

iii	456	425	1	0	0	1	3	3
	125	81	0	1	1	-3	1	1
	81	44	1	-3	-1	4	1	1
	44	37	-1	4	2	-7	1	1
	37	7	2	-7	-3	11	9	1
iv	34	10	1	0	0	1	3	3
	10	4	0	1	1	-3	2	11

Finally, we get $x = 2,527$ and $y = 183$, therefore, $v = GCD(x, y) = GCD(2,527, 183)$ which can be calculated easily using Euclidian's algorithm.

3 LEAST COMMON MULTIPLE (LCM)

LCM is well known elementary number theory algorithm that is defined as the smallest multiple integer of two numbers. The very basic method to calculate LCM is using prime factorization method (PF-LCM).

PF-LCM method depends on fact that each integer number can uniquely written as a product of prime numbers raised to different powers. Then, we can calculate LCM by taking all common factors with greatest power.

An example of the algorithm is shown below:

$$40 = 2^3 \cdot 5, 26 = 2 \cdot 13 \rightarrow LCM(40, 26) = 2^3 \cdot 5 \cdot 13 = 520$$

Alternatively, LCM can be efficiently calculated by using the GCD reduction method [9]. LCM is usually calculated using GCD from the formula:

$$LCM(a, b) = \frac{ab}{GCD(a, b)} = \left(\frac{a}{GCD(a, b)} \right) b$$

Where GCD can be calculated using different algorithms without need to factor the numbers.

4 CONCLUSIONS

GCD/LCM operations are essential number theory algorithms that commonly used as underlying operations of many computing processors. The efficient utilization of such algorithms contributes in the overall leverage of system execution. For instance, Lehmer's

algorithm reduces the large integers to common base (b) with fast convergence rate at complexity of $O(n/\log(n))$ which make it faster than other algorithms. Eventually, Lehmer's algorithm can be efficiently implemented (in hardware or software) to compute both GCD and LCM operations for any two large integer numbers.

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