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Two–Dimensional Analytical Modeling of Permanent Magnet Induction Generator (*PMIG)*

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Abstract — This paper presents the analytical modeling of permanent magnet induction generator *(PMIG)* used in green energy *(i.e. Wind energy)*. This analysis, is based on analytical modeling in determining the essential parameters for the magnetic field in the PMIG. A 2-D field analysis is presented which takes the permanent magnet leakage, the distribution of the MMF across the magnet height, equivalent width determination, flux density components, and its fundamentals by depending on Helm-Holtz, and Maxwell equations with using the separation of variables technique, and Fourier expansion for gap flux density. Thus, solving a set of PDE equations at boundary conditions at machine regions will provide a set of hyperbolic equations allowing determining the explicit magnetic field coefficients. Then, the expressions of the global quantities *(MMF, cogging torque, and electromagnetic torque)* are deduced from the expressions of the analytical study for the magnetic field distribution.

The presented analytical modeling helps, as a first aim, to explore rapidly the search space of potentially optimal prototypes.

Keywords— Analytical modeling; Arkkio torque; Cogging torque; Fourier series; Maxwell stress tensor; Permanent magnet induction generator.

I. INTRODUCTION

A. *History*

RECENTLY, wind generation and micro-hydro plants have been introduced as green energy sources, and as competitive forms of clean energies that protect the environment.

Induction generators extensively used due to its main advantages from lacking to frequency control and due to its effective initial and maintenance costs [1]. However, the IG needs a magnetizing current as a feedback from the grid, that cause a decrease in terms of the power factor and efficiency. Hence, the performance of IG needs to have a compensation to the previous terms. Thus, it is possible that the PM excitation would be suppressed to decrease the magnetizing current and improve the power factor and efficiency.

In order to understand the performance of the PMIG, it is necessary to perform its analytical model. However, the core, and eddy losses are not considered.

Permanent magnets used widely in the industrial applications, especially for generators where carbon steel permanent magnets used firstly in the PMIG manufacturing. Where it concern a simple structure and an effective cost in terms of its main raw materials. Knowing that the PMIG do not need power electronic converters (i.e. *Direct – Drive Generator*). In general, modular permanent magnet induction generators are competitive in terms of their performance and maintenance cost compared to other generators [6].

In general, the PMIG have three main advantages a high efficiency generator, loss less generating power in high voltage direct current transmission system (*HVDC)*, improvement in transient and steady state compared with the conventional induction generator due to the permanent magnets usage.

Furthermore, the PMIG is more preferable as offshore wind turbines, due to the constant wind speed.

B. *General Overview*

Analytical models compared to the classical modeling *(Finite Element Analysis)* is more preferred due to its less consuming time during simulation, furthermore due to its accuracy in the obtained results.

This paper focuses on solving the Maxwell's equation in the PMIG armature, permanent magnet rotor, and air gap regions for their electromagnetic field variation by solving the corresponding coefficients at boundary conditions. And in our modeling we assume a smooth stator surface with infinite permeability, and anisotropic permanent magnet. Furthermore, this paper sheds light in studying the effective gap length by depending on Carter's factor, and in determining the current sheet density in each region, without forgetting the Fourier expansion for gap flux density, and for the cogging torque production.

This paper attempts to provide analytical model to ease the analysis and design of PMIG on MATLAB software as a simulation tool for the electrical engineers.

II. ANALYTICAL MODEL DESIGN

A. 2D Modeling and Assumptions

In this study, we assume that the edge effects are ignored. In order to simplify the three-dimensional assumption, it is reduced into two-dimensional assumption, and supposing the magnetic potential vector $\vec{A}$ only have a z-component, furthermore the PM will be composed into a very thin strips,

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in order to facilitate the modeling as shown in fig.1.

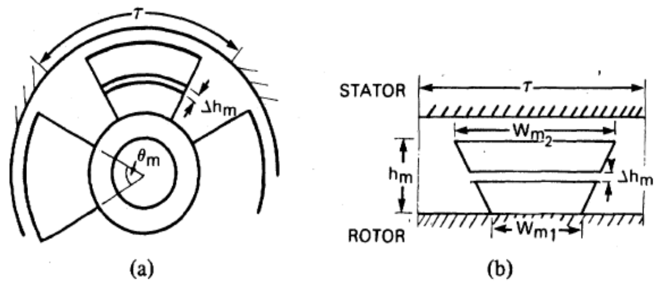


Fig. 1: (a) Curved magnet, (b) Thin strips decomposition of PM.

The model primarily is formulated in two-dimensional cartesian coordinates (X, Y), and in polar form in some cases.

In this model, some assumptions are taken into consideration, where:

- The stator core, and teeth are supposed to have zero reluctivity; on the other hand the flux saturation taken into consideration.
- The 2-D model is set to cover only the regions where the air
gap and magnet regions fluxes are compared with respect to each other.
- Eddy current effects are not taken into consideration.

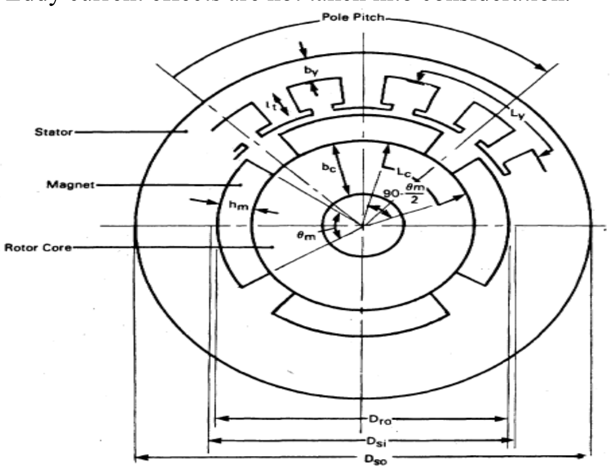


Fig. 2: 2-D PMIG cross-sectional view.

B. Problem Description

In order to perform our modeling, we assume that the PMIG decomposed into the three regions as mentioned before. By considering the magnetic vector potential has only an axial component.

In order to study the magnetic vector potential $\overrightarrow{A}(z)$, we will introduce the differential form of Maxwell equation by:

$$\nabla \wedge \vartheta \cdot \nabla \wedge A = J - \sigma \cdot \frac{\partial A}{\partial t} - \sigma \cdot \nabla \vartheta + \nabla \wedge Hc \quad \text{Eq. (1)}$$

Where, J is the current density in [A/m²], ϑ *is the* reluctivity, and Hc is the coercive force in [A/m].

For the simplicity in understanding the modeling approach, we suppose that the slots are equidistant, and by decomposing the gap into periodic segmentation considered in this study. Also, the slotted stator has a classical configuration with U - shaped teeth, where the PMs are located on the rotor surface.

$$B(x.y) = \begin{bmatrix} \overrightarrow{ex} & \overrightarrow{ey} & \overrightarrow{ez} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & Az \end{bmatrix} \quad \text{Eq. (2)}$$

- Effective air gap length δ'', and its Carter's coefficient: According to Carter's principle about the air gap length, shows that the effective air gap length of the generator will be greater than the actual length, by considering the distance between magnets, slot pitch, and equivalent slot opening; such that these factors lies in the Carter's factor Kc. In order to introduce Carter's factor, we will assume that the magnetic flux density function is approximated to a rectangular function, so that the density equal to zero at the slot opening, while it is constant at the stator teeth, as shown in fig.3 [2].

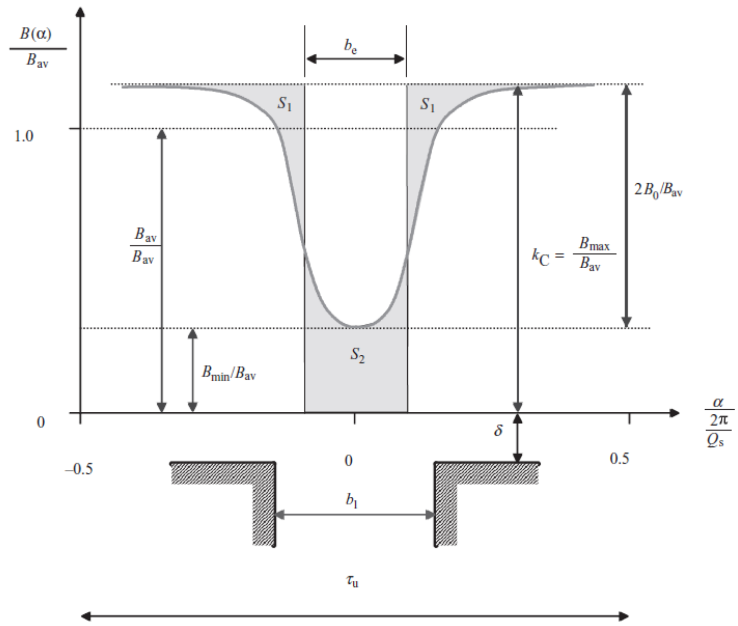


Fig. 3: Flux density distribution along a slot pitch.

C. Mathematical approach for flux density.

For a two-dimensional problem, the second order P.D.E of Laplace equation in Cartesian coordinate, will be expressed as:

$$\frac{\partial^2 A}{\partial^2 x} + \frac{\partial^2 A}{\partial^2 y} = 0. \quad \text{Eq. (3)}$$

Thus, its implicit solution in the permanent magnet, and air gap regions is on the following manner; knowing that the solution function will have a sinusoidal signal, due to its relation with current sheets [3].

$$Ai = \{Ci \cdot sinh(ay) + Di \cdot cosh(ay)\} \cdot cos(ax) \quad \text{Eq. (4)}$$

Where, $i = I'.I''.II$ *represents the permanent magnets, and air gap regions respectively as shown in Fig. 4;* $a = \frac{\pi}{\tau}$.

Now, in order to evaluate the flux density components in each region, so we tend to integrate them at the boundary conditions as shown in fig. 4.

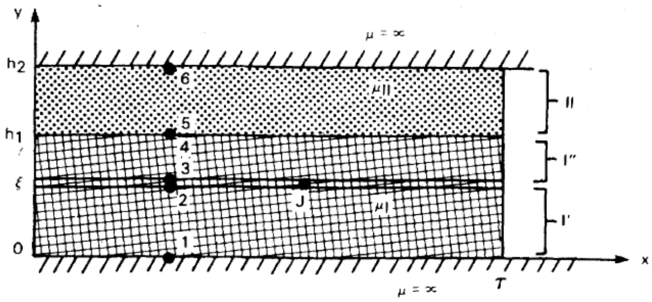


Fig. 4: Region boundaries for magnets and stator fields.

Thus the coefficients $Ci.$ *and* Di substituted in the following equations, in order to obtain the fundamental y-components of $\vec{B}$ are:

$$ByI' = \frac{\mu 1\cdot \vec{J}}{N}\Big\{cosh[a(h1-\varepsilon)] + \frac{\mu I}{\mu II} sinh[a(h1-\varepsilon)] \cdot tanh(a[h2-h1]\Big\} \cdot cosh(ay) \quad \text{Eq. (5)}$$

$$ByII' = \frac{\mu 1\cdot \vec{J}}{N}\Big\{cosh[a(h1-y)] + \frac{\mu I}{\mu II} sinh[a(h1-y)] \cdot tanh(a[h2-h1]\Big\} \cdot cosh(a\varepsilon) \quad \text{Eq. (6)}$$

$$ByII = \frac{\mu 1\cdot \vec{J}}{N} \cdot \{\cosh(a\varepsilon)\cdot \cosh[a(h2-y)]\} \cdot / \cosh[a(h2-h1)] \quad \text{Eq. (7)}$$

Where, $\boldsymbol{N} = \mathbf{sinh}(\boldsymbol{ah1}) + \frac{\boldsymbol{\mu I}}{\boldsymbol{\mu II}}\mathbf{cosh}(\boldsymbol{ah1}) \cdot \mathbf{tanh}[\boldsymbol{a}(\boldsymbol{h2} - \boldsymbol{h1})]$ Eq.(8)

Now, we can deduce the x-component of flux density in each region by:

$$Bxi = \frac{1}{a} \cdot \frac{\partial Byi}{\partial y} \quad \text{Eq. (9)}$$

D. Stator Flux Density Distribution

In order to obtain the armature flux density distribution, we set ε to zero, to evaluate the flux density from both magnet, and air gap regions.

1) *For the magnetic region.*

$$(\boldsymbol{By})\boldsymbol{m} = \frac{\boldsymbol{\mu 0}\cdot\overrightarrow{\boldsymbol{Js1}}}{\boldsymbol{\mu m}} \cdot \{\mathbf{cosh}[\boldsymbol{a}(\boldsymbol{hm} + \boldsymbol{\delta''} - \boldsymbol{y})] / \mathbf{cosh}(\boldsymbol{a} \cdot \boldsymbol{hm})\} \quad \text{Eq. (10)}$$

2) *For the Air Gap region.*

$$(By)g = \frac{\mu 0\cdot\overrightarrow{Js1}}{N1} \cdot \Big\{\cosh[a(\delta''-y)] + \frac{\mu 0}{\mu m} \cdot \sinh[a(\delta''-y)] \cdot \tanh(hm)\Big\} \quad \text{Eq. (11)}$$

Furthermore, since the magneto-motive force of the air gap is variable due to the interference of its flux density, we tend to express the average flux density for the armature, and magnets regions in the following manner.

$$\begin{cases} (\overrightarrow{\boldsymbol{Ba}})\boldsymbol{avg} := \frac{\mu 0\cdot\overrightarrow{Js1}}{N1\cdot a\cdot\delta''}\Big\{\mathbf{sinh}(\boldsymbol{a\delta''}) + \frac{\mu 0}{\mu m}\cdot \mathbf{tanh}(\boldsymbol{a}\cdot\boldsymbol{hm})\cdot[\mathbf{cosh}(\boldsymbol{a\delta''}) - \mathbf{1}]\Big\} \\ (\overrightarrow{\boldsymbol{Bm}})\boldsymbol{avg} := \frac{\mu m\cdot\overrightarrow{Jm}}{N2\cdot a\cdot\delta''}\cdot\Big\{\frac{\sinh(a\cdot hm)}{a\cdot hm}\Big\}\cdot \mathbf{tanh}(\boldsymbol{a}\cdot\boldsymbol{\delta''}) \end{cases} \quad \textbf{Eq. (12)}$$

$$\begin{cases} \boldsymbol{N1} = \mathbf{sinh}(\boldsymbol{a\delta''}) + \frac{\boldsymbol{\mu 0}}{\boldsymbol{\mu m}}\cdot\mathbf{cosh}(\boldsymbol{a\delta''})\cdot\mathbf{tanh}(\boldsymbol{a}\cdot\boldsymbol{hm}) \\ \boldsymbol{N2} = \mathbf{sinh}(\boldsymbol{a}\cdot\boldsymbol{hm}) + \frac{\boldsymbol{\mu m}}{\boldsymbol{\mu 0}}\cdot\mathbf{cosh}(\boldsymbol{a}\cdot\boldsymbol{hm})\cdot\mathbf{tanh}(\boldsymbol{a}\cdot\boldsymbol{\delta''}) \end{cases} \quad \text{Eq. (13)}$$

And $\overrightarrow{Js1}$ is the amplitude of the fundamental component of current sheet representing the MMF in three phase windings, (stator) in [A/m²], expressed as: $Js1 = \frac{3\sqrt{2}\cdot Nw\cdot Kw}{P\cdot\tau}\cdot is$; such that is is the stator load current in [Ampere], Nw, and Kw are the number of turns per winding, and the winding factor respectively.

E. Magnetomotive Force (MMF).

In order to have simplicity during studying the magneto motive force exerted between the PMs and the gap regions, we tends to decompose the magnet into n strips by differential form, And by Fourier analysis of current sheet pulse, thus we have the fundamental component by:

$$\boldsymbol{\Delta Jm1} = \{\frac{4}{\tau}\frac{\boldsymbol{Br}\cdot\boldsymbol{\Delta\varepsilon}}{\boldsymbol{\mu m}}\mathbf{sin}\left(\boldsymbol{\alpha e}\cdot\frac{\pi}{2}\right)\}\cdot\mathbf{cos}(\frac{\pi}{2}\boldsymbol{x}) \quad \text{Eq. (14)}$$

Where, $\alpha e = \frac{Wm}{\tau} = \frac{360°}{2\cdot m\cdot q}$, m is the number of phases, q is the number of slots per pole, and $\Delta\varepsilon$ is the electric pulse width in [seconds].

Hence, in order to take into consideration, the total MMF for the magnet, the equivalent equation is:

$$\overrightarrow{\text{M1}} = \sum_{i=1}^{n}(\Delta\overrightarrow{\text{M1}})\text{i} = \sum_{i=1}^{n}\frac{\text{M}}{\text{n}}\cdot\sin(\theta\text{m1} + \text{i}\cdot\Delta\theta) \quad \text{Eq. (15)}$$

And in order to control the MMF variation by changing the widths correspondingly, will tends to determine an expression for the equivalent magnet width by:

$$Wm = \frac{2\tau}{\pi}\cdot\sin^{-1}\Big\{\frac{\cos\theta m1 - \cos\theta m2}{\theta m2 - \theta m1}\Big\} \quad \text{Eq. (16)}$$

Where, $M = \frac{4Br\cdot hm}{\pi\cdot\mu m}$, $\theta m1 = \frac{Wm1}{\tau}\cdot\frac{\pi}{2}$, $\Delta\theta = \frac{(\theta m2 - \theta m1)}{n}$ and $\theta m2 = \frac{Wm2}{\tau}\cdot\frac{\pi}{2}$

F. Electro Magnetic Torque

i. *Maxwell Stress Tensor.*

In an electrical rotating machine, most of the studies concentrates on the electromechanical parameters between rotor and stator interaction, unless on the electromagnetic torque which plays a primary role in the conversation of energy [4].

The Maxwell stress tensor is the most familiar method to determine the torque, in the numerical analysis for electrical machines.

The total electromagnetic torque given by (16):

$$Te = \oint_S r \wedge \sigma \cdot dS = \oint_S r \wedge \left\{\vartheta 0(B \cdot n)B - \frac{1}{2\mu 0}B^2 n\right\} dS = \frac{1}{\mu 0} r \int_S B\alpha \cdot Br \cdot dS = \vartheta 0 \cdot \int_{r1}^{r2}\left\{\int_0^{2\pi} r \cdot Br \cdot B\alpha \cdot d\alpha\right\} \cdot dr$$
Eq. (17)

Where, Br and Bα are the radial and angular components of flux density respectively in [Tesla].

ii. Arkkio's Method.

In order to study the electromagnetic torque along the air gap, thus Arkkio developed a dependable method on that of Maxwell, by integrating the above formula along the radial direction over the difference in radii between stator and rotor, that's it:

$$Te(r_s - r_r) = \int_{r_r}^{r_s} Te \cdot dr = \frac{1}{\mu 0}\int_{Sag} rBrB\alpha dS$$

$$\boldsymbol{Te = \frac{1}{\mu 0(r_s - r_r)}\int_{Sag} rBrB\alpha dS} \quad \text{Eq. (18)}$$

Where, T_e is the electromagnetic torque in [N.m], r_s and r_r are the stator, and rotor radii respectively.

G. Electromechanical Equations.

In order to perform the analysis for the *PMIG* electromagnetic torque, thus consider the first order differential equation by:

$$\boldsymbol{Te - Tl = J \cdot \frac{d\omega(mech)}{dt}} \quad \text{Eq. (19)}$$

Where, $\boldsymbol{T_l}$ is the load torque [N.m], and the expression of the angular speed in [radian/sec] is:

$$\boldsymbol{\omega(t) = \frac{To}{b} \cdot \left(1 - e^{\frac{-t}{\tau_m}}\right)} \quad \text{Eq. (20)}$$

Now, the mechanical displacement $\emptyset m(t)$ in [radian] can be deduced by integrating the angular speed with time, therefore:

$$\boldsymbol{\emptyset m(t) = \int_{t1}^{t2} \omega(t) \cdot dt = \frac{To}{b}\left[t + \tau_m . e^{\frac{-t}{\tau_m}}\right]} \quad \text{Eq. (21)}$$

Where *T0* is the generator torque [N.m], b is the viscous friction coefficient in [N.m.sec.] and, *J* is the shaft inertia in [Kg.m], and τ_m is the time constant, expressed by $\tau_m = \frac{J}{b}$.

H. Fourier series for air gap flux density.

In order to determine the magnetic flux density in the air gap of the *PMIG,* we apply the Fourier series expansion [5], assuming that there is no magnetic field effects occurred by the stator teeth.

We define a ratio between the width of the magnet, and the pole – pitch of the rotor core by "α_{p-p}"; such that: $\alpha_{p-p} = \frac{\alpha(arc)}{\alpha(pitch)}$, and $0 \leq \alpha_{p-p} \leq 1$.

Hence, the Fourier expansion at an electrical angle θe for the flux density is expressed by:

$$\boldsymbol{Bg(\theta e) = \frac{2Bg.peak}{\pi}\sum_{kh=1}^{\infty}\frac{1-(-1)^{kh}}{kh} \cdot cos[kh \cdot (\frac{1-\alpha_{p-p}}{2}) * 180°]\, sin(kh \cdot \theta e)} \quad \text{Eq. (22)}$$

Where, Bg_peak is the peak value of the air gap flux density in [Tesla], and K_h is the K-th harmonic of flux density.

Furthermore, we should spotlight on the saturation of the flux density in the generator core, such that when the saturation case takes place, the well-defined formula: $B = \mu \cdot H$, can't be more applied, since the main parameters in this formula will vary independently, however will tend to the general approximated formula for the B-H curve by:

$$\boldsymbol{B(H) = \frac{2}{\pi}h\infty * tan^{-1}(\frac{\pi}{2} \cdot \frac{H}{h\infty})} \quad \text{[Tesla]} \quad \text{Eq. (23)}$$

Where, $h\infty$ is a constant represents the type of the permanent magnet material.

I. Fundamental MMF of air gap.

The rectangular air – gap MMF of the *PMIG,* can be resolved into a Fourier series comprising a fundamental component and a series of odd harmonics, thus it can be expressed as:

$$\text{Fag} = \frac{4}{\pi}(\frac{Kw \cdot Nph}{poles}) \cdot i_a \cdot \cos(\frac{poles \cdot \theta_e}{2}) \quad \text{Eq. (24)}$$

Therefore, once the air gap MMF is known, then the air gap flux intensity can be determined by dividing the fundamental component of the gap MMF by the gap length.

$$H_{ag1} = \frac{4}{\pi}(\frac{Kw \cdot Nph}{g \cdot poles}) \cdot i_a \cdot \cos(\frac{poles \cdot \theta_e}{2}) \quad \text{Eq. (25)}$$

J. Generated Voltage:

As the rotor turns, the flux linkage varies co-sinusoidal with the angle between the magnetic axes of the stator coil and rotor, thus the flux linkage in one phase of the PMIG winding is:

$$\lambda a = K_w N_{ph} \emptyset_p \cos((\frac{poles}{2})\omega_t = K_w N_{ph} \emptyset_p \cos(\omega \cdot t)$$
Eq. (26)

Thus, by depending on Faraday's law, the induced voltage can be deduced by:

$$e_a = \frac{d\lambda}{dt} = -\omega \cdot K_w . N_{ph} \cdot \emptyset \sin(\omega t) \quad \text{Eq. (27)}$$

K. Cogging in the Presence of a Permanent Magnet.

The cogging torque, represents the interference between the permanent magnets, and the stator teeth, that is when a magnet rotates and reaches the slot opening, the magnet reluctance will vary due to its long path in the slot, hence results in producing cogging torque; which is the interest of designers to minimize its value, by varying the sizes of the slot opening, or by its horse shoe [6].

Mathematically, this torque will represent a given value from the total torque of the machine, regarding to the mutual torque, and the excitation torque. The cogging torque for PMIG machines can be expressed by depending on Fourier expansion as:

$$T_{cogg} = \sum_{n=1.2.3\cdots}^{\infty} T_{nc} \cdot \sin(N_c \cdot n \cdot \theta) \quad \text{Eq. (28)}$$

Where T_{cogg} is the summation for each cogging torque element produced by the equivalent slot opening, that is deduced from Carter's factor; where N_c is the least common multiple of the pole number, and the equivalent slot openings number N_s, during one mechanical revolution. T_{nc} is the amplitude of its Nc^{th} harmonic; in other words they shall be determined by Fourier transformation, and θ is the mechanical displacement between the rotor and armature regions.

The MMF for the PM is rectangular takes a rectangular wave, that consequently produces a rectangular wave form for its flux density. For the purpose of simplicity, we introduce a correction factor between the ratio of fundamental to total flux of the PMIG. However, for a rectangular flux density waveform having an equivalent width *Wm,* we have:

$$B_l = \frac{4}{\pi} B_m \sin\left(\frac{Wm}{\tau}\frac{\pi}{2}\right) \quad \text{Eq. (29)}$$

Thus, the correction factor C_f between the total flux per unit length, and the fundamental component is expressed by:

$$C_f = \frac{\phi_l}{\phi_t} = \frac{4}{\pi}\frac{\sin(\theta_e \cdot \frac{\pi}{2})}{(\theta_e \frac{\pi}{2})} \quad \text{Eq. (30)}$$

III. MATLAB Modeling Validation.

The following graphs show the speed of the PMIG machine, under load conditions, in both start – up, and steady states; so that the steady speed 1500rpm for four poles reached at 102 s.

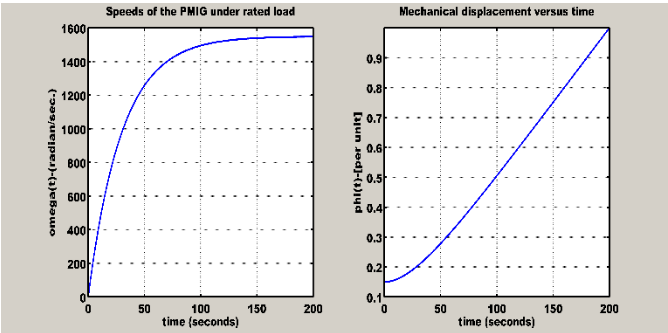
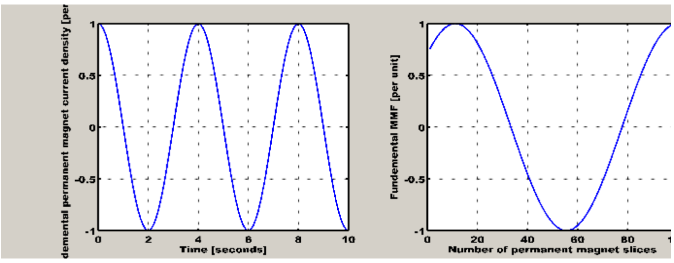


Fig. 5:Variation of speed, and mechanical displacement versus time.



(a) (b)

Fig. 6: (a) Fundamental magnet current density (p.u) as a function of time, and (b) The fundamental MMF versus PM Slices.

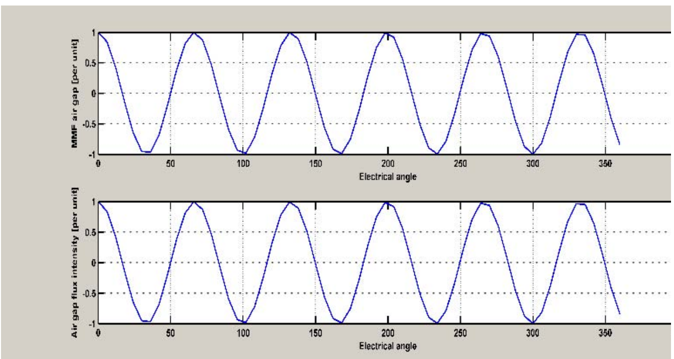
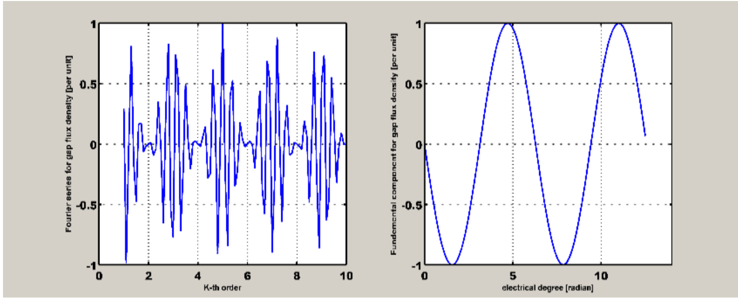


Fig. 7: MMF, and the flux intensity in the air gap versus the electrical angle in degree.

Furthermore, the Fourier series for the gap flux density will be investigated in Fig. 8.



(a) (b)

Fig. 8: (a) Fourier expansion for gap flux density, (b) Fundamental component graphs.

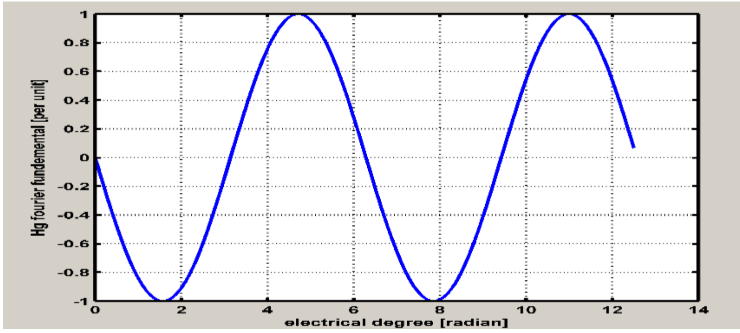


Fig. 9: Fourier series for flux intensity.

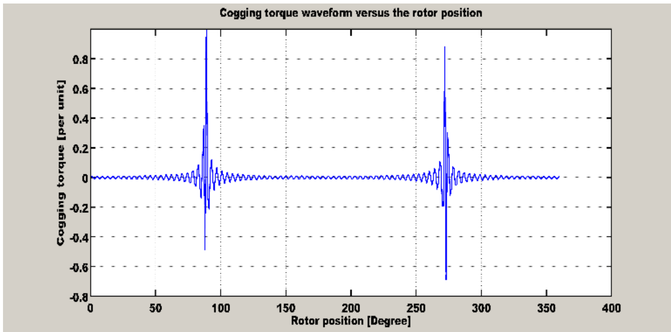


Fig. 10: Fourier expansion for the cogging torque waveform as a function of rotor position.

Note that Fig. 10, shows the cogging torque of the PMIG that consists from 16 PM poles, and 9 flux gaps.

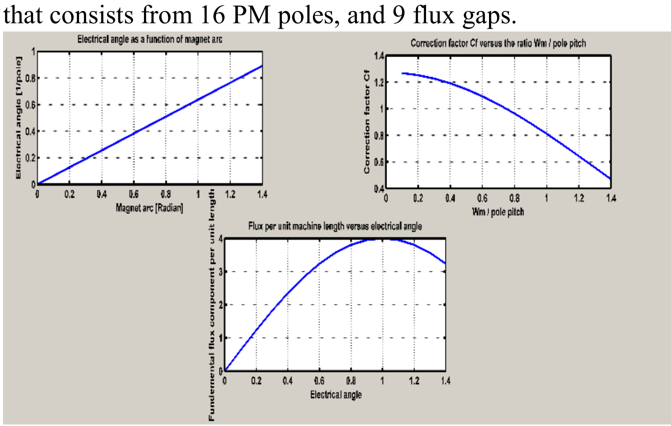


Fig. 11: Main parameters of the machine regarding the correction factor.

Fig. 11, shows the variation of the electrical angle as a function of the magnet arc that increases linearly, also the correction factor between the fundamental and the total flux density per machine length, that provides the user to design the corresponding flux variation to the machine, as well as the fundamental flux component versus the electrical angle.

IV. CONCLUSION.

In this paper, we presented the equations needed for PMIG modeling using Maxwell's equations, Fourier expansion for flux density, air gap flux density, geometric design for permanent magnets, and their effect, Carter's coefficient and its interference on air gap length, hence an analytical approach was executed to study the dynamic magnetic fields of the PMIG machine, taking into consideration the PM geometry, in other words its curved shape.

Furthermore, this analytical modeling allows the user to make a convenient analysis of the machine via compromising the correction factor between the fundamental and the total harmonics for PMIG flux density. Such compromise permits machine to be produced with a less cogging torque compared to the conventional class of induction generators.

As a result, it provides a clear estimation for the startup, and steady-state performances, under variety of operating status.

In addition, the results of the PMIG qualities lead to a low maintenance, and a convenient solution for offshore wind turbines, as well as, the results show that the studied machine will not need power converters , and complicated control.

V. FUTURE WORK

For the next vision, we attempt to mitigate the cogging torque produced by the PMIG by changing the magnet shape, and size, as a first approach, and by depending on the air gap segmentation as a second approach. Furthermore, we will validate the analytical results obtained in MATLAB, by the Finite Element Analysis *(FEA)*.

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