International Journal of Digital Information and Wireless Communications (IJDIWC) 1(4): 718-725
The Society of Digital Information and Wireless Communications, 2011(ISSN 2225-658X)

New Hexagonal Geometry in Cellular Network Systems

Adil Belhaj[1]*, Abdelouahed Jraifi[2,3]†

[1] Centre of Physics and Mathematics, CPM-CNESTEN, Rabat, Morocco

[2] Lab Physique de Hautes Energies: Modélisation et Simulation, Group of Canal Propagation Radio,

Faculté des Sciences, Université Mohammed V -Agdal Rabat, Morocco

[3] Maroc Telecom Rabat, Morocco

July 29, 2012

Abstract

Motivated by the existence of two frequency bands and inspired from Lie algebra classification, we propose a new geometric cellular shape with double hexagonal structure. Based on a two dimensional root system of the exceptional Lie algebra G_2, this cellular network model involves two hexagons of unequal side length at angle 30°. In this new hexagonal cellular networks, the principal hexagonal unit cell contains twelve coverage limits in contrast of the usual structure involving only six ones. Within the G_2 hexagonal structure, two frequency bands can be now placed at the same sheet in contrast to the single hexagonal structure where it can appear only one frequency band. This double hexagonal structure can bring improvements for the operators and may allow them to enhance many new factors. In particular, it can be used to enhance security energy and spacial flexibility in cellular networks.

KeyWords: Wirless communications, Interference, Hexagonal models, Lie algebras.

*belhaj@unizar.es
†jraifi@@gmail.com

1 Introduction

Recently, development of wireless communication models and networks have been considered as a subject of great interest in telecom systems [1, 2, 3, 4]. A special focus is to study intercell interferences in the modern cellular networks. In particular, many techniques for modeling such interferences, between telecom operators, are well developed in many works [5, 6, 7]. It has been shown that the solution of this problem can be articulated around the following approaches:

- The use of received band filtering
- The respect of guard band between assigned channels
- Separation of the site antennas
- Physical optimization including Tilt and Azimuth.

In connection with these activities, some efforts have been devoted to study the deployment of telecom sites. This is due to insufficient areas for installing base stations in higher building and higher density population. This issue has been extensively studied using various engineering methods and approaches [8, 9, 10].

An additional problem can arise due to the effect of signal radio on human health. To overcome such problems many technical methods have been explored using different computations ways based on appropriate geometric shapes. Usually, some of such geometries are too hard to handle analytically for dealing with the corresponding cellular network models. However, to facilate the calculation, from mathematical point of view, wireless connection in base station modeling requires a hexagonal structure on cellular networks [11, 12, 13]. Indeed, it has been shown that the conceptional hexagonal structure is adopted to wireless connection due to its simplicity and realistic behaviors. This hexagonal configuration is frequently employed in planning and analyzing of wireless networks due to its flexibility and convenience. In particular, it has been found that it can be used to mitigate coverage gaps compared with other flat geometries appearing in cellular networks.

On the other hand, due to insufficient rooms to implement based station systems on the same hexagonal area with two frequency bands, it is important to look new shearing solution for base stations. Based on this serious problem, we will try to introduce a new hexagonal geometry dealing with the placement modeling of the two base stations with different frequency bands at the same localization. Our main motivation is the connection between hexagonal structure and Lie algebras [14]. Indeed, the hexagonal structure arises naturally in the study of su(3) root system. Inspired from this connection, we propose a new flat geometry involving a double hexagonal structure based on a two dimensional root system of the exceptional Lie algebra G_2. In this telecom representation, it arises two hexagons of unequal side length at angle 30°. In this new hexagonal cellular network, the principal hexagonal unit cell contains twelve coverage limits in contrast of the usual structure(su(3) hexagon) involving only six ones. The G_2 hexagonal structure can be motivated from the fact in telecom systems arise two frequency bands. Each frequency band can be associated with a single hexagonal structure. In G_2 cellular shape, the two frequency bands can be collocated on the same area in contrast to the single hexagonal structure where it can appear only one frequency band. We expect that this new geometric cellular shape can be used in a special situation where it involves a higher density population in the absence of rooms for installing new base stations. This double hexagonal structure can bring new important physical resource improvements for the operators and may allow them to enhance many new parameters. In particular, it can be used to enhanced security energy and spacial flexibility in cellular networks.

2 Single hexagonal structure in telecom systems

In wireless communication systems, the cellular concept plays an important role in solving the problem of spectral congestion and capacity. The cellular concept is obtained by replacing high cell power transmitter (large cell) with many small cells. In this configuration, each cell provides coverage to only a small area.

As the demand for services increases, the number of cells must be increased to improve the user capacity. To realize a total coverage, the choice of a structure which can overlaid upon a map without leaving gaps or creating overlapping area is necessary. From mathematical point of view, there are there choices: a square, an equilateral triangle and regular hexagon. A cell shape is designed to serve mobiles within foot print. In wireless communication, we use the hexagonal concept to model coverage because of the following main reasons:

- Hexagonal model cover an entire area without overlapping,
- Hexagon has the largest area of the above mentioned geometries,
- Hexagonal system can be considered as the semi-realistic model.

Motivated by the role placed by hexagonal geometry in cellular network communications, we will propose a new geometry based on hexagonal root systems of an exceptional Lie algebra called G_2[14]. Before going ahead, let us show the first relation between cellular hexagonal shape and Lie algebras which may be useful for our proposed geometry. Indeed, the hexagonal structure appears naturally in the study of the root system of Lie algebra su(3). In this Lie algebra, there are two simple vector roots α_1 and α_2 of equal length, and at 120° angle:

$$\widehat{(\alpha_1, \alpha_2)} = 120^\circ. \tag{2.1}$$

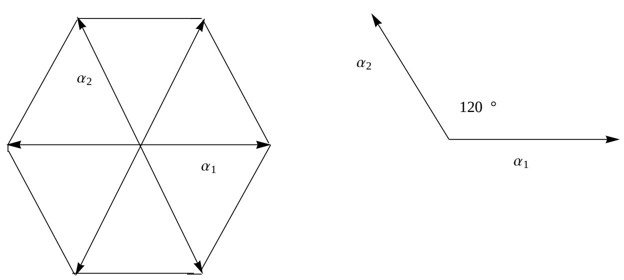

Figure 1: su(3) hexagon

Roughly speaking, we will see that the above root system can be associated with radio system in wireless communication. In fact, an unit hexagonal cell in telecom systems is associated with the six nonzero vector roots of su(3) Lie algebra. These roots are obtained from the simple ones (α_1 and α_2) with the sum and the opposed forming the well known single hexagon. Each root is associated with a coverage limit. For this algebra, all roots have equal length which is associated with the coverage radius. Since the base station at the center of each hexagonal cell, it should be associated with the zero roots corresponding to Cartan subalgebra.

The full picture of network can be obtained by using the fact that su(3) hexagons tessellate the full plane making a hexagonal cellular shape for communication systems. In what follows we go beyond such a structure by using a new symmetry used in high energy physics.

3 Double hexagonal geometry in telecom systems

Inspired from Lie algebras classification and motivated by the existence of two frequency bands in wireless communication systems, we implement a new hexagonal structure in telecom system. In particular, we bring a new geometry in telecom system involving a double hexagonal structure associated with G_2 Lie algebras. We refer to this structure as G_2 hexagons. It is recalled that this symmetry is an exceptional Lie algebra with rank 2. This structure appears in many placed in physics in particular in a seven real dimensional manifold, playing a crucial role in the M-theory compactification leading to four dimensional models with only four supercharges[16, 17]. In mathematics, G_2 symmetry is known by the group of automorphisms for the octonionic algebra given by $t_i t_j = -\delta_{ij} + c_{kij} t_k$, where c_{kij} are constants of the structure.

From the root system classification, G_2 contains a special hexagonal structure. In particular, we have two hexagons of unequal side length generated by two simple unequal roots α_1 and α_2 at angle 150°

$$\widehat{(\alpha_1, \alpha_2)} = 120° + 30° = 150°. \tag{3.2}$$

These two simples roots are constrained by

$$|\alpha_2|^2 = 3|\alpha_1|^2. \tag{3.3}$$

Having introduced the mathematical backgrounds of G_2 hexagons, now we engineer a double hexagonal structure in cellular network communications. To do so, consider first a double hexagonal unit cell. The converge limits will be associated with the root system of G_2 symmetry. In practice, each unit cell consists of 12 coverage limits instead of six appearing in single hexagonal structure [11]. Inspired from a root system classification, the unit cell shape involves two hexagons of unequal side length at angle 30°. Each simple root of G_2 symmetry generates a single hexagon. The small one is generated by the root set $\{\pm\alpha_1, \pm(\alpha_1 + \alpha_2), \pm(2\alpha_1 + \alpha_2)\}$. All these six root have equal side length. This length is associated with a coverage radius of the small hexagon, which may be associated with su(3) hexagon corresponding to a single frequency band. The big one is generated by the following equal side length $\{\pm\alpha_2, \pm(3\alpha_1 + \alpha_2), \pm(3\alpha_1 + 2\alpha_2)\}$ describing the second coverage radius.

A close inspection reveals that one can make a nice correspondence between rank 2 root systems and cellular network systems. It is recalled that there are four possible angles between two the simple roots. They are given by 90°, 120°, 135° and 150°. However only two algebras involve hexagonal structures which are su(3) and G_2 corresponding to the angle 120° and 150° respectively. As in the single hexagonal structure, G_2 hexagons tessellate the full plane making a double hexagonal cellular shape in network communications.

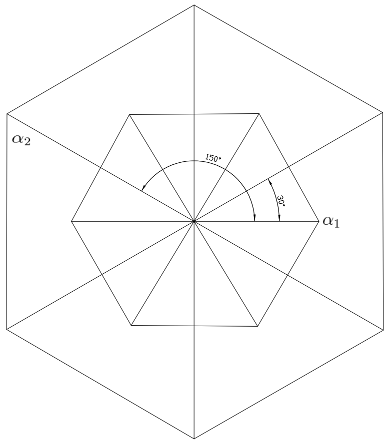

Figure 2: G_2 hexagons.

Roughly speaking, we complete the analyze of the correspondence between the above G_2 and cellular network systems. Our result is summarized in the following table:

Rank two Lie algebras	cellular network systems
Root systems	Radio systems
Non zero roots	coverage limiting
zero roots	base statitions
Simple roots	frequency bands
Simple roots with equal length	one frequency band
Simple roots with unequal length	two frequency bands

The introduction of the double hexagonal structure in cellular network systems can be supported by the existence of two frequency bands. Each frequency band can be associated with a single hexagonal structure. In this G_2 cellular shape, the frequency bands can be collocated on the same area in contrast to the single hexagonal structure where it can appear only one frequency band without shearing the same base station. It is worth noting that this new geometric cellular shape can be used in special situation where it involves either a higher density population or a missing space room to implement new base stations. Based on this problem, the double hexagonal structure can bring new promoting improvement for the operators. In particular, due to expensive investment in the network access for optimizing the infrastructure, the operators usually try to minimize the cost of deployment sites. We believe that double hexagonal structure will be of great interest in the sense that it could be used to estimate important factors including rate of overall capex.

4 Conclusion and open questions

Based on a two dimensional root system, we have proposed a double hexagonal structure in telecom systems. We have refereed to this structure as G_2 hexagons in communication networks. In this cellular network communication, it appears two hexagons of unequal side length at angle 30°. The principal hexagonal unit cell shape contains twelve vertices associated with coverage limiting in contrast of the usual structure involving only six ones. In this cellular shape, the frequency bands can be explored and shear the same base station placed at the center of G_2 hexagons. This double hexagonal structure can turn on some lights to improve the operator quality and may allow one to enhance extra parameters. In particular, it can be explored to enhanced security energy and spacial flexibility in cellular networks. Based on this nice correspondence between root systems and radio systems of cellular networks, it has been shown that it is possible to realize new cellular structures which may provide gains by engineering appropriate models. It should be intersecting to think about them.

Our paper comes up with many open questions. At natural question can be addressed to see the role played by the angle 30° required by Lie symmetry. Moreover, it should be interesting to give analytic network models based on G_2 cellular shape appearing a double hexagonal structure. We think that these questions deserve deeper study. We hope to report on them in future works.

Acknowledgments: AB would like to thank L. J. Boya, M.P. Garcia del Moral and A. Segui for discussion on G2 related topics.

References

[1] T. S. Rappaport, *Wireless Communications: Principles and Practice*, (2nd Ed.), Prentice Hall, 2002.

[2] J. W. Mark, W. Zhuang, *Wireless Communications and Networking*, Prentice Hall India, 2006.

[3] G. Haring, Raymond Marie, R. Puigjaner, K. Trivedi, *Loss Formulas and Their Applicatio to Optimization for Cellular Networks*, IEEE Transactions on Vehicular Technology, Vol. **50**, No. 3 (2001)664.

[4] Z. N. Chen, K. M. Luk, *Antennas for Base Stations in Wireless Communication*, McGraw-Hill Professional, 1 edition, 2009.

[5] A. Jraifi, R. A. Laamara, A. Belhaj, and E. H. Saidi, *A proposal solutio for intereference inter-operators*, Progress In Electromagnetics Research C, Vol. **12** (2010)15.

[6] A. Jraifi, *A theoretical prediction of users in WCDMA interefence*, Journal of Theoretical and Applied Information Technology, Vol. **35**, No 2 (2012)163.

[7] P. Stravroulakis, *Interference Analysis and Reduction for Wireless System*, Artech House, 2003.

[8] S. Katiyar, R. K. Jain, N. K. Agrawal, *R.F. Pollution Reduction in Cellular Communication*, arXiv:1204.2101.

[9] P. Madhusudhanan, J. G. Restrepo, Y. Liu, T. X Brown, K. R. Baker, *Multi-tier Network Performance Analysis using a Shotgun Cellular System*, arXiv:1110.3267.

[10] P. Marsch and G. P. Fettweis, *Static clustering for cooperative multi-point (comp) in mobile communications*, Proceedings IEEE Int. Communications (ICC) Conf. (2011)16.

[11] K. B. Baltzis, *A geometrical-based model for cochannel interference analysis and capacity estimation of CDMA cellular systems*, EURASIP Journal on Wireless Communications and Networking, Vol.**1** (2008)791374.

[12] K. Huang, J. G. Andrews, *A Stochastic-Geometry Approach to Coverage in Cellular Networks with Multi-Cell Cooperation*, arXiv:1103.4223.

[13] Y. Zhuang, J. Pan, *Random Distances Associated with Hexagons*, arXiv:1106.2200.

[14] J. E.Humphreys, *Introduction to Lie Algebras and Representation Theory*. Sauteur-Verlag, New York, 1978.

[15] N. Jacobson, *Lie Algebras*, J. Wiley and Sons, New York 1962.

[16] A. Belhaj, *Manifolds of G_2 Holonomy from N=4 Sigma Model*, J.Phys.**A35**(2002)8903-8912.

[17] A. Belhaj, M.P. Garcia del Moral, A. Restuccia, A. Segui, J.P. Veiro, *The Supermembrane with Central Charges on a G2 Manifold*, J.Phys.**A42**5(2009)325201, arXiv:0803.1827.