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Utilization of the Fast Discrete Curvelet Transform in the OFDM System

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ABSTRACT

With the high demand for high data rates, there is also a need to develop more efficient wireless communication systems so Orthogonal frequency-division multiplexing (OFDM) was one of the options available to provide the required demand. The work done in this paper can be considered an effort in this direction. As it known, the inverse fast Fourier transform (IFFT) and the fast Fourier transform (FFT) are utilized in conventional OFDM systems for data modulation and demodulation respectively, but in this work a new OFDM system based on discrete fast curvelet conversion (FDCT)) Has been proposed.
As an alternative to FFT in the proposed model, the FDCT via Unequispaced fast Fourier transform (USFFT) has been used as well as the FDCT via Wrapping. In the 10 – tap Rayleigh channel, the results showed that the curvelet based OFDM system has higher spectral efficiency and low out – of – band (low – side loop) energy compared to the conventional OFDM system. With regard to spectral efficiency and power spectral density (PSD) the results showed that each of the FDCT conversions used in this work yields better results than conventional OFDM. On the other hand, and in the terms of the computational complexity, the given system gives more the computational complexity compared to conventional OFDM system. As a final point, and in the terms of the Bit Error Rate (BER) performance metric parameter using Binary Phase Shift Keying (BPSK), it can be deduced that the given system is outperformed the conventional OFDM and it performs almost same as the theoretical BER performance.

KEYWORDS

OFDM, BER, PSD, FFT, DCT, WPT, DTCWT, Curvelet Transforms.

1 INTRODUCTION

The idea begin by using synchronous orthogonal subcarriers in order to obtain high spectrum efficiency with minimum noise and interference [1] pulse waveforms, these theoretical implementation of OFDM system started using discrete Fourier transform (DFT) as part of the modulation and demodulation process [2], while the practical application of this system was using FFT where there are many papers dealing with OFDM system based on FFT and presented many treatments for many associated problems such as PAPR and others [2-5].
On the other hand, there are many works that have suggested instead of the FFT the discrete cosine transform (DCT) can be used in the OFDM system. As the case of the DFT, the DCT fulfilled the cyclic convolution and moreover, the OFDM system based on DCT is outperform the conventional OFDM system under certain condition [6-7].
Other papers highlighted Wavelet transform (WT) as an alternative to the FFT in the OFDM system. The first group of these papers proposed instead of FFT the use of discrete Wavelet transform (DWT) in the OFDM system. The positive sign of is that it is better spectral containment in perfect reconstruction filter banks than in the DFT [8-14], the second group provided the Wavelet packet transform (WPT) as alternative transform. In addition to supporting the multi-rate services, the proposed system is more robust than the conventional system, in general Carrying multi-rate services and limiting non-stationary noise are the two new features of proposed system [15-20]. The third group used Complex version of WT i.e. complex wavelet transform (CWT) and complex wavelet packet transform (CWPT). Because of the spectrally contained nature of the CWT and CWPT, the new system gives better results than the conventional system in term of average bit error probability [21-22].
The last fourth group introduced the dual tree complex wavelet transform (DTCWT) [23-25] as an alternative to the use of FFT in the OFDM

system. In term of the PAPR, BER and PSD the new system gives better results compared to conventional OFDM system. These improvements are attributed to two distinct properties of the DTCWT filters – the unique impulse response and shift-invariance of the filters [26-33]. Figure 1 shows the OFDM system model using different types of the previous proposed transform.

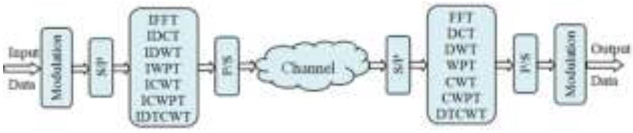

Figure 1. OFDM system for the various types of previous proposed transform.

Recently there are works that introduced the Curvelet transform [34-38] as a new alternative instead of previous transfers and investigate the PAPR in the proposed new system using both fast discrete Curvelet transform (FDCT) via Unequispaced fast Fourier transform (USFFT) and FDCT via Wrapping [39]. This work is an extension of the previous work as it will address the proposed OFDM system based on FDCT via USFFT and Wrapping. In this work, some performance parameter will be investigated such as the PSD, spectral efficiency, computational complexity and the BER in the Raleigh channel.

2 CONVENTIONAL OFDM SYSTEM

The conventional OFDM system describes herein uses a simplified replica with the following assumptions: N is number of transmitted subcarriers, M modulation symbols in the m-th data frame, $a^m[k]$ where k=0,1,2,…, N-1, mapping interval $(0,T)$, T is the symbol duration, the frequency $f_0 = 1/T$, the Nyquist rate $1/T$ at N time instances is $t = nT/N$, n = 0, 1, 2, . . . , N-1, then the discrete form x(n) of the continues transmitted signal x(t) is:

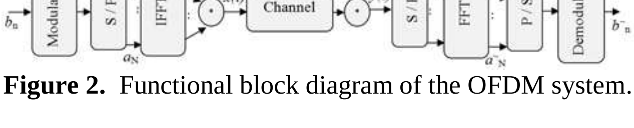

Figure 2. Functional block diagram of the OFDM system.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} a[k]e^{j2\pi kn/N} \tag{1}$$

Figure 2, shows a functional block diagram of an OFDM system. Consider x[n] is the transmitted OFDM signal which is the IDFT of the modulated input sequence b(n). As it known, in the practical implementation of the OFDM system the IFFT is used. The FFT is used on the receiver side for decoupling the subcarrier followed by a demodulator to detect the transmitted signal [1-5], assuming that the total channel delay is $N_g \geq L$, the channel coefficient $h(n,l)$, w(n) is the zero mean Gaussian random variable with variance $N_0/2$, then after removing the cyclic prefix (CP), the received signal y(n) is given by

$$y(t) = \sum_{l=0}^{L} H(n,l)\, x(n-l) \;+\; w(n) \tag{2}$$

The BER reduction is another performance metric parameter in wireless communication. To measure the noise robustness of OFDM scheme, a useful performance tool i.e. the relationship of the BER as a function of energy per bit to noise power spectral density ratio (Eb/N0) is used for different levels of noise and different types of channels [1-5]. The BER performance of the OFDM system is normally compared with the theoretical BER. Where, the theoretical BER performance of the BPSK is given by

$$P_e = Q\left(\sqrt{2(E_b/N_0)}\right) \tag{3}$$

3 CURVELET TRANSFORM

Curvelets have been introduced in 2000 [34], curvelet transform has been manly applied in the image processing. Moreover, it has been applied to other areas including: seismic processing, fluid mechanics, differential equations (PDEs) and compressed sensing or compressive sampling (CS). It has been approved that the curvelets transform is very effective in many field and give good performance compared to other type of transform [34-38]. The curvelets via USFFT, and the curvelets via wrapping [35] are considered as most faster, simpler, and less redundant of the FDCTs versions. Therefore, this work used these two types of the FDCTs.

3.1 Unequispaced FFT FDCT

In the FDCT via USFFT, taking the axes of the parallelogram with respect to an equispaced grid aligned, the DFT is demonstrated as a trigonometric polynomial and is taking in the interior each parallelepipedal region. So, there is a diverse sampling grid for each scale/orientation amalgamation. The forward transform is specified in closed form and is invertible. For the vector x(t); $-n/2 \le t \le n/2$ of size n, with a set of points (f_k); $1 \le k \le m$. The Fourier transform (FT) of x(t) and the inverse version $X(f_k)$ are given in equations 4 and 5 respectively:

$$X(f_k) = \sum_{t=-n/2}^{n/2} x(t) e^{-j2\pi f_k t} \quad (4)$$

$$x(t) = \sum_{k=1}^{m} X(f_k) e^{j2\pi f_k t} = \quad (5)$$

Using the non-uniform FFTs strategy to overcome the computation complexity problem, the strategy is then to convolve $X(f_k)$ with a short filter $H(f)$ to make it approximately band limited, then sample the result on a regular grid and apply the FFT, and de-convolve the output to correct for the convolution with $X(f)$ [36].
The other solution to overcome the computation complexity problem is using the USFFTs strategy, the idea is to compute intermediate Fourier samples on a finer grid and use Taylor approximations to compute approximate values of $X(f_k)$ at each node f_k. The algorithm operates as follows:
Using the zero padding on the signal x(t) to create x^(t) of size D*n* of-D$n/2 \le t \le$ D$n/2$

$$x^\wedge(t) = \begin{cases} x(t) & -n/2 \le t \le n/2 \\ 0 & elswhere \end{cases} \quad (6)$$

Then make L copies of x^(t) and multiply each copy by $(-it)^l$ obtaining

$$x^{\wedge l}(t) = (-it)^l x^\wedge(t), \quad l = 0,1,2,\cdots L-1 \quad (7)$$

Next take the FFT of $x^{\wedge l}(t)$ thus obtain the $X^{\wedge l}(f_k)$ with spacing $2\pi/n$, namely, $X^{\wedge l}(2\pi k/nD)$. Finally, given an arbitrary point f, where f_o is the closest fine grid point of f, evaluate an approximation of $X(f)$ by

$$X(f) \approx Y(f_0) := X(f_0) + X'(f_0)(f - f_0) + \cdots + X^{(L-1)}(f_0)\frac{(f-f_0)^{(L-1)}}{(L-1)!} \quad (8)$$

3.2 Wrapping FDCT

In this version, uses periodization instead of interpolation; this enable that the Fourier samples can be localized in rectangular region in which the IFFT can be applied. The forward transformation is determined in a closed form and is reversible with reverse in closed form. And for a specific scale, this only agrees with two Cartesian sampling networks, the first one is for all angles in the northwestern quadrants, and the second one is for the north-south quadrant. The curvelet transforms which are figured by wrapping is as geometrically realistic to the continuous transform as the sampling on the grid agrees. Using the algorithm of the FFT and the diagram that illustrate the data flow for the forward and inverse wrapping FDCT, the wrapping FDCT is implemented as shown in Figure 3.

First, the data are transformed into the frequency domain by the FFT. Second, it's multiplied with a set of window functions. Then by using the IFFT, the curvelet coefficients were obtained from windowing data. Because the window functions are zero excluding the support regions of elongated wedges; the regions that need to be transformed by the IFFT will be much smaller than the original data. On the wrapping FDCT and before being applied to IFFT algorithm, the FFT coefficients on these areas will be wrapped or placed into a rectangular form. The size of the rectangle is generally not an integer portion of the size of the a native data. This process is equivalent to filtering and sub-sampling the curvelet sub-band by rational numbers in two dimensions. Finally, the computational complexity of both algorithms for computing L FFTs of length Dn followed by m evaluation of the Taylor polynomial is only of $O(n \log n + m)$ [35].

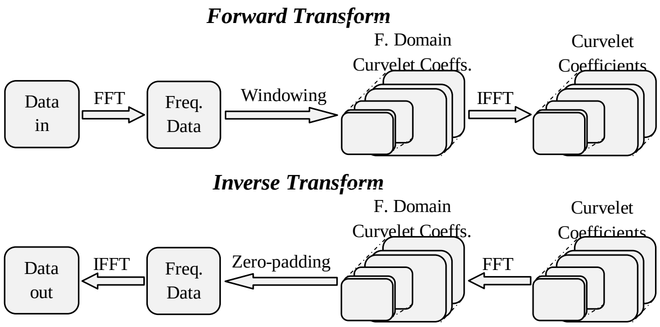

Figure 3. Data flow for the Forward and inverse wrapping FDCT.

4 OFDM SYSTEM BASED ON FDCT

Similar to the conventional OFDM system based on FFT, Figure 4 shows a functional block diagram of OFDM based on FDCT. In the proposed system at the transmitter side an IFDCT block is used while the FDCT is used at the receiver side.

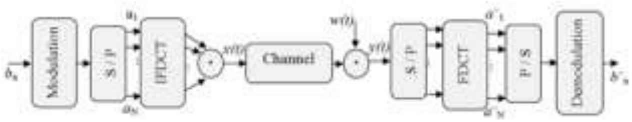

Figure 4. Functional block diagram of the OFDM system based on FDCT.

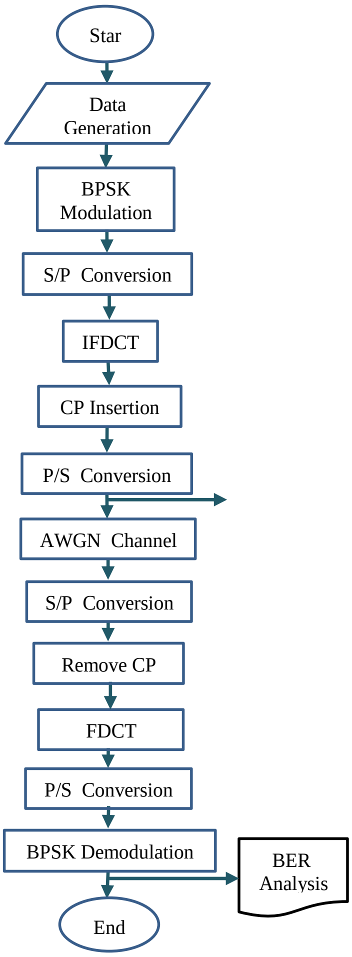

Figure 5. Simulation procedures of the OFDM system based on FDCT.

The simulation procedures used in this work are summarized in the flowchart shown in Fig. 5. The simulations carried on this work were implemented using MATLAB®. Simulations were performed in MATLAB® for the proposed system using FDCT/IFDCT functions and compared with conventional OFDM using FFT/IFFT functions. The BER simulation was carried out in this work in the Rayleigh 10-tap channel. Table.1 summarizes the simulation parameters that were used in this work.
The PSD, spectral efficiency and the BER performance for the transmitted signal of the proposed system were estimated through the simulation in a 10 – tap Rayleigh channel using BPSK as documented in Table 1. The computational complexity of the planned model was also investigated and compared with computational complexity of the conventional OFDM system.

Table 1. Simulation Parameters

Simulation Parameters	
Modulation	BPSK
Channel	10 – tape Rayleigh
number of subcarriers (N)	64
number of symbols	10^4
Cyclic Prefix	¼
number of data subcarriers	64
number of bit per OFDM symbol	8
FDCT	USFFT, wrapping

5 RESULTS AND DISCUSSIONS

Among the performance metric parameters of the considered system such as PAPR, PSD, the accuracy of channel estimation, computational complexity, spectral efficiency, data rate, and sensitivity to synchronization, the focus in this work will be on BER, spectral efficiency, PSD, and computational complexity. Other parameters, although important, but can be considered in the future work related to the proposed system.

5.1 SPECTRAL EFFICIENCY

In terms of spectral efficiency, the significant reduction in side-lobes levels is the main driver behind the recent trend of using curvelet filters in OFDM systems. curvelet filters provide better spectral containment than their Fourier counterparts. When the orthogonality between the OFDM carriers is missing when the signal is transmitted over an irregular or a non-uniform channel, the amount of interference between the carriers in the curvelet systems is much lower than the Fourier systems, where the side-lobes contain less energy.
The improved spectral containment reduces inter-carrier interference (ICI). Reducing the ICI without the need for a cyclic prefix (CP) is an

attractive feature of curvelet OFDM. It permits data rates to be pushed past those of Fourier OFDM, which relies heavily on the CP and 1-tap equalizer to mitigate the effects of ICI.
The main side-lobe of the Fourier filter has a magnitude 15 dB smaller than the main lobe, on the other hand, for the FDCT USFFT filter, the first side-lobe has a magnitude 47 dB below the main lobe, and 45 dB below the main lobe for the FDCT wrapping filter, as shown in Figure 6 and Figure 7, respectively. i.e., the first side-lob in the proposed system is approximately 30 dB attenuated from the main than in the conventional OFDM system.
The results show that curvelet based OFDM system has higher spectral efficiency and providing robustness ICI than the conventional OFDM system, because of the low out-of-band energy (low side-lobes).

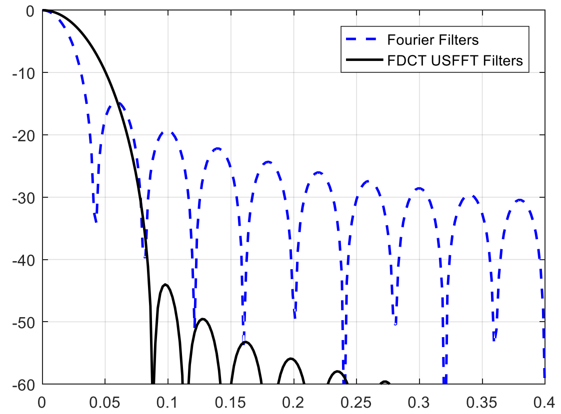

Figure 6. The frequency response of Fourier and FDCT USFFT for OFDM transmission.

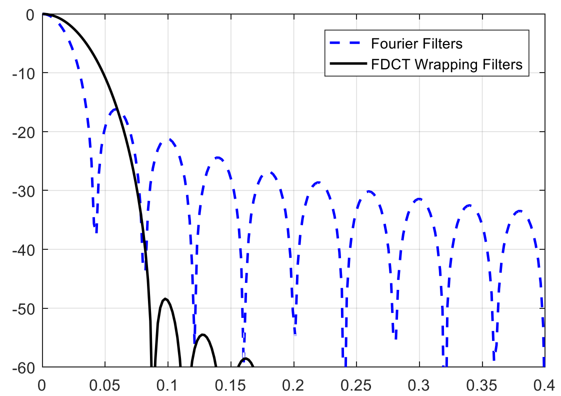

Figure 7. Frequency response of Fourier and FDCT wrapping for OFDM transmission.

5.2 Bit Error Rate

To measure the noise robustness of the proposed system, the relationship of the BER as a function of the energy per bit to noise power spectral density ratio (Eb/No) performance is a useful performance tool. The BER performances are shown in Figure 8 and Figure 9 using FDCT via USFFT and FDCT via wrapping respectively, with same simulation parameters of the above section when signaling with BPSK in a 10-tap Rayleigh channel.
For the results shown in Figure 8 and Figure 9 the red curve represents the BER performance of the proposed system, the black curve represents the BER for conventional OFDM and the blue curve represents the theoretical BPSK. These figures indicate that the BER performance of the considered system nearly matches the theoretical BER performance in the Rayleigh channel.

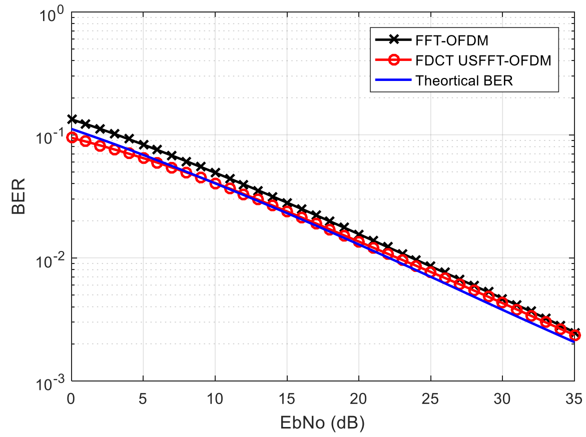

Figure 8. BER performance of OFDM system based on FDCT via USFFT.

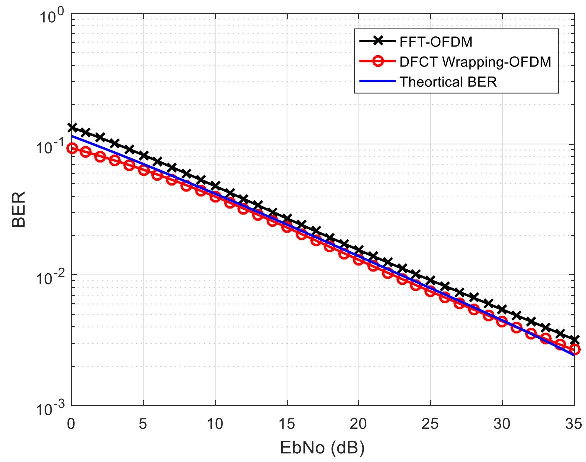

Figure 9. BER performance of OFDM system based on FDCT via wrapping.

5.3 Power Spectrum Density

Further, to demonstrate the similarities and dissimilarities between the conventional OFDM and the proposed system, their PSD characteristics are shown in Figure 10. The first curve (solid blue line) represents the PSD of the conventional OFDM, the second curve (solid red line) represents the PSD for the FDCT USFFT based OFDM system, and the third curve (solid black line) represents the PSD for the FDCT wrapping based OFDM system.
The spectral re-growth begins below 37 dB for the conventional OFDM systems, while the spectral re-growth begins below 50 dB and 52 dB for the proposed system based on FDCT via USFFT and FDCT via wrapping, respectively. It can be deduced that the proposed system is relatively showing better spectrum characteristics in terms of more low out of band attenuation (more suppression of out of band attenuation) than the conventional OFDM.

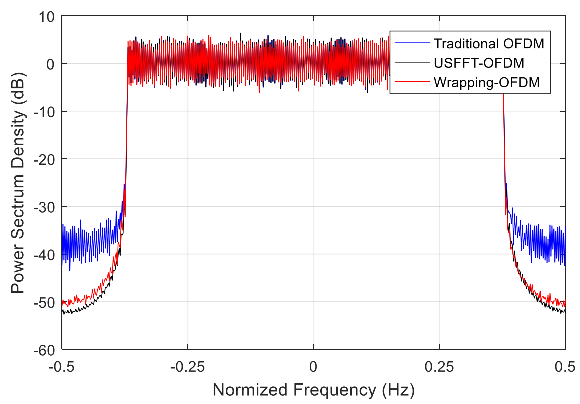

Figure 10. PSD for the proposed and conventional OFDM systems.

5.4 Computational Complexity

Computational complexity is an significant matter in recent communication system. As a result of the high data rates required in modern applications, low complexity is crucial. For the purpose of comparison, the computational complexity of conventional OFDM system will be taken into account.
Recall the computational complexity of Fourier, both Fourier and WPT, have a computational complexity of:

$$O(n \log n) \quad (9)$$

where, n is the rank of the transform, or the number of sub-channels, while the computational complexity for DWT is:

$$O(n) \quad (10)$$

Since the DTCWT use two DWT (upper and lower parts), the computational complexity for DTCWT is:

$$O(2n) \quad (11)$$

The complexity of the DTCWT is in less order comparing by the complexity of FFT and WPT. Finally, the computational complexity of both FDCT algorithms for computing L FFTs of length Dn followed by m evaluation of the Taylor polynomial is:

$$O(n \log n + m) \quad (12)$$

Thus it can be deduced that the complexity of the FDCT is in higher-order compared by the complexity of Fourier.

6 CONCLUSION

In this work, FDCT was used as alternative of FFT in the proposed OFDM system. The results of PSD indicate that the recommended system gives enhanced results than the conventional OFDM system. Using BPSK, BER performance was then examined in the 10 – tap Rayleigh channel; simulation results indicate that the BER performance of the proposed system is approximately same as the theoretical BER of BPSK in the Rayleigh channel. The results also show that the curvelet based OFDM system has higher spectral efficiency and low out-of-range energy (low side lobes) than the conventional OFDM system. Finally, in terms of computational complexity, the results show that the proposed system has higher computational complexity than the conventional OFDM.
Over all the performance results of the proposed system based on FDCT over the conventional OFDM, the computational complexity is consider as one of the drawback of the proposed system, thus this issue give a direction for future research activities in order to reduce the computational complexity of the proposed system. Generally, there are many possibilities for the future research activities in proposed model, and the using FDCT in OFDM system open many research feasibilities for the further improving OFDM system performance.

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