The numerical results obtained from the derived models show that increasing the number of mesh routers, arrival rate and load leads to increase in queuing delay and queue length. In addition, it is observed that correlated arrivals modeled by MMPP/M/m and MMPP/BP/m queue systems exhibit a higher queuing delay and queue length as compared to Poisson arrivals modeled by M/D/1 queue system especially at high values of mesh routers, arrival rates and load.

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REFERENCES


