RELIABILITY EVALUATION OF MULTI-SOURCE MULTI-SINK STOCHASTIC-FLOW NETWORKS UNDER QUICKEST PATH AND SYSTEM CAPACITY CONSTRAINTS

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Abstract
A multi-source multi-sink stochastic-flow network consists of a set of nodes, including sources nodes s_i that supply resources and sink nodes tj where demands are realized, and a set of directed arcs that carry resource flows. This paper presents an algorithm to determine the probability, called system reliability ($R_{d_{wj}C_{ij}T}$), that for each source–sink pair (s_i, t_j), d_{wj} (the demand for resource *w* at sink node t_j) units of data can be sent from s_i to t_j through a valid group of paths on the network under two constraints: the transmission time along each valid path does not exceed a specified upper bound T and the maximal capacity of that path is not less than a specified lower bound C_{ij} (required capacity). Based on the minimal paths, the proposed algorithm generates all the lower boundary points for (d_{wj}, C_{ij}, T), which can be used to calculate $R_{d_{wj}C_{ij}T}$ by applying the inclusion-exclusion rule. Finally, we can calculate the system reliability for the multi-source multi-sink stochastic-flow network.

Keywords
Time and capacity constraints, minimal path, multi-source multi-sink stochastic-flow networks, system reliability.

1 INTRODUCTION

A single-source single-sink flow network consists of a set of nodes, including a source node that supplies resources and a sink node at which resources are demanded, and a collection of directed arcs that carry resource flows between them, where each arc or node has several operational states. The system reliability of such a network is the probability that the maximum throughput of the network is not less than the given demand d (for a single commodity). Several algorithms based on minimal paths (MPs) have been presented to evaluate the system reliability, such as by Lin, Jane, and Yuan [1] and Lin [2,3]. Furthermore, Lin [4] and Lin [5] evaluated the system reliability under a budget constraint C. These reports presented efficient algorithms to generate all lower boundary points for (d , C) based on MPs. The system reliability can then be calculated in terms of these lower boundary points by applying the inclusion-exclusion rule. In addition, Lin studied the system reliability of flow networks under budget constraints [6], with multiple commodities [7], and under transmission time constraints [8].

The idea of considering path capacity and a minimum required system capacity C_s in the reliability evaluation was introduced by Aqqarwal, Chopra, and Bajwa [9]. They stated that a system is good if and only if it can successfully achieve the required capacity for transmission from the source node to the sink node. In addition, they defined a group as a set of branches such that the success of these branches ensures system success, as defined above. All such groups can be identified from knowledge of the MPs of the system graph.

A multi-source multi-sink stochastic-flow network is an extension of the concept to multiple sources and sinks on the same network. For such networks, Lin and Yeh [10] developed an optimization algorithm that combined a genetic algorithm, the MP method, and the recursive sum of disjoint products to solve the optimal double-resource assignment problem with maximal network reliability.

In this paper, we will extend the idea of using C_s to multi-source multi-sink flow networks. The system reliability for each source–sink pair (s_i, t_j) in a multi-source multi-sink stochastic-flow network, denoted by ($R_{d_{wj}C_{ij}T}$), is defined as the probability that d_{wj} (demand for resource *w* at sink node t_j) units of data can be sent from s_i to t_j through a valid group of paths on the network such that the transmission time of each valid path does not exceed the specified upper bound T and the maximal capacity of that path is not less than the specified lower bound C_{ij} (required capacity). Based on MPs from s_i to t_j, the proposed algorithm generates all lower boundary points for (d_{wj}, C_{ij},T), which can be used to calculate $R_{d_{wj}C_{ij}T}$ by applying the inclusion-exclusion rule. Finally, we generate all the lower boundary points for (D, C_S, T), where D is the set of all d_{wj} and C_S is the minimum required system capacityfor multi-source multi-sink network, and calculate the system reliability $R_{D,C_S,T}$ of a multi-source multi-sink stochastic-flow network by using the inclusion-exclusion rule.

2 NOTATION and ASSUMPTIONS

2.1 Notation

G (*A*, *N*, *M*, *S*, *T*) a multi-source multi-sink stochastic-flow network containing a set of arcs **A=** $\{a_e | 1 \le e \le n\}$ and a set of nodes N. The maximum capacity of each arc a_e is denoted by $M = \{M_1, M_2, ..., M_n\}$, where M_e is an integer.

S $\{s_1, \ldots, s_q\}$: set of source nodes.

T $\{t_1, \ldots, t_\theta\}$: set of sink nodes.

D $\{d_{w,j} | 1 \le w \le m, 1 \le j \le \theta\}$, where $d_{w,j}$ is the demand for resource *w* at sink node t_j.

$MP_{i,j,k}$ The k^{th} MP from s_i to t_j.

MPS $\{MP_{i,j,k} | 1 \le i \le \sigma, 1 \le j \le \theta, 1 \le k \le k_{i,j}\}$: a set of all MPs, where $k_{i,j}$ represents the number of MPs from s_i to t_j.

np Total number of MPs contained in MPS.

X Capacity vector defined as X= (x_1, x_2, …x_e,, …, x_n).

$R_{d_{wj}C_{ij}T}$ The system reliability for the given demand d_{wj} under T and C_{ij} constraints.

2.2 Assumptions

i. The capacity of each arc a_e is an integer-valued random variable, which takes values $0 < 1 < 2 < \ldots < M_e$ according to a given distribution.

ii. The capacities of the arcs are statistically independent.

iii. The flow along a path does not exceed its maximum capacity.

3 PROPOSED ALGORITHM FOR COMPUTING $R_{d_{wj}C_{ij}T}$

3.1 Definition of Lower Boundary Points For (d_{wj}, C_{ij}, T)

If X is a minimal capacity vector such that the network can send d_{wj} units of data from the source to the sink within T units of time under system capacity C_{ij}, then X is called a lower boundary point for (d_{wj}, C_{ij}, T).

3.2 Generate All Lower Boundary Points For (d_{wj}, C_{ij}, T).

In the following steps, for the k^{th} MP $MP_{i,j,k}$ from s_i to t_j, $MP_{i,j,k} = \{a_{j1}, a_{j2}, \ldots, a_{jn}\}$, we will show how to find the minimal capacity vector X^j = (x_1, x_2,…, x_i,…, x_n) such that the network sends d units of data within T units of time under maximum system capacity C_s.

1. For all mp_{ijk}, examine the path capacities,
   $C_{i,j,k} = \min\{C_i | a_i \in mp_{i,j,k}\}$, k = 1, 2, …, $k_{i,j}$. …(1)
2. For all $mp_{i,j,k}$, calculate the transmission time of the path, $T_{i,j,k}$:
   $$T_{i,j,k} = \sum_{i=1}^{n} \{l_i \mid a_i \in mp_{j,j,k}\} + \lceil d_{wj} / C_{ij} \rceil \quad \ldots(2)$$
3. Determine the valid group paths,

$V_{i,j}$ = {$mp_{i,j,k}$| $C_{i,j,k} \geq C_s$ and $T_{i,j,k} \leq T$, k = 1, 2, …, $k_{i,j}$}.

4. Generate the system capacity vector $X^{i,j}$ = (x_1, x_2,…, x_e,…, x_n) for each $mp_{i,j,k}$ that belongs to $V_{i,j}$ as follows:

$$x_e = \begin{cases} C_{ij} & \text{if } a_i \in mp_{i,j,k} \\ 0 & \text{otherwise} \end{cases} \quad \ldots(3)$$

where x_e is an element of $X^{i,j}$.

3.3 Evaluation of $R_{d_{wj}C_{ij}T}$

If $X_1^{i,j}, X_2^{i,j}, \ldots, X_q^{i,j}$ are the collection of all (d_{wj}, C_{ij}, T)-$mp_{i,j}$, then the system reliability $R_{d_{wj}C_{ij}T}$ is defined as follows:

$$R_{(d_{wj},C_{ij},T)} = \Pr\{\bigcup_{u=1}^{q}\{Y \mid Y \geq X_u^{i,j}\}\} \quad \ldots(4)$$

where $\Pr\{Y\} = \Pr\{y_1\} \cdot \Pr\{y_2\} \cdot \ldots \cdot \Pr\{y_n\}$. We use the inclusion-exclusion rule described by Janan [11] to calculate $R_{d_{wj}C_{ij}T}$ as follows:

If $A_1 = \{Y \mid Y \geq X_1^{i,j}\}$, $A_2 = \{Y \mid Y \geq X_2^{i,j}\}$, …, $A_q = \{Y \mid Y \geq X_q^{i,j}\}$, then apply the inclusion-exclusion rule to calculate $R_{d_{wj}C_{ij}T}$ by using the relation,

$$R_{(d_{wj},C_{ij},T)} = \sum_{i} \Pr\{A_i\} - \sum_{i \neq j} \Pr\{A_i \cap A_j\} + \sum_{i \neq j \neq k} \Pr\{A_i \cap A_j \cap A_k\} - \ldots + (-1)^{q-1} \Pr\{A_1 \cap A_2 \cap \ldots \cap A_q\} \quad \ldots(5)$$

4 AN ILLUSTRATIVE EXAMPLE

As an example, we consider the network in Fig. 1, which has two source and two sink nodes. The arcs are numbered from a_1 to a_{14}; their capacities and the corresponding probabilities are listed in Table 1 [12], and the lead time of each arc is shown in Table 2.

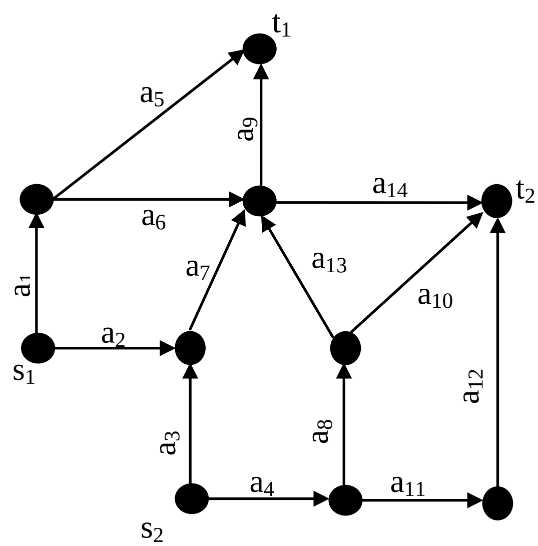

FIGURE 1. Two-source two-sink computer network

Table 1. Arc data for network in Fig. 1

Arc	0	1	2	3	4	5	6	7	8	9	10	11
a_1	0.001	0.001	0.003	0.004	0.005	0.005	0.006	0.007	0.010	0.015	0.060	0.150
a_2	0.001	0.003	0.003	0.004	0.005	0.007	0.007	0.008	0.009	0.010	0.943	0.000
a_3	0.002	0.002	0.003	0.006	0.007	0.007	0.010	0.012	0.015	0.017	0.919	0.000

a_4	0.001	0.001	0.002	0.003	0.005	0.008	0.010	0.011	0.012	0.015	0.015	0.016
a_5	0.001	0.002	0.009	0.012	0.020	0.040	0.050	0.060	0.806	0.000	0.000	0.000
a_6	0.001	0.002	0.002	0.005	0.010	0.012	0.015	0.017	0.020	0.025	0.891	0.000
a_7	0.001	0.001	0.002	0.005	0.008	0.010	0.012	0.015	0.015	0.017	0.020	0.022
a_8	0.001	0.002	0.005	0.005	0.007	0.008	0.010	0.012	0.015	0.015	0.016	0.020
a_9	0.001	0.001	0.002	0.002	0.003	0.004	0.005	0.008	0.009	0.010	0.011	0.015
a_{10}	0.002	0.003	0.005	0.006	0.007	0.009	0.012	0.015	0.941	0.000	0.000	0.000
a_{11}	0.002	0.002	0.003	0.005	0.007	0.008	0.010	0.011	0.020	0.030	0.902	0.000
a_{12}	0.001	0.002	0.003	0.005	0.008	0.009	0.010	0.012	0.015	0.040	0.895	0.000
a_{13}	0.001	0.001	0.003	0.005	0.005	0.010	0.011	0.017	0.018	0.020	0.025	0.031
a_{14}	0.001	0.001	0.002	0.002	0.003	0.005	0.007	0.009	0.016	0.021	0.024	0.025

12	13	14	15	16	17	18	19	20	21	22	23	24
0.733	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.020	0.025	0.856	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.025	0.030	0.817	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.884	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.016	0.017	0.019	0.020	0.857	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.853	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.030	0.035	0.040	0.060	0.719	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2. Lead time for each arc

Arc	Lead time	Arc	Lead time
a_1	2	a_8	1
a_2	1	a_9	3
a_3	3	a_{10}	1
a_4	3	a_{11}	2
a_5	1	a_{12}	2
a_6	2	a_{13}	1
a_7	2	a_{14}	2

In the following, we will show how to calculate $R_{d_{11}C_{11}T}$, i.e., the reliability from the source node s_1 to the sink node t_1. There are three MPs:

$mp_{1,1,1} = \{a_1 , a_5 \}$, $mp_{1,1,2} = \{a_1 , a_6, a_9 \}$,
$mp_{1,1,3} = \{a_2 , a_7, a_9 \}$.

Given d_{11} = 11 and T = 9, Tables 3 and 4 summerize the values of C_{11k} and $T_{1,1,k}$ for each path $mp_{1,1,k}$ for the different values of C_{11} using the algorithm in Section 3.2. In additon, under each table, we show the value of V_{ij}, the corresponding X vectors, and the system reliability $R_{d_{11}C_{11}T}$.

Table 3: Values of $C_{1,1,k}$ and $T_{1,1,k}$ when C_{11} = 10

$mp_{1,1,k}$	$C_{1,1,k}$	Transmission Time $T_{1,1,k}$
$mp_{1,1,1}$	8	4
$mp_{1,1,2}$	10	9
$mp_{1,1,3}$	10	8

According to Table 3, $V_{1,1} = \{mp_{1,1,2},mp_{1,1,3}\}$ because both $C_{1,1,2}$ and $C_{1,1,3}$ are equal to C_{11}. Furthermore, $T_{1,1,2}$ equals T and T_5 is less than T.

Thus, we have

$X_2^{1,1}$ = (10 0 0 0 0 10 0 0 10 0 0 0 0 0) and $X_3^{1,1}$ = (0 10 0 0 0 0 10 0 10 0 0 0 0 0).

Let $A_2 = \{Y | Y \geq X_2^{1,1}\}$ and $A_3 = \{Y | Y \geq X_3^{1,1}\}$.

Then, the system reliability $R_{11,10,9} = \Pr\{A_2 \cup A_3\} = 0.93327$ using the inclusion-exclusion rule, where

$$\Pr\{A_2\} = \Pr\{Y \geq (10\,0\,0\,0\,0\,10\,0\,0\,0\,10\,0\,0\,0\,0\,0)\}$$
$$= \Pr\{x_1 \geq 10\} \times \Pr\{x_2 \geq 2\} \times \Pr\{x_3 \geq 0\} \times$$
$$\times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 0\} \times \Pr\{x_6 \geq 10\} \times$$
$$\times \Pr\{x_7 \geq 0\} \times \Pr\{x_8 \geq 0\} \times \Pr\{x_9 \geq 10\} \times$$
$$\times \Pr\{x_{10} \geq 0\} \times \Pr\{x_{11} \geq 0\} \times \Pr\{x_{12} \geq 0\} \times$$
$$\times \Pr\{x_{13} \geq 0\} \times \Pr\{x_{14} \geq 0\}$$
$$= 0.943 \times 1 \times 1 \times 1 \times 1 \times 0.891 \times 1 \times 1 \times 0.955 \times$$
$$\times 1 \times 1 \times 1 \times 1 \times 1 = 0.8024$$

$$\Pr\{A_3\} = \Pr\{Y \geq (0\,10\,0\,0\,0\,0\,0\,10\,0\,10\,0\,0\,0\,0\,0)\}$$
$$= \Pr\{x_1 \geq 0\} \times \Pr\{x_2 \geq 10\} \times \Pr\{x_3 \geq 0\} \times$$
$$\times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 0\} \times \Pr\{x_6 \geq 0\} \times$$
$$\times \Pr\{x_7 \geq 10\} \times \Pr\{x_8 \geq 0\} \times \Pr\{x_9 \geq 10\} \times$$
$$\times \Pr\{x_{10} \geq 0\} \times \Pr\{x_{11} \geq 0\} \times \Pr\{x_{12} \geq 0\} \times$$
$$\times \Pr\{x_{13} \geq 0\} \times \Pr\{x_{14} \geq 0\}$$
$$= 1 \times 0.943 \times 1 \times 1 \times 1 \times 1 \times 0914 \times 1 \times 0.955 \times$$
$$\times 1 \times 1 \times 1 \times 1 \times 1 = 0.8231$$

$$\Pr\{A_2 \cap A_3\} = \Pr\{Y \geq (\,10\,10\,0\,0\,0\,0\,10\,10\,0\,10\,0\,0\,0\,0\,0)\}$$
$$= \Pr\{x_1 \geq 10\} \times \Pr\{x_2 \geq 10\} \times \Pr\{x_3 \geq 0\} \times$$
$$\times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 0\} \times \Pr\{x_6 \geq 0\} \times$$
$$\times \Pr\{x_7 \geq 10\} \times \Pr\{x_8 \geq 0\} \times \Pr\{x_9 \geq 10\} \times$$
$$\times \Pr\{x_{10} \geq 0\} \times \Pr\{x_{11} \geq 0\} \times \Pr\{x_{12} \geq 0\} \times$$
$$\times \Pr\{x_{13} \geq 0\} \times \Pr\{x_{14} \geq 0\}$$
$$= 0.943 \times 0.943 \times 1 \times 1 \times 1 \times 0.891 \times 0.914 \times$$
$$\times 1 \times 0.955 \times 1 \times 1 \times 1 \times 1 \times 1 = 0.69159$$

Table 4. Values of $C_{1,1,k}$ and $T_{1,1,k}$ when C_{11} = 5

$mp_{1,1,k}$	$C_{1,1,k}$	Transmission Time $T_{1,1,k}$
$mp_{1,1,1}$	8	6
$mp_{1,1,2}$	10	10
$mp_{1,1,3}$	10	9

According to Table 4, $V_{1,1} = \{mp_{1,1,1},mp_{1,1,3}\}$ because both $C_{1,1,1}$ and $C_{1,1,3}$ are greater than C_{11}. Furthermore, both $T_{1,1,1}$ and $T_{1,1,3}$ are less than T.

Thus, we have $X_1^{1,1}$ = (5 0 0 0 5 0 0 0 0 0 0 0 0 0) and $X_3^{1,1}$ = (0 5 0 0 0 0 5 0 5 0 0 0 0 0).

Let $A_1 = \{Y | Y \geq X_1^{1,1}\}$ and $A_3 = \{Y | Y \geq X_3^{1,1}\}$.

Then, the system reliability $R_{11,5,9} = \Pr\{A_1 \cup A_3\} = 0.9976$ using the inclusion-exclusion rule, where

$$\Pr\{A_1\} = \Pr\{Y \geq (5\,0\,0\,0\,5\,0\,0\,0\,0\,0\,0\,0\,0\,0)\}$$
$$= \Pr\{x_1 \geq 5\} \times \Pr\{x_2 \geq 2\} \times \Pr\{x_3 \geq 0\} \times$$
$$\times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 5\} \times \Pr\{x_6 \geq 0\} \times$$
$$\times \Pr\{x_7 \geq 0\} \times \Pr\{x_8 \geq 0\} \times \Pr\{x_9 \geq 0\} \times$$
$$\times \Pr\{x_{10} \geq 0\} \times \Pr\{x_{11} \geq 0\} \times \Pr\{x_{12} \geq 0\} \times$$
$$\times \Pr\{x_{13} \geq 0\} \times \Pr\{x_{14} \geq 0\}$$
$$= 0.986 \times 1 \times 1 \times 1 \times 0.956 \times 1 \times 1 \times 1 \times$$
$$\times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 0.9426$$

$$\Pr\{A_3\} = \Pr\{Y \geq (0\,5\,0\,0\,0\,0\,5\,0\,0\,5\,0\,0\,0\,0\,0)\}$$
$$= \Pr\{x_1 \geq 0\} \times \Pr\{x_2 \geq 5\} \times \Pr\{x_3 \geq 0\} \times$$
$$\times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 0\} \times \Pr\{x_6 \geq 0\} \times$$
$$\times \Pr\{x_7 \geq 5\} \times \Pr\{x_8 \geq 0\} \times \Pr\{x_9 \geq 5\} \times$$
$$\times \Pr\{x_{10} \geq 0\} \times \Pr\{x_{11} \geq 0\} \times \Pr\{x_{12} \geq 0\} \times$$
$$\times \Pr\{x_{13} \geq 0\} \times \Pr\{x_{14} \geq 0\}$$
$$= 1 \times 0.984 \times 1 \times 1 \times 1 \times 1 \times 0.983 \times$$
$$\times 1 \times 0.991 \times 1 \times 1 \times 1 \times 1 \times 1 = 0.9586$$

$$\Pr\{A_1 \cap A_3\} = \Pr\{Y \geq (\,5\,5\,0\,0\,5\,0\,5\,0\,5\,0\,0\,0\,0\,0)\}$$
$$= \Pr\{x_1 \geq 5\} \times \Pr\{x_2 \geq 5\} \times \Pr\{x_3 \geq 0\} \times$$
$$\times \Pr\{x_4 \geq 0\} \times \Pr\{x_5 \geq 5\} \times \Pr\{x_6 \geq 0\} \times$$
$$\times \Pr\{x_7 \geq 5\} \times \Pr\{x_8 \geq 0\} \times \Pr\{x_9 \geq 5\} \times$$
$$\times \Pr\{x_{10} \geq 0\} \times \Pr\{x_{11} \geq 0\} \times \Pr\{x_{12} \geq 0\} \times$$
$$\times \Pr\{x_{13} \geq 0\} \times \Pr\{x_{14} \geq 0\}$$
$$= 0.986 \times 0.984 \times 1 \times 1 \times 0.956 \times 1 \times 0.983 \times$$
$$\times 1 \times 0.991 \times 1 \times 1 \times 1 \times 1 \times 1 = 0.9036$$

Table 5 summarizes the values of $R_{d_{wj}C_{ij}T}$ for each value of C_{ij} and shows the values of $C_{i,j,k}$, C_{ij}, and $T_{i,j,k}$ for each pair (s_i, t_j) corresponding to each path $mp_{i,j,k}$.

Table 5. Values of $R_{d_{wj}C_{ij}T}$ for different values of d_{wj} and C_{ij}

(s_i, t_j)	$mp_{i,j,k}$	$C_{i,j,k}$	C_{ij}	Transmission Time $T_{i,j,k}$	Value of $T_{i,j,k}$	$R_{d_{wj}C_{ij}T}$	The value of $R_{d_{wj}C_{ij}T}$
(1,2)	$mp_{1,2,1}$ $mp_{1,2,2}$	10 10	10	$T_{1,2,1}$ $T_{1,2,2}$	8 7	$R_{12,10,9}$	0.91241
			5	$T_{1,2,1}$ $T_{1,2,2}$	9 8	$R_{12,5,9}$	0.98991
(2,1)	$mp_{2,1,1}$ $mp_{2,1,2}$	10 12	10	$T_{2,1,1}$ $T_{2,1,2}$	9 9	$R_{7,10,9}$	0.92187
			5	$T_{2,1,1}$ $T_{2,1,2}$	10 10	$R_{7,5,9}$	0.00000
(2,2)	$mp_{2,2,1}$ $mp_{2,2,2}$ $mp_{2,2,3}$ $mp_{2,2,4}$	10 12 8 10	10	$T_{2,2,1}$ $T_{2,2,2}$ $T_{2,2,3}$ $T_{2,2,4}$	6 8 6 8	$R_{10,10,9}$	0.97605
			5	$T_{2,2,1}$ $T_{2,2,2}$ $T_{2,2,3}$ $T_{2,2,4}$	7 9 7 9	$R_{10,5,9}$	0.99963

Finally, the system reliability $R_{D,Cs,T}$ of the multi-source multi-sink flow network can be calculated for different values of C_s by using the inclusion-exclusion rule according to the generated set of all lower boundary points for (D, C_S, T). Table 6 shows the set of all lower boundary points for (D, C_S, T) and the corresponding value of $R_{D,Cs,T}$.

Table 6: Values of $R_{D,C_s,T}$

(C_S, T)	Set of lower boundary points for (D, Cs, T)	The corresponding value of $R_{D,C_S,T}$
(10, 9)	0 10 0 0 0 0 10 0 10 0 0 0 0 0 10 0 0 0 0 10 0 0 0 0 0 0 0 0 10 0 10 0 0 0 0 10 0 0 0 0 0 0 0 0 0 0 10 0 0 0 10 10 0 0 0 10 0 0 0 10 0 0 0 10 0 0 0 0 0 0 10 0 0 0 10 0 0 0 10 0 0 0 0 10 10 0 0 0 10 0 0 0 0 0 0 10 10 0	0.997016
(5, 9)	0 0 0 0 0 0 0 0 0 5 0 0 0 5 0 0 0 0 0 5 0 5 0 0 0 0 5 0 5 0 0 0 0 0 0 0 5 0 0 0 0 5 5 0 0 0 0 0 0 5 0 0 0 0 5 0 5 0 0 0 0 0 0 0 0 0 0 5 0 0 5 5 0 0 0 0 5 0 0 0 5 0 0 0 0 0 0 0 5 0 5 0 0 0 5 0 0 0 0 0 0 5 0 0 0 0 0 0 5 5 0 0	0.99999

5 TIME ANALYSIS

The algorithm in Section 3.2 needs O(m_{ij} n) time to generate all the lower boundary points for (d_{wj}, C_{ij}, T) in the worst case, where n is the number of arcs and m_{ij} is the number of MPs from s_i to t_j. The algorithm in Section 3.3 needs O(m_{ij}^2n) time to evaluate the system reliability in the worst case by using the inclusion-exclusion rule. Therefore, the total time of the algorithm is (O(m_{ij}n) + O(m_{ij}^2n)) to calculate the system reliability $R_{d_{wj}C_{ij}T}$ in the worst case. Thus, the total time of the algorithm is (O(np*n) + O(np^2 *n)) for all pairs (s_i, t_j) in the worst case, where *np* is total number of MPs through the multi-source multi-sink network.

6 DISCUSSION AND COMPARISON

In comparison with the algorithm presented in [8], the algorithm in [8] evaluated the system reliability $R_{d,T}$ in terms of all lower boundary points for (d; T) in the case of single-source single-sink flow network to a given demand d (single commodity). Takes into account both the transmission time and system capacity constraints, the algorithm presented in this paper to evaluate the system reliability $R_{D,C_S,T}$ in the case of multi-source multi-sink network for multiple commodities.

7 CONCLUSIONS AND FUTURE WORK

This paper presented an algorithm to evaluate the reliability of a stochastic-flow network with multiple sources and sinks. The algorithm is based on MPs to generate all lower boundary points for (d_{wj}, C_{ij}, T), which are used to calculate the reliability $R_{d_{wj}C_{ij}T}$ from source s_i to sink t_j by using the inclusion-exclusion rule.

Finally, the algorithm has been used to evaluate the system reliability $R_{D,C_S,T}$ of a multi-source multi-sink flow network.

For the future work, the algorithm can be extended to evaluate the system reliability for the multi-source multi-sink network under the transmission cost constraint.

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