

## Application of Hybrid Wavelet-Fractal Compression algorithm for Radiographic Images of Weld Defects

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### ABSTRACT

Based on the standard fractal transformation in spatial domain, simple relations may be found relating coefficients in detail subbands in the wavelet domain. In this work we evaluate a hybrid wavelet-fractal image coder, and we test its ability to compress radiographic images of weld defects. A comparative study between the hybrid coder and standard fractal compression technique have been made in order to investigate the compression ratio and corresponding quality of the image using peak signal to noise ratio. Numerical experiments using radiographic images of weld defects illustrate the superior performance of the hybrid coder compared to standard fractal algorithm.

### KEYWORDS

Fractal compression, Discrete wavelet transform, Wavelet-Fractal coder, Radiographic images of weld defects, Compression ratio, Peak signal to noise ratio.

### 1 INTRODUCTION

Fractal coding is a lossy compression technique. The method consists of the representation of image blocks through the contractive transformation coefficients, using the self-similarity

concept. This type of compression provides a good scheme for image compression with fast decoding and competitive rate-distortion curves [1] [2], but it suffered from a large encoding time, difficulties to obtain high quality of decoded images and blocking artefacts at low bitrates. These drawbacks can be avoided if fractal coding is performed in the wavelet domain. Many works combined wavelets with fractal coding to improve a visual quality for compression at low bitrate [3][4][5]. Moreover, the hybrid wavelet-fractal coder can help to speedup the runtime of standard fractal compression algorithm, with its less computational complexity [6][7].

In this paper, hybrid and standard fractal algorithms have been evaluated by applying them on radiographic images of weld defects. Radiographic testing is one of the most common method of non-destructive testing (NDT) used to detect defects within the internal structure of welds [8]. The radiographic films are examined by interpreters, of which the task is to detect, recognize and quantify eventual defects and to accept or reject them by referring to the non destructive testing codes and standards. The detection of the defects in a radiogram is sometimes very difficult, because of the

bad quality of the films, the weld thickness, and the weak sizes of defects. In recent years there has been a marked advance in the research for the development of an automatic system to detect and classify weld defects by using digital image processing and pattern recognition tools [9]. Radiographic images like any other digital data require compression in order to reduce disk space needed for storage and time needed for transmission. The lossless image compression methods can reduce the file only to a very limited degree. The application of lossy compression techniques allow to obtain much higher compression ratios with a good quality of reconstructed images.

The organisation of the paper is as follow: an overview of a basic fractal coding scheme is given in section 2. Section 3 describes fractal coding in wavelet domain. Discussion and comparison of the results obtained with studied methods are given in section 4. Section 5 contains the conclusion.

## 2 FRACTAL IMAGE CODING ALGORITHM

Benoit Mandelbrot [10] coined the term fractal to describe a geometric figure, often characterized as being self similar, irregular, fractured and fragmented. Fractal structure is an infinite structure made up of similar forms and patterns that occurs in many different sizes. Mathematically, the self similarity in the images can be searched among determined regions in the image through finding the amount of rotation, scale of the region of interest compared to other regions.

Fractal image coding is based on the theory of iterated function systems (IFS)

and collage theorem [11]. Fractal block coder, as describe by Jacquin [12], assume that image redundancy can be efficiently exploited through self transformability on a block wise basis.

In standard fractal block coder, the image  $I$  is partitioned twice into a set of non overlapping  $B \times B$  range blocks  $R_i$ , and larger  $2B \times 2B$  domain blocks  $D_j$  which can overlap.

The task of fractal coder is to find a domain block  $D_j$  from the same image for every range blocks  $R_i$  such that a transformation of the domain block is a good approximation of the range block.

The contractive mapping from the domain block to range block is given by:

$$\hat{R}_i = W_i(D_j) = \alpha_i \Gamma_i(S_i(D_j)) + o_i \quad (1)$$

where  $S_i, \Gamma_i, \alpha_i$  and  $o_i$  represent the spatial contraction, isometric transformation, contrast scaling and luminance offset, respectively. Isometric transformation  $\Gamma_i$  includes identity, horizontal and vertical flip, diagonal reflection along the first axis and the second axis, and  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$  rotations.

The map  $W_i$  is chosen in order to minimize the distance between the range block  $R_i$  and its approximation  $\hat{R}_i$ . Typically, we want to minimize the MSE distance:

$$d(R_i, \hat{R}_i) = \frac{1}{B^2} \left[ \sum_{l=1}^B \sum_{m=1}^B (R_i(l, m) - \hat{R}_i(l, m))^2 \right] \quad (2)$$

Where  $l, m$  denote the pixel position of range block.

Fractal code of the range block  $R_i$  is composed of  $\Gamma_i, \alpha_i$  and position of the domain block  $D_j$ .

In fact, the goal of fractal encoding scheme is to define the image  $I$  as the fixed point of a transformation  $W: F \rightarrow F$

from a complete metric space  $F$  of images to it self. An approximation  $\hat{I}$  of the image  $I$  is obtained as the fixed point of the transformation  $W$ .

$$\hat{I} = \lim_{n \rightarrow \infty} W^{0n}(I_0), \quad \forall I_0 \in F. \quad (3)$$

The coding and the decoding procedures may be summarized as follows:

Encoding:

1. Get an image  $I$ .
2. Partition the image into sub-blocks  $R_i$ .
3. Partition the image into larger domain blocks called  $D_j$ .
4. For each range block  $R_i$ , find a domain block  $D_j$  and a transformation  $W_i$ .
5. Pack the fractal codes.

Decoding: (find the fixed point)

1. Start with any initial image  $I_0$ .
2. Apply  $W = \cup W_i$  iteratively until the result converges.

### 3 FRACTAL CODING IN THE WAVELET DOMAIN

#### 2.1 Discrete Wavelet Transform

Since the concept of multi-resolution analysis was put forward by Mallat [13], wavelet transform has always been a precedent in image processing fields. Wavelets are obtained from a single prototype wavelet called mother wavelet  $\psi(t)$  by dilation and shifting as follow:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (4)$$

Where  $a$  and  $b$  are the scaling and the shifting parameters respectively. These parameters allow the wavelet functions to represent signals in multiple levels of time-frequency resolutions.

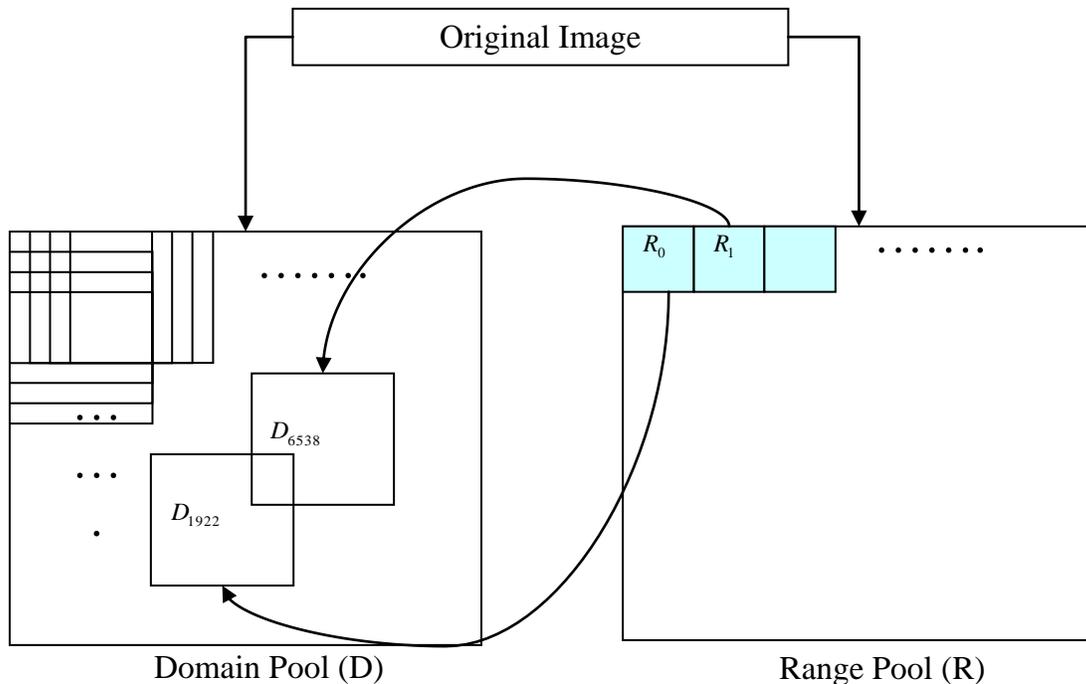


Figure 1. Contractive mapping in spatial domain.

The discrete wavelet transform (DWT) can be implemented by passing the signal through a combination of low pass and high pass filters and down sampling by a factor of two to obtain a single level of decomposition. Multiple levels of the wavelet transform are performed by repeating the filtering and down sampling operation on low pass branch outputs [14].

The 1-D wavelet transform can be extended to a two dimensional wavelet transform using separable filters. The 2-D transform is performed by applying a 1-D transform along the rows then along the columns. The 2-D wavelet decomposition of a signal is shown in figure 2.

In the figure,  $\tilde{H}(x)$  and  $\tilde{G}(x)$  represent the low pass and high pass filter responses, respectively. The filter outputs at level 2 are given by  $a$ ,  $d^H$ ,  $d^V$ ,  $d^D$ . The low pass filter output from the

previous level serves as the input for the next level of decomposition.

The discrete wavelet transform of an image provides a set of wavelet coefficients, which represent the image at multiple scales. The input image is decomposed into four subimages (or subbands) LH, HL, HH, LL, where the pair letters denotes the row-column filtering operations performed to obtain the subimage. For instance, subimage LH is obtained by low-pass filtering the rows and high-pass filtering the columns, followed by a factor two subsampling in each direction. This procedure can be iterated to obtain a multilevel decomposition of the image. Figure 3 demonstrates the decomposition process. The image is decomposed into different space (horizontal, vertical and diagonal directions) and different frequencies sub-images by multi-resolution analysis.

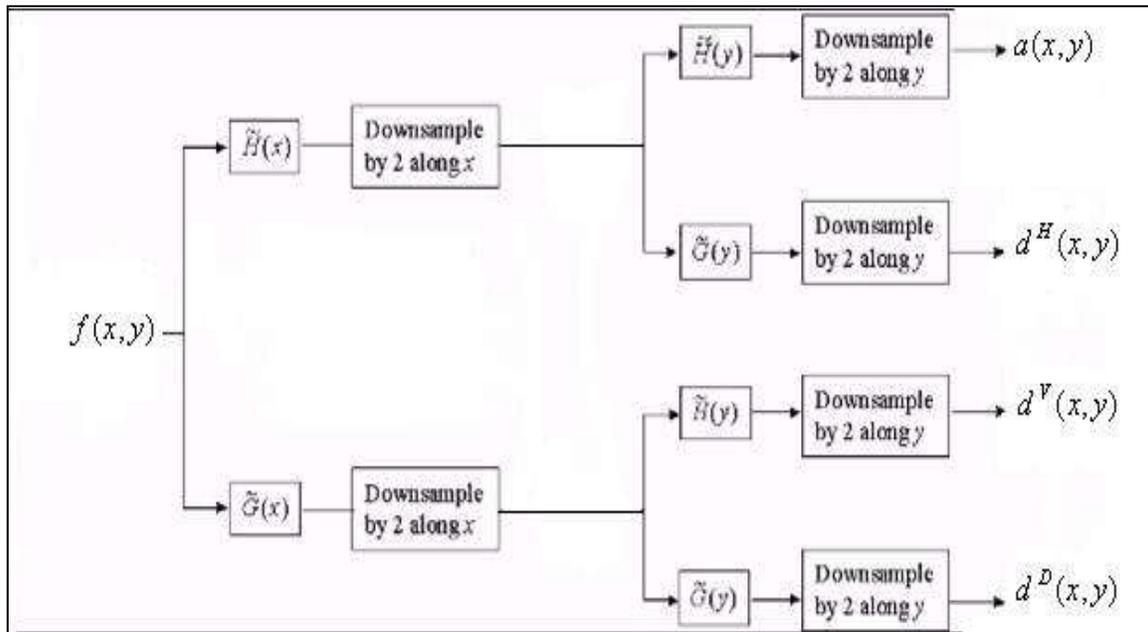


Figure 2. 2-D wavelet decomposition.

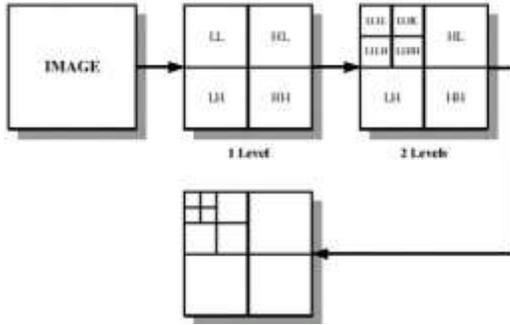


Figure 3. 2D DWT for image.

## 2.2 Hybrid Wavelet-Fractal Image Coder:

The motivation for wavelet-fractal image compression stems from the existence of self similarities in the multi-resolution wavelet representation. In fact, fractal image coding in the wavelet domain has quite different characteristics from the spatial domain coders and can be interpreted as the prediction of a set of wavelet coefficients in the higher frequency subbands from those in the lower ones.

Wavelet transform data can be organized into a subtree structure that can be efficiently coded. The oriented wavelet subtree is kind of structure with tree shape, it is composed by wavelet coefficients with different resolution, same direction and same relative space. The wavelet coefficients of image after wavelet transform can compose three kinds of oriented wavelet subtrees: the horizontal direction wavelet subtree which has low frequency in horizontal direction and high frequency in vertical direction, the vertical direction wavelet subtree which has high frequency in horizontal direction and low frequency in vertical direction; the diagonal direction wavelet subtree which has both high frequency in horizontal and vertical direction, shown in figure 4.

The contractive mapping operations carried out in the spatial domain have direct analogy in the wavelet domain. The averaging and subsampling operation  $S$  matches the size of the domain tree with that of the range tree. If we use Haar wavelet transform, the subsampling operation is equivalent to moving up the domain block tree by one scale in the wavelet domain, since the Haar transform is exactly the same as combined averaging and subsampling operations. The isometric transformation  $\Gamma$  is done within each subband. The contrast scaling factor  $\alpha$  is multiplied with each wavelet coefficient of domain tree to reach its correspondence in the range tree. Note that, in wavelet domain, an additive constant is not required as the wavelet tree does not have a constant offset.

Mathematically, the wavelet-fractal mapping can be written as [15][16]:

$$\hat{R} = W(D) = \alpha * \Gamma(S(D)) \quad (5)$$

where  $R$  is a range tree, and  $D$  is the domain tree.

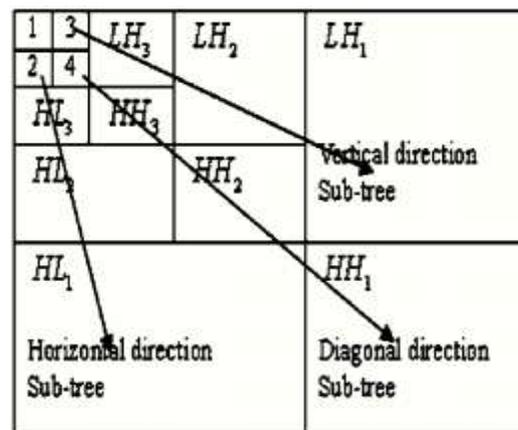


Figure 4. Wavelet subtree.

We consider  $x=(x_1,x_2,\dots,x_n)$  the ordered set of coefficients of a range tree and  $y=(y_1,y_2,\dots,y_n)$  the ordered set of coefficients of a subsampled domain tree. Then the mean squared error is:

$$MSE = \sum_{i=1}^n (x_i - \alpha * y_i)^2 \quad (6)$$

The optimal  $\alpha$  is then:

$$\alpha = \frac{\sum_{t=1}^n x_t * y_t}{\sum_{t=1}^n y_t^2} \quad (7)$$

The encoded parameters are the position of the domain tree, isometric operator  $\Gamma$  and the scaling factor  $\alpha$ .

The main steps of wavelet-fractal image coding algorithm are described as follow:

Encoding process:

- 1. Take an image as input.
- 2. Calculate the N-level DWT (Haar).
- 3. Partition the H, V, D components of the i th level into domain blocks of size  $2B \times 2B$ .
- 4. Partition the H, V, D components of the (i+1)th level into range blocks of size  $B \times B$ .
- 5. Find the best matching domain block tree for each range block tree
- 6. Save the mapping information.

Decoding process:

- 1. Get any initial image
- 2. DWT (Haar) of the image, for all N levels
- 3. Fractal decoding

- 4. Inverse DWT (Haar) for all N levels
- 5. Repeat the process for k iterations.

#### 4 EXPERIMENTAL RESULTS AND ANALYSIS

The two studied algorithms were tested on various radiographic images of weld defects. The Jacquin's fractal coding algorithm will be referred as FRAC, whereas the hybrid wavelet-fractal coder referred as WFC. The performance of the decoded images is evaluated by the PSNR value. The PSNR is calculated using the equation given below:

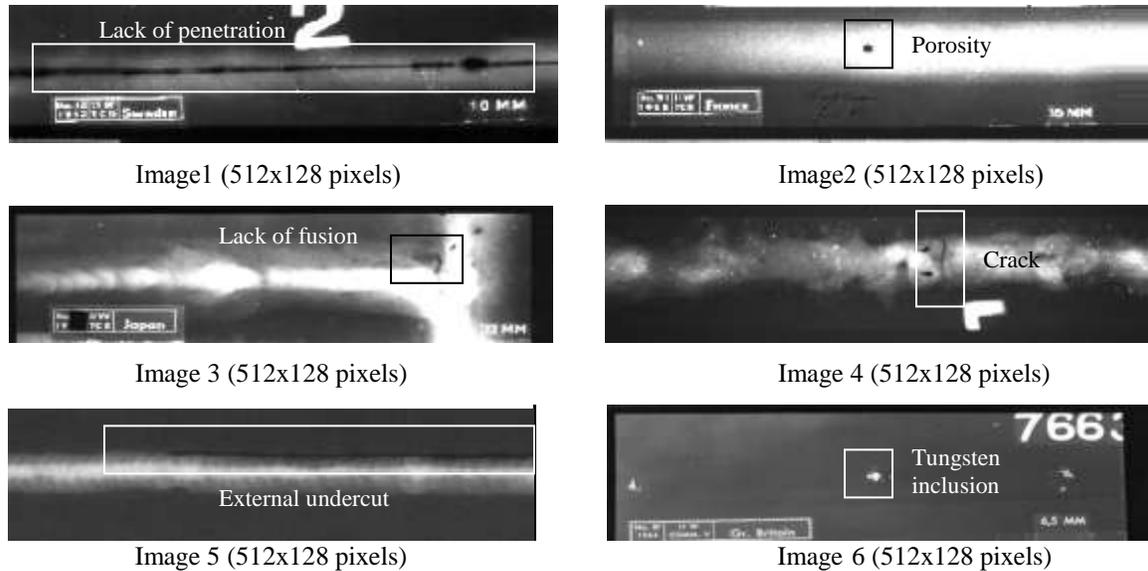
$$PSNR = 10 \log \frac{255^2}{\frac{1}{MN} \sum_{i=1}^M \sum_{j=1}^N (I(i, j) - I_0(i, j))^2}$$

where,  $I_0(i, j)$  is the original image and  $I(i, j)$  is the decompressed one,  $M$  and  $N$  are the number of columns and rows in the image. The amount of compression is calculated by compression ratio given by:

$$CR = 100 \left( 1 - \frac{bpp}{8} \right) \%$$

where  $CR$  is compression ratio and  $bpp$  is bits per pixel.

Simulation results were obtained by using six radiographic images representing several weld defects: Lack of penetration, Porosity, Lack of fusion, Crack, External undercut and Tungsten inclusion, shown in figure 5. Radiographic images used in the experiments are provided by international institute of welding (IIW).



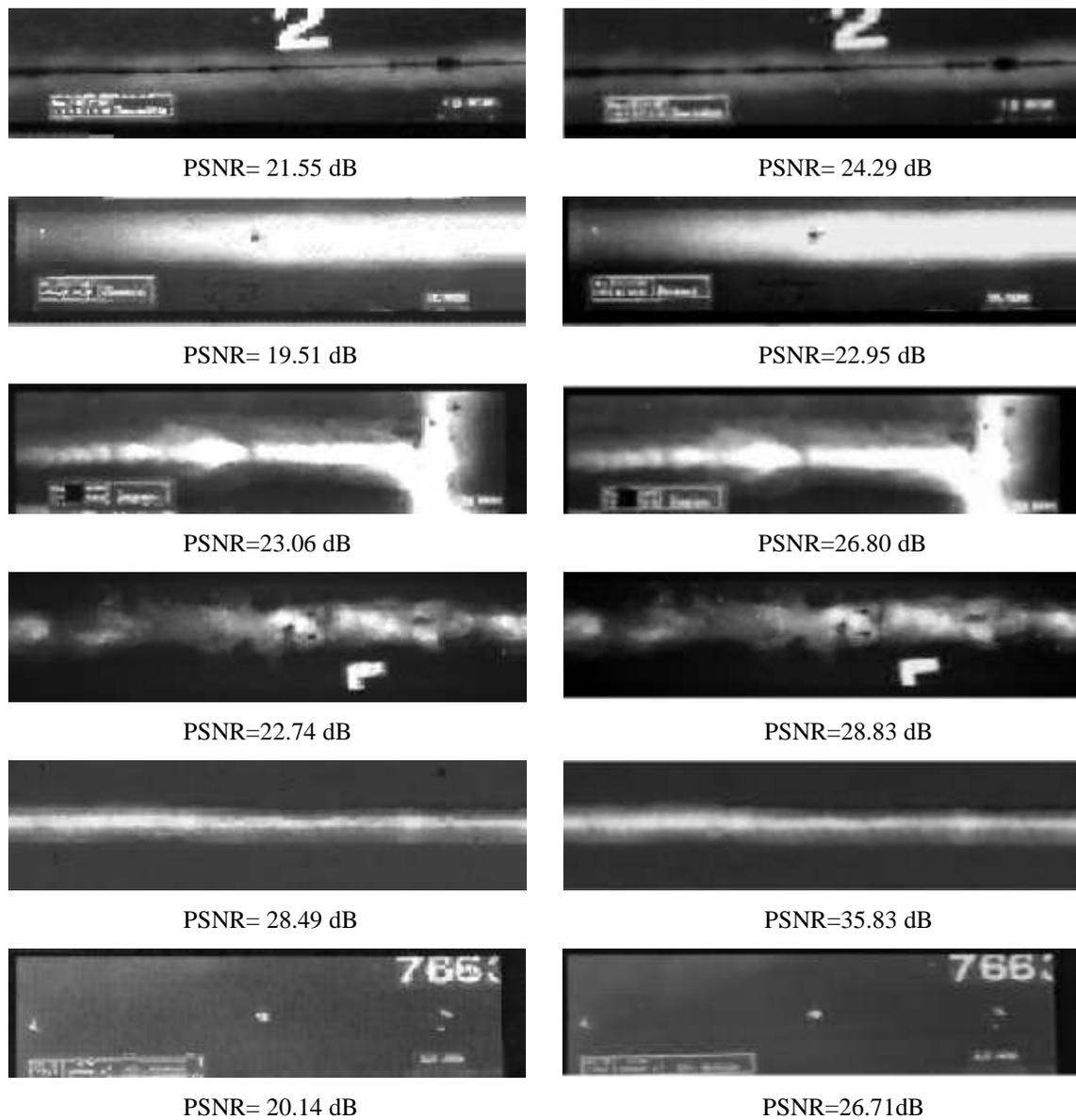
**Figure 5.** Original radiographic images.

The fractal image compression experiments were performed by keeping range size as eight. The domain pool consists of the blocks of the partitioned image with atomic block size  $16 \times 16$ . In wavelet-fractal image compression algorithm, first we decompose the image by 5-level Haar wavelet transform. Then, the block sizes of  $8 \times 8$ ,  $4 \times 4$ ,  $2 \times 2$  and  $1 \times 1$  were used from the high frequency subbands to low frequency subbands, and searched for the best pair with the same block size  $8 \times 8$ ,  $4 \times 4$ ,  $2 \times 2$  and  $1 \times 1$  within the downsampled images in the subbands with one level less. The pair matching is performed between the subbands of the 1, 2, 3, and 4 levels as domain pool and downsampled of subbands of 2, 3, 4, and 5 levels as range block, respectively. The calculation of scale factor is performed through equation (7). We note that isometric operation have not been considered here. Visual results are shown in figure 6 for the compression ratio 86%. Performance comparison for different images is shown in table 1. Figure 7 shows the

graphical representation comparisons of PSNR value vs. bitrate.

By examining the reconstructed images, a significant improvement of subjective quality is achieved by wavelet-fractal coder, avoiding blurring and causing no blocking effects for the overall images, which confirms PSNR objective measures obtained. In the case of images 1, 2, 3, and 6, we remark some spurious regions, but we can distinguish the defects (Lack of penetration, Porosity, Lack of fusion, and Tungsten inclusion respectively). For images 4 and 5, the defects (Crack and external undercut respectively) are put in obviousness by the WFC algorithm. However with Jacquín's algorithm we can not differentiate the defects from the welded joint.

Based on performance comparison shown in table 1 and the rate-distortion curve in figure 7, we can say that the hybrid wavelet-fractal coder significantly outperforms the Jacquín's algorithm.



**Figure 6.** Compression results (PSNR) at 86%. Left: by standard fractal coding, right: by wavelet-fractal coder.

**Table 1.** Performance comparison for different images.

Bite rate (bpp)	PSNR (dB)											
	Image 1		Image 2		Image 3		Image 4		Image 5		Image 6	
	FRAC	WFC	FRAC	WFC	FRAC	WFC	FRAC	WFC	FRAC	WFC	FRAC	WFC
0.96	20.13	21.70	9.45	17.35	19.61	19.88	21.30	21.45	27.89	34.19	19.52	21.70
1.12	21.55	24.29	19.51	22.95	23.06	26.80	22.74	28.83	28.49	35.82	20.14	26.71

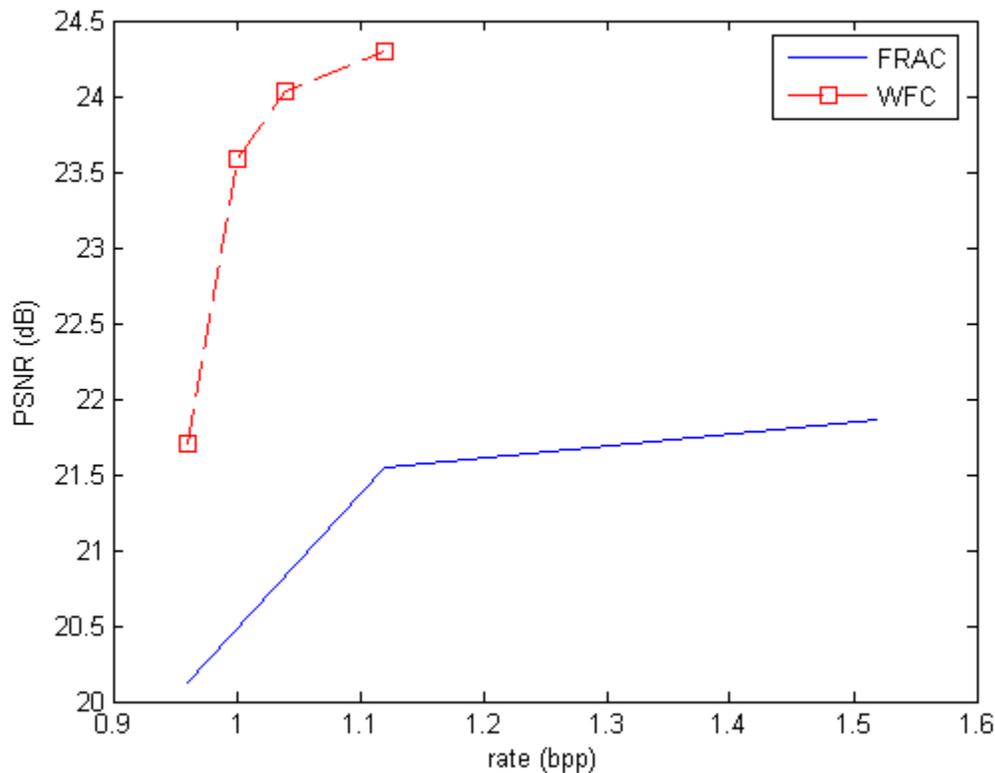


Figure 7. PSNR vs. bitrate.

## 5 CONCLUSION

In this paper we have evaluated a hybrid wavelet-fractal coder on radiographic images of weld defect. The wavelet-fractal coder has been compared to the standard fractal algorithm. Simulation results demonstrate a gain in PSNR objective measure with good compression ratio percentage. We can conclude that the hybrid coder can be utilized for radiographic images compression, but the algorithm requires some improvements to provide competitive PSNR values, which is one of our future research focuses.

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