International Journal of Digital Information and Wireless Communications (IJDIWC) 3(3): 206-214
The Society of Digital Information and Wireless Communications, 2013 (ISSN: 2225-658X)

Elliptic Jes Window Form 2 in Signal Processing

Claude Ziad Bayeh[1, 2]
[1]Faculty of Engineering II, Lebanese University
[2]EGRDI transaction on mathematics (2012)
Lebanon
Email: claude_bayeh_cbegrdi@hotmail.com

ABSTRACT

The *Elliptic Jes window form 2* is an original study introduced by the author in Mathematics and in Signal Processing in 2012. Similar to other windows used in signal processing such as: Hamming, Hanning, Blackman, Kaiser, Lanczos, Tukey and many other windows, the main goal of introducing the *Elliptic Jes window form 1*is to improve the convergence of the Fourier Series at the discontinuity. The different points between the proposed window function and the previous ones are: -The proposed window function is variable in form; it can take more than 6 different forms by varying only one parameter.-It can help the Fourier series to converge more rapidly compared to the traditional ones. –It can be used in both analog design of filters and digital design of filters. –It is used to truncate the Fourier series with a variable window shape that keep the necessary information about the signal even after truncation.

In fact, the *Elliptic Jes window form 2* is an application of the Elliptic Trigonometry in Signal Processing. The Elliptical Trigonometry is an original study introduced also by the author in mathematics in 2004, and it has an ultimate importance in all fields related to the Trigonometry topics such as Mathematics, Electrical engineering, Electronics, Signal Processing, Image Processing, Relativity, Physics, Chemistry, and many other domains. The Elliptical Trigonometry is the general case of the traditional trigonometry in which an Ellipse is used instead of a Circle, so the Elliptical Trigonometry functions are much more important compared to the traditional trigonometry functions. Therefore, all topics related to the traditional trigonometry will be ultimately improved by using the Elliptical Trigonometry functions including Signal Processing and Specifically the design of windows and filters. As a consequence, the *Elliptic Jes window form 2* will replace all traditional window functions.

KEYWORDS

Window functions, Signal processing, Mathematics, Elliptical trigonometry, Trigonometry, Fourier series, Truncated series.

1 INTRODUCTION

In mathematics and in signal processing, a window function (also known as an apodization function or tapering function) is a mathematical function that is zero-valued outside of some chosen interval [1-3]. For instance, a function that is constant inside the interval and zero elsewhere is called a *rectangular window*, which describes the shape of its graphical representation [6-15]. When another function or a signal (data) is multiplied by a window function, the product is also zero-valued outside the interval: all that is left is the part where they overlap; the "view through the window". Applications of window functions include spectral analysis, filter design, and beamforming [4-5], [28] and [33].

A more general definition of window functions does not require them to be identically zero outside an interval, as long as the product of the window multiplied by its argument is square integrable, that is, that the function goes sufficiently rapidly toward zero.

In typical applications, the window functions used are non-negative smooth "bell-shaped" curves, though rectangle, triangle, and other functions are sometimes used. Briefly, a modification of Fourier coefficients using window functions improves the convergence of the series at the discontinuity.

In this paper, the author introduced a new window function using an Elliptical Trigonometric function such as *Elliptic Jes* function [16-17]. This new trigonometry is also introduced by the author and it can be considered as the basis of the new generation of Signal Processing, Electronics and Electrical systems based on variable signals [17]. The new window function based on the Elliptical Trigonometry has huge advantages over the traditional window functions based on the traditional trigonometry. This will be discussed in this paper.

2 BRIEF INTRODUCTION TO THE "ELLIPTIC JES" FUNCTION

The *Elliptic Jes* function is a function of the Elliptical Trigonometry which is defined in the papers [16-17]. If we compare this function to the Cosine function of the traditional trigonometry we find that the *Elliptic Jes* function is the general case of the cosine function and it is defined as following:

$$Ejes_b(x) = \frac{ang_x(x)}{\sqrt{1+\left(\frac{a}{b}\mathrm{Cter}(x)\right)^2}} \qquad (1)$$

-With $ang_x(x)$ is the angular function related to the (ox) axis is defined [35], for $K \in \mathbb{Z}$, as:

$$ang_x(\beta(x+\gamma)) = \begin{cases} +1 \;\; for \;\; (4K-1)\frac{\pi}{2\beta} - \gamma \le x \le (4K+1)\frac{\pi}{2\beta} - \gamma \\ -1 \;\; for \;\; (4K+1)\frac{\pi}{2\beta} - \gamma < x < (4K+3)\frac{\pi}{2\beta} - \gamma \end{cases} \qquad (2)$$

With:

β is the frequency of the function

γ is the translation of the function on the axis (ox).

x is the a variable parameter $x \in]-\infty; +\infty[$

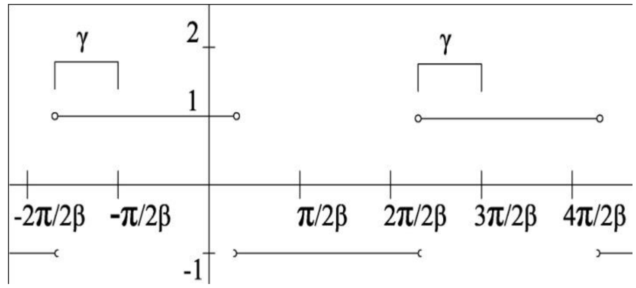


Figure 1, The $ang_x(\beta(x+\gamma))$ waveform.

In fact:

$$ang_x(x) = \begin{cases} +1 \;\; for \;\; (4K-1)\frac{\pi}{2} \le x \le (4K+1)\frac{\pi}{2} \\ -1 \;\; for \;\; (4K+1)\frac{\pi}{2} < x < (4K+3)\frac{\pi}{2} \end{cases}$$

For x going from -∞ to +∞ the sign of the function changes into two values +1 and -1 only,

For $x = (2K+1)\frac{\pi}{2}$, it changes from:

$$\begin{cases} -1 \; to +1 \;\; for \; x = -\frac{\pi}{2} + 2K\pi = (4K-1)\frac{\pi}{2} \\ +1 \; to -1 \;\; for \; x = +\frac{\pi}{2} + 2K\pi = (4K+1)\frac{\pi}{2} \end{cases}$$

• Particular case: for $\beta = 1$ and $\gamma = 0$, the expression (2) becomes:

$$ang_x(x) = \begin{cases} +1 \;\; for \;\; \cos(x) \ge 0 \\ -1 \;\; for \;\; \cos(x) < 0 \end{cases} \qquad (3)$$

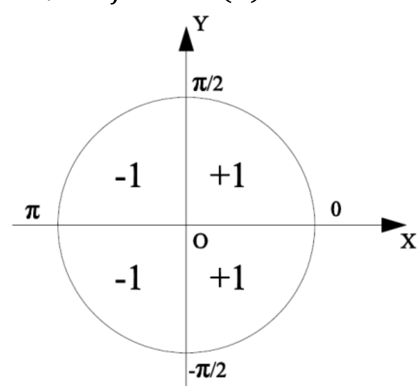
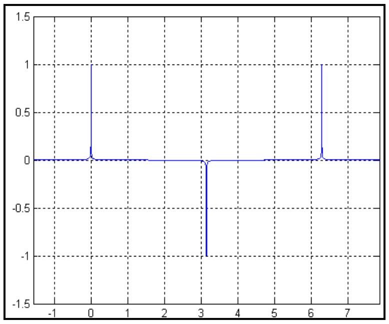


Figure 2, The $ang_x(x)$ on the unit circle.

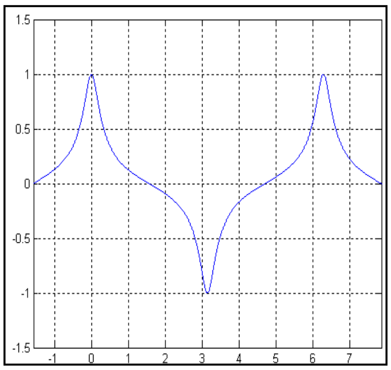
-And $\mathrm{Cter}(x) = \tan(x)$ is the *Circular Trigonometric Ter* function which is equivalent to the tangent of the traditional trigonometry.

• Multi form signals made by $Ejes_b(x)$:

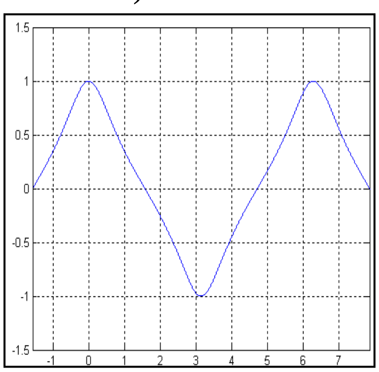
Figures 3.a to 3.f represent multi form signals obtained by varying one parameter (b).



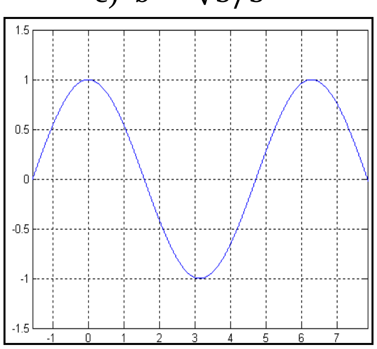
a) $b = 0.001$



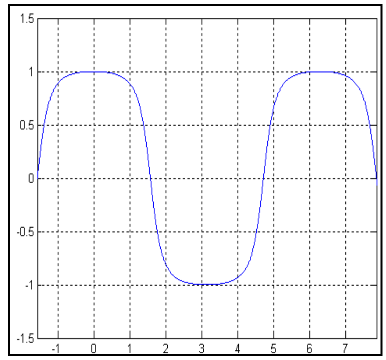
b) $b = 0.2$



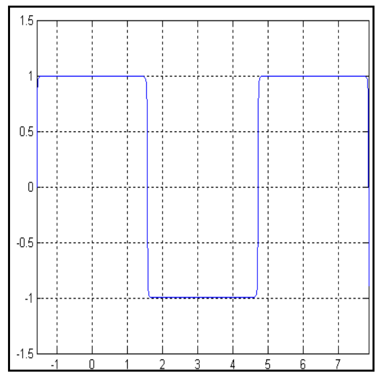
c) $b = \sqrt{3}/3$



d) $b = 1$



e) $b = 3$



f) $b = 90$

Figure 3, multi form signals of the function $Ejes_b(x)$ and for different values of $b > 0$.

Important signals obtained using this function:
Impulse train with positive and negative part, elliptic deflated, quasi-triangular, sinusoidal, elliptical swollen, square signal, rectangular signal…
These types of signals are widely used in power electronics, electrical generator, signal processing and in transmission of analog signals.

3 "ELLIPTIC JES" WINDOW FORM 2 FUNCTION

The *Elliptic Jes window form 2* function is the application of the *Elliptic Jes* function in signal processing. It takes the following forms:

$$w = \frac{1}{2}\left(1 - Ejes_b\left(\frac{2\pi n}{M-1}\right)\right) =$$

$$\frac{1}{2}\left(1 - \frac{ang_x\left(\frac{2\pi n}{M-1}\right)}{\sqrt{1+\left(\frac{a}{b}\mathrm{Cter}\left(\frac{2\pi n}{M-1}\right)\right)^2}}\right) \qquad (4)$$

With $0 \leq n \leq M-1$ and $M \in \mathbb{N}$

And

$$w_k = \frac{1}{2}\left(1 + Ejes_b\left(\frac{k\pi}{n}\right)\right) \quad (5)$$

With $-n \leq k \leq n$ and $n \in \mathbb{N}$

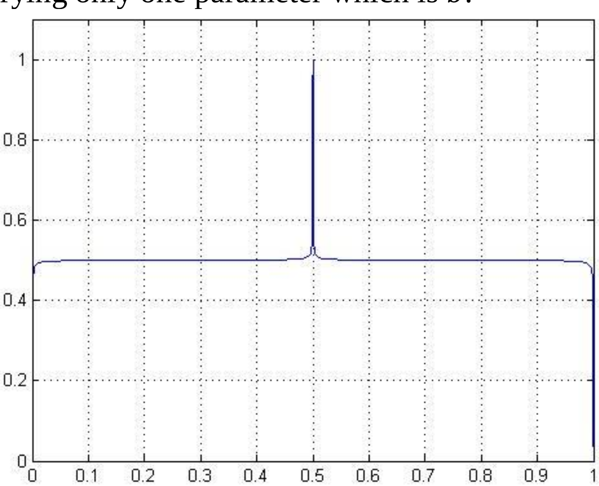
So the truncated Fourier series using the *Elliptic Jes window form 2*takes the following form:

$$S_n(\theta) = \sum_{k=-n}^{n} w_k c_k e^{jk\theta}$$

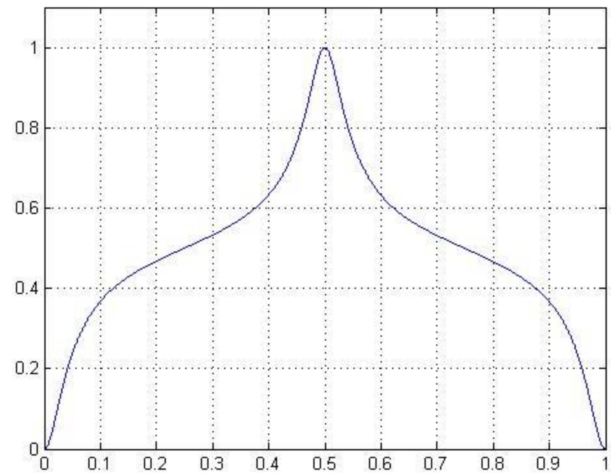
$$= \sum_{k=-n}^{n} \frac{1}{2}\left(1 + Ejes_b\left(\frac{k\pi}{n}\right)\right) c_k e^{jk\theta} \quad (6)$$

3.1 Variable shapes of window formed by Elliptic Jes window form 2

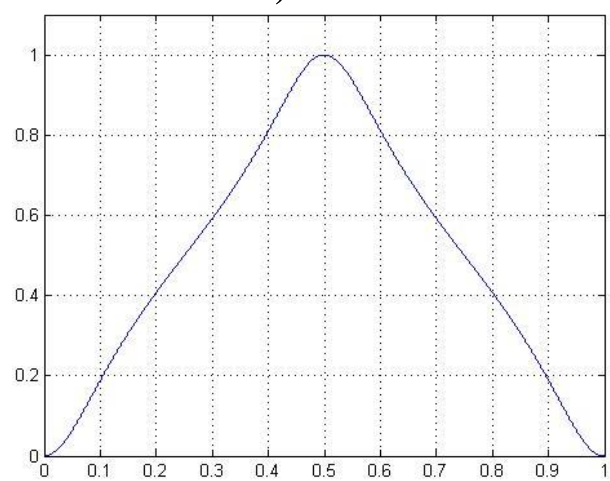
The formed shapes of this function can be drawn using MATLAB. In the figures 4.a to 4.f, different shapes of the window function are formed by varying only one parameter which is b.



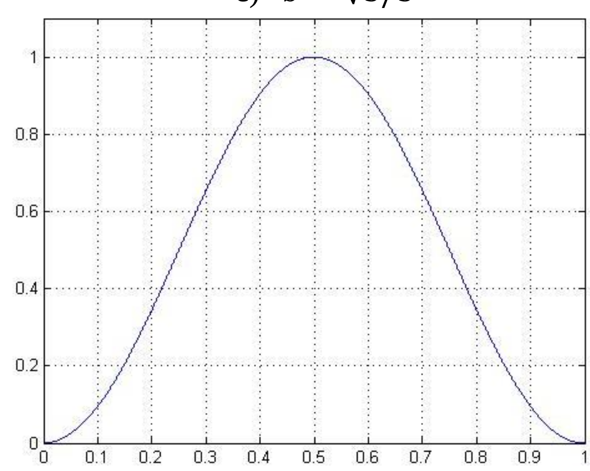
a) $b = 0.001$



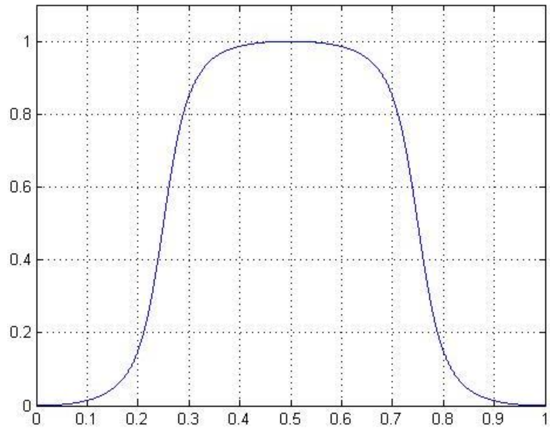
b) $b = 0.2$



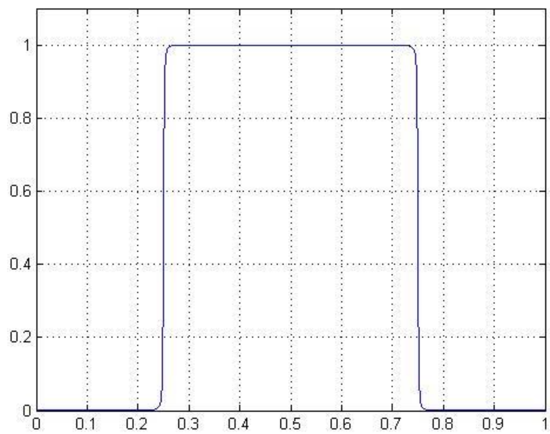
c) $b = \sqrt{3}/3$



d) $b = 1$



e) $b = 3$



f) $b = 90$

Figure 4, multi form signals of the function *Elliptic Jes window form 2* and for different values of $b > 0$.

In fact, this window is very important as it has variable amplitude that can be changed as we wish over a period or a half period. Applications of window functions include spectral analysis, filter design, beamforming and telecommunications. A more general definition of window functions does not require them to be identically zero outside an interval, as long as the product of the window multiplied by its argument is square integrable, that is, that the function goes sufficiently rapidly toward zero.

3.2 Programming the function Elliptic Jes window form 2 using MATLAB

```
%--------------------------------
%Elliptic Jes Window form 2
%Introduced by Claude Ziad Bayeh in
2012-06-21
clc
close all
M=2;
a=1; x=0:0.0001:M-1;
fprintf('---Elliptic Jes Window form 2
Introduced by Claude Ziad Bayeh in 2012-
06-21---\n');
fprintf('-------------------\n');
repeat='y';
while repeat=='y'
    b=input('determine the form of the
Elliptic trigonometry: b=');
    fprintf('b is a variable can be
changed to obtain different signals
\n');
    %b is the intersection of the
Ellipse and the axe y'oy in the positive
part.
    if b<0,
       b
       error('ATTENTION: ERROR b must be
greater than Zero');
    end;

Ejes=(1./(sqrt(1.+((a/b).*tan(x)).^2))).
*angx(x); % the Elliptic Jes "Ejes"

Emar=(1./(sqrt(1.+((a/b).*tan(x)).^2))).
*angx(x).*tan(x).*a/b; % the Elliptic
Mar "Emar"

    % Elliptic Jes Window form 2
    EjesW2=1./2.*(1-
((1./(sqrt(1.+((a/b).*tan(2.*pi.*x)).^2)
)).*angx(2.*pi.*x)));

    plot(x,EjesW2);
    axis([0 M-1 0 1.1]);
    grid on;
    fprintf('Do you want to repeat
?\nPress y for ''Yes'' or any key for
''No''\n');
    repeat=input('Y/N=','s');
    clc
    close all
end; %End while
%--------------------------------
```

3.3 Advantages of the function Elliptic Jes window form 2 over the traditional window functions

Similar to other windows used in signal processing such as: Hamming, Hanning, Blackman, Kaiser, Lanczos, Tukey and many other windows, the main goal of introducing the *Elliptic Jes window form 2* is to improve the convergence of the Fourier Series at the discontinuity.

The advantages of the new window function over the traditional windows are:

-The proposed window function is variable in form; it can take more than 6 different forms by varying only one parameter.

-It can help the Fourier series to converge more rapidly compared to the traditional ones.

–It can be used in both analog design of filters and digital design of filters.

–It is used to truncate the Fourier series with a variable window shape that keep the necessary information about the signal even after truncation.

4 EXISTING WINDOW FUNCTIONS

There are many window functions used to converge a Fourier series at the limit [1-2] and [39]. Such as:

4.1 Rectangular window

The rectangular window (sometimes known as the boxcar or Dirichlet window) is the simplest window, equivalent to replacing all but N values of a data sequence by zeros, making it appear as though the waveform suddenly turns on and off:

$$w(n) = \begin{cases} 1 \; for \; 0 \le n \le N-1 \\ 0 \qquad elsewhere \end{cases} \qquad (7)$$

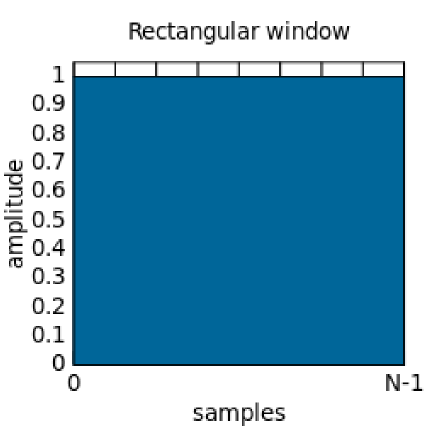


Figure 5, Rectangular window function (from Wikipedia).

Other windows are designed to moderate these sudden changes because discontinuities have undesirable effects on the discrete-time Fourier transform (DTFT) and/or the algorithms that produce samples of the DTFT.

4.2 Triangular window

The triangular window is defined by:

$$w(n) = 1 - \left| \frac{n - \frac{N-1}{2}}{\frac{N+1}{2}} \right| \qquad (8)$$

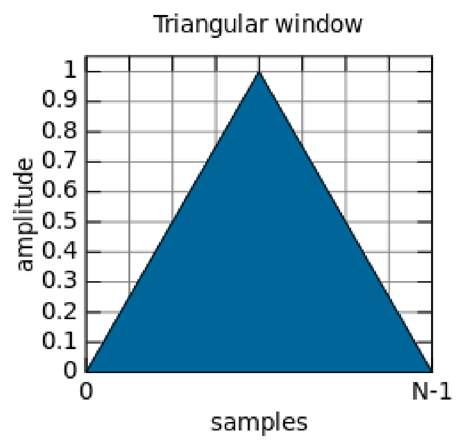


Figure 6, Triangular window function (from Wikipedia).

The end samples are positive (equal to $2/(N + 1)$). This window can be seen as the convolution of two half-sized rectangular windows (for N even), giving it a main lobe width of twice the width of a regular rectangular window. The nearest lobe is −26 dB down from the main lobe.

4.3 Welch window

The Welch window consists of a single parabolic section:

$$w(n) = 1 - \left(\frac{n - \frac{N-1}{2}}{\frac{N+1}{2}} \right)^2 \qquad (9)$$

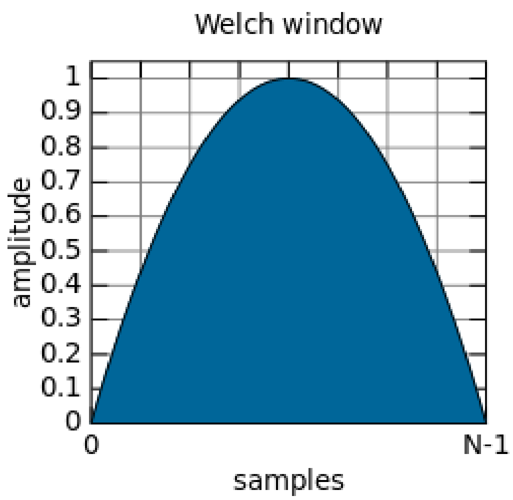


Figure 7, Welch window function (from Wikipedia).

The defining quadratic polynomial reaches a value of zero at the samples just outside the span of the window.

4.4 Hann (Hanning) window

The Hann window also known as the Hanning is defined by:

$$w(n) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{N-1}\right)\right) \qquad (10)$$

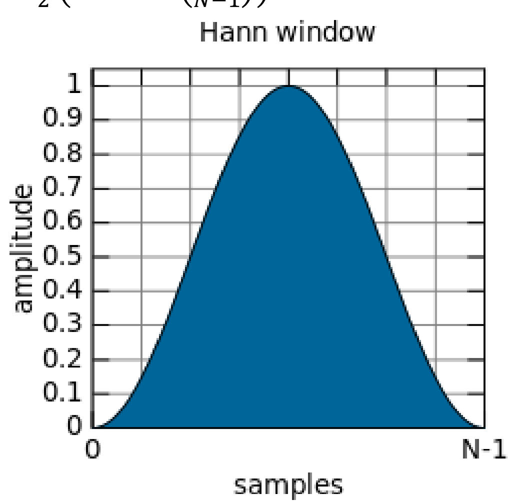


Figure 8, Hann window function (from Wikipedia).

4.5 Tukey window

The Tukey window, also known as the *tapered cosine window*, can be regarded as a cosine lobe of width $\alpha N/2$ that is convolved with a rectangular window of width $(1 - \alpha/2)N$

$$w(n) = \begin{cases} \frac{1}{2}\left(1 + \cos\left(\pi\left(\frac{2n}{\alpha(n-1)} - 1\right)\right)\right) & for\ 0 \le n \le \frac{\alpha(N-1)}{2} \\ 1 & for\ \frac{\alpha(N-1)}{2} \le (N-1)\left(1 - \frac{\alpha}{2}\right) \\ \frac{1}{2}\left(1 + \cos\left(\pi\left(\frac{2n}{\alpha(n-1)} - \frac{2}{\alpha} + 1\right)\right)\right) & for\ (N-1)\left(1 - \frac{\alpha}{2}\right) \le n \le (N-1) \end{cases} \qquad (11)$$

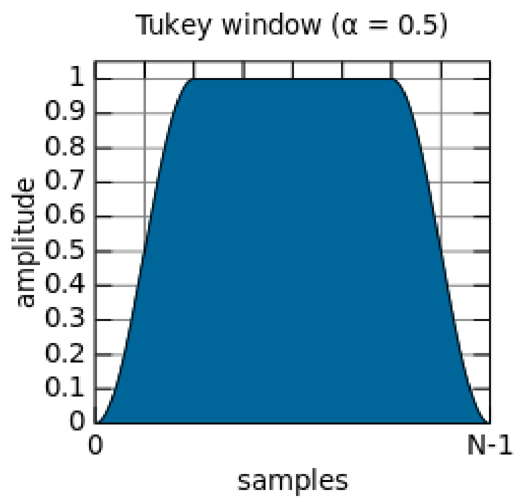


Figure 9, Tukey window function (from Wikipedia).

And so on…

The main purpose of developing these windows is to obtain the smoother form that helps the attenuation of the desired signal in the extremity of the window function at the same time obtaining the minimum amplitude of the side-lobes and maximum width of the main lobe (refer to figure 10). This is not possible with the existing window functions. So there is a compromise to do.

The disadvantage of these window functions is that their frequency response doesn't converge to zero outside the interval, and moreover, their amplitudes are not negligible.

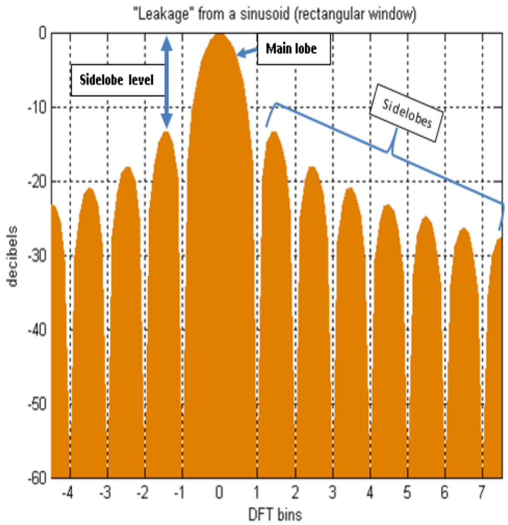


Figure 10, window function in the frequency domain (from Wikipedia).

This problem is resolved with the proposed window function by the author based on the Elliptical Trigonometry in which we can regulate the shape of the window to obtain a very smooth form within the window function and obtain at the same time a wider main lobe and very small side lobes in magnitude.

5 CONCLUSION

In this paper, the author introduced a new window function based on the Elliptical Trigonometry. This new window function has many advantages as cited in the previous sections. The main goal of introducing this new window function is to improve the convergence of the Fourier Series at the discontinuity. We have seen a brief introduction about the Elliptical Trigonometry in this paper, for additional information about the Elliptical Trigonometry and the Angular function please refer to the published papers [16-17] and [35-38].

The new window function has enormous applications in mathematics and in signal processing and precisely in the design of analog and digital filters.

6 REFERENCES

1. H. Baher, "Signal processing and integrated circuits", *Published by John Wiley & Sons Ltd.*, ISBN: 9780470710265, (2012).
2. J. G. Proakis, Dimitris G. Manolakis, "Digital Signal Processing, Principles, Algorithms, and Applications" Fourth edition, *Pearson International Edition*, ISBN: 0-13-228731-5.
3. N. Wirth, "Digital Circuit Design", *Springer*, ISBN: 3-540-58577-X.
4. N. Senthil Kumar, M. Saravanan, S. Jeevananthan, "Microprocessors and Microcontrollers", *Oxford University Press*, ISBN-13: 978-0-19-806647-7.
5. G. Wade, "Signal Coding and Processing" Second Edition, *Cambridge University Press*, ISBN: 0-521-42336-8.
6. S. J. Orfanidis, "Introduction to Signal Processing", *Rutgers University*, ISBN 0-13-209172-0, 2010.
7. B. Gold and C. M. Rader, "Digital Processing of Signals", *McGraw-Hill*, New York, (1969).
8. A. V. Oppenheim and R. W. Schafer, "Discrete-Time Signal Processing", *Prentice Hall*, Englewood Cliffs, NJ, (1989).
9. A. V. Oppenheim and R. W. Schafer, "Digital Signal Processing", *Prentice Hall*, Englewood Cliffs, NJ, (1975).
10. L. R. Rabiner and B. Gold, "Theory and Application of Digital Signal Processing", *Prentice Hall*, Englewood Cliffs, NJ, (1975).
11. S. K. Mitra and J. F. Kaiser, eds., "Handbook of Digital Signal Processing", *Wiley*, New York, (1993).
12. T. W. Parks and C. S. Burrus, "Digital Filter Design", *Wiley*, New York, (1987).
13. A. Antoniou, "Digital Filters: Analysis and Design", 2nd ed., *McGraw-Hill*, New York, (1993).
14. D. F. Elliott, "Handbook of Digital Signal Processing", *Academic Press*, New York, (1987).
15. L. R. Rabiner and C. M. Rader, eds., "Digital Signal Processing", *IEEE Press*, New York, (1972).
16. C. Bayeh, M. Bernard, N. Moubayed, Introduction to the elliptical trigonometry, *WSEAS Transactions on Mathematics*, Issue 9, Volume 8, (September 2009), pp. 551-560.
17. N. Moubayed, C. Bayeh, M. Bernard, A survey on modeling and simulation of a signal source with controlled waveforms for industrial electronic applications, *WSEAS Transactions on Circuits and Systems*, Issue 11, Volume 8, (November 2009), pp. 843-852.
18. M. Christopher, *From Eudoxus to Einstein: A History of Mathematical Astronomy*, Cambridge University Press, (2004).
19. E. W. Weisstein, *Trigonometric Addition Formulas*, Wolfram MathWorld, (1999-2009).
20. P. A. Foerster, *Algebra and Trigonometry: Functions and Applications*, Addison-Wesley publishing company, (1998).

21. F. Ayres, *Trigonométrie cours et problèmes*, McGraw-Hill, (1991).
22. R. C. Fisher and Allen D.Ziebur, *Integrated Algebra and Trigonometry with Analytic Geometry*, Pearson Education Canada, (2006).
23. E. Demiralp, Applications of High Dimensional Model Representations to Computer Vision, *WSEAS Transactions on Mathematics*, Issue 4, Volume 8, (April 2009).
24. A. I. Grebennikov, Fast algorithm for solution of Dirichlet problem for Laplace equation, *WSEAS Transactions on Computers Journal*, 2(4), pp. 1039 – 1043, (2003).
25. I. Mitran, F.D. Popescu, M.S. Nan, S.S. Soba, Possibilities for Increasing the Use of Machineries Using Computer Assisted Statistical Methods, *WSEAS Transactions on Mathematics*, Issue 2, Volume 8, (February 2009).
26. Q. Liu, Some Preconditioning Techniques for Linear Systems, *WSEAS Transactions on Mathematics*, Issue 9, Volume 7, (September 2008).
27. A. I. Grebennikov, The study of the approximation quality of GR-method for solution of the Dirichlet problem for Laplace equation. *WSEAS Transactions on Mathematics Journal*, 2(4), pp. 312-317, (2003).
28. R. Bracewell, *Heaviside's Unit Step Function. The Fourrier Transform and its Applications*, 3rd edition, New York: McGraw-Hill, pp. 61-65, (2000).
29. M. Abramowitz and Irene A. Stegun, eds, *Handbook of mathematical functions with formulas, graphs and mathematical tables*, 9th printing, New York: Dover, (1972).
30. V. Kantabutra, On hardware for computing exponential and trigonometric functions, *IEEE Transactions on Computers*, Vol. 45, issue 3, pp. 328–339, (1996).
31. H. P. Thielman, A generalization of trigonometry, *National mathematics magazine*, Vol. 11, No. 8, (1937), pp. 349-351.
32. N. J. Wildberger, *Divine proportions: Rational Trigonometry to Universal Geometry*, Wild Egg, Sydney, (2005).
33. C. W. Lander, *Power electronics*, third edition, McGraw-Hill Education, (1993).
34. Claude Bayeh, “Introduction to the Rectangular Trigonometry in Euclidian 2D-Space”, WSEAS Transactions on Mathematics, ISSN: 1109-2769, Issue 3, Volume 10, (March 2011), pp. 105-114.
35. C. Z. Bayeh, “Introduction to the Angular Functions in Euclidian 2D-space”, *WSEAS TRANSACTIONS on MATHEMATICS*, ISSN: 1109-2769, E-ISSN: 2224-2880, Issue 2, Volume 11, (February 2012), pp.146-157.
36. C. Z. Bayeh, “Introduction to the General Trigonometry in Euclidian 2D-Space”, *WSEAS TRANSACTIONS on MATHEMATICS*, ISSN: 1109-2769, E-ISSN: 2224-2880, Issue 2, Volume 11, (February 2012), pp.158-172.
37. C. Bayeh, “Application of the Elliptical Trigonometry in industrial electronic systems with analyzing, modeling and simulating two functions Elliptic Mar and Elliptic Jes-x”, WSEAS Transactions on Circuits and Systems, ISSN: 1109-2734, Issue 11, Volume 8, (November 2009), pp. 843-852.
38. C. Bayeh, “A survey on the application of the Elliptical Trigonometry in industrial electronic systems using controlled waveforms with modeling and simulating of two functions *Elliptic Mar* and *Elliptic Jes-x”,* in the book “ Latest Trends on Circuits, Systems and Signals”, publisher WSEAS Press, ISBN: 978-960-474-208-0, ISSN: 1792-4324, (July 2010), pp.96-108.
39. Wikipedia, “Window function”.