Degree of Diffusion in Real Complex Networks

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ABSTRACT

A measure of information diffusion in real complex networks is the degree of diffusion. In this work, we assess the effect of information diffusion of seven different directed and undirected real complex networks and for several undirected generated networks; random, small world, and scale-free networks. The degree of diffusion α has been used to measure the diffusion and the adoption rates of different complex networks. It is defined as the percentage between the adopters and non-adopters in a network during the diffusion process. The results showed that the degree of diffusion α of undirected networks is different than directed networks. All the obtained results showed that in real networks, randomization does not exist. The behavior of these networks is determined based on the network's members and the interactions between them. Therefore, most of the real networks should be classified as small-world, scale-free networks, or what we defined as small-world random, small-world scalefree networks.

KEYWORDS

Degree of diffusion; diffusion; directed networks; reverse diffusion; undirected networks

1 INTRODUCTION AND BACKGROUND

Nowadays, information diffusion in real networks is taking the researchers' attention in many areas such as biological, technological, and sociological. Diffusion is the process of spreading information through a network. Many techniques have been proposed to discover the diffusion and innovation in different networks. Several studies were conducted to analyse the diffusion in business area [1][2][3]. Floortje and Carolina studied in [1] the spread of information in a social network where the network consists of agents (consumers) that

are introduced to a new product and they should take a decision to buy or not the new product depending on their interests and their neighbours' decisions in social networks. Another consumer decision-making diffusion model has been introduced in [2] where the diffusion and the adoption probability were taken based on the external marketing activities and the impact of the internal influence of the individuals (consumers) in their personal networks. Moreover, a brandlevel competition diffusion model was suggested in [3] and applied on the China Mobile Communication Industry markets. The subscribers of China Mobile and China Unicom companies have been involved in the study. The researchers investigated the competition's impact of brand diffusion on China Mobile and China Unicom. A Predictive Model for the temporal dynamics of information diffusion was developed in [4] to predict the diffusion in online social networks. This model applied on a Twitter dataset and showed that the model is useful and effective in determining the diffusion value in online social networks.

On the other hand, some researches proposed genetic techniques to determine the diffusion in networks as in [5][6]. A Genetics-based Diffusion Model (GDM) has been proposed in [5], which have the ability to create various objects with different relationships distributed in social networks. In GDM, the nodes represented as chromosomes and the communications between the nodes represented as genes. Another genetic technique has been introduced in [6], which is called Genetic Algorithm Diffusion Model (GADM). GADM has been used to show the information flow for large dynamic social network produced from e-mail headers.

The degree of diffusion α has been used to determine the diffusion of information in social

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networks [7]. It has been determined by the calculation of the percentage of adopters and non-adopters in the network through different penetration depths during the diffusion process. We selected our previously developed techniques in this work and applied it on different types of generated and real networks.

In general, the networks are divided into three types of models depending on the network structure and characteristics: Random, Small world, and Scale-free network models. Erdos and Renyi proposed in 1959 an important model for random networks [8][9][10]. This model is a probabilistic method, which discusses the random network behaviour based on probability. It can be used to generate a network of any size. The model consists of *N* nodes. A pair of nodes is connected in this model based on a probability p. The resultant network is a simple network.

Small-world networks have been characterized between regular networks, with high clustering and high diameter, and random networks, with low clustering and small diameter. But in (Watts and Strogatz, 1998), Watts & Strogatz analysed small-world networks in 1998 characterized them as networks with high clustering as the regular networks and low diameter as the random networks. In [11], Watts and Strogatz showed that natural networks, such as neural network of the worm Caenorhabditis elegans network, and artificial networks, such as the power grid of the western United States network, are highly clustered and have very low average shortest path between two nodes (low diameter). To generate a WS small-world network, if the network consists of N nodes, each node will be linked to a certain number of neighbouring nodes (2k). Then in the next level, with a rewiring probability P, each link will be rewired to a random node in the network.

In some real networks, the nodes have different degrees and that is because some nodes have more connections with other nodes than others. This type of network is called scale-free network where the number of nodes increased throughout the lifetime of the network. This what Barabasi and Alberts proved in their work [12]. Barabasi and Alberts defined scale-free networks as networks which has an initial population M_0 and where the

new nodes are added and connected to the initial population. P_c is the probability of a new node to have a connection with an existing node in the network. Real networks can be of different sizes and connectivity. Real networks, such as biological, sociological, and technological networks also can be considered as one of the three described networks models. For instance, yeast protein-protein interaction network is a random network [13], dolphin social network is a scale-free network, Caenorhabditis elegans worm's neural network [11], us power grid [11], and positive sentiment social network [16] are small-world networks.

In our previous works [7][17], we studied the degree of diffusion to calculate the adoption rate and applied it on different types of complex networks. In [7], we used several directed networks whereas in [17], we used different directed and undirected real networks and different generated undirected networks: random network, scale-free network, and small-world network. Then the results in [17] have been compared with the results obtained in [7]. The results showed that the degree of diffusion a of undirected networks is different than directed networks. For instance, the average number of degree of diffusion was 148 for directed random network where it was 1.9705 for undirected random network. All the obtained results showed that in real networks, randomization does not exist. The behaviour of those networks is determined based on the network's members and the interactions between them. Therefore, most of the real networks should be classified as smallworld, scale-free networks, or what we defined as small-world random, small-world scale-free networks.

The rest of the paper is organized as follows: the entropy and cyclic entropy is presented in section 2. The degree of diffusion model is explained in section 3 along with the explanation of diffusion and reverse diffusion process, the experiments, and the results. Then, in section 4, the real networks used in this paper are explained in details with their network graphs. Section 5 shows the results and a discussion about the results. Section 6 and section 7 contain the conclusion and the future work respectively.

2 ENTROPY AND CYCLIC ENTROPY

The information-theoretical definition of entropy is "the minimum number of bits you need to fully describe the microscopic configuration and detailed state of the system" [18]. This definition of entropy is related to the traditional thermodynamic definition of entropy.

Although the concept of entropy originated in thermodynamics and statistical mechanics, it has found applications of numerous subjects such as communications, economics, information science and technology, linguistics, music, etc. The probability characteristic of entropy leads to its use in communication theory as a measure of information. The absence of information about a situation is equivalent to an uncertainty associated with the nature of the situation. This uncertainty is the entropy of the information about the particular situation.

Let k be a set of discrete random variable that takes the following values $k=\{1...k...N\}$ with probabilities $p=\{P(1),...,P(i),...P(N)\}$ respectively

such that $P(k) \ge 0$ and $\bigcap_{k=1}^{N} P(k) = 1$. There exists a

measure of randomness, heterogeneity and uncertainties known as *entropy*, *H* defined as

$$H(p) = - \mathop{\tilde{a}}_{k=-1}^{N} p(k) \ln(p(k))$$
 (1)

In a complex network context, P(k) may represent the degree distribution of links or the remaining degree (outward links) or cycles of size k in the network. Degree distributions of links are the most common representation of P(k) in the literature. Different models of networks such as random, small-world, scale-free, exponential, uniform and many others are usually represented and constructed as degree distribution models of actual networks; such as software, social, biological, circuits ... etc. network. Simple degree distribution describes the connectedness of the network; hence the entropy will be a measure of heterogeneity, uncertainty of network connectedness [19].

3 DEGREE OF DIFUSSION MODEL

This section contains the explanation of the degree of diffusion model. The diffusion and the reverse diffusion processes are described in part A and part C respectively. The results that were obtained in [7][17] are summarized in Section 3.4.

3.1 Diffusion Process

The adoption process in a network consists of several levels where in each level there will be new adopters. A single adopted/infected node starts the nodes adoption/infection operation in the diffusion process. This node will spread the infection to its direct neighbours and they will become adopters. Then, the new adopters will also infect their direct neighbours in the next level...etc. This process will stop when all the nodes become adopters or when there are no more nodes that can be reached.

3.2 Penetration Depth and Adoption Rate

Each level in the diffusion process is called penetration depth that represents (n)equilibrium level between the adopter phase and the non-adopter phase in the diffusion process. Penetration depth is the first parameter that should be considered in the process of determining the diffusion in a network. The maximum number of penetration depth n_{max} has been defined as the maximum distance between the selected source node and the last adopter node in the diffusion process [7]. The value of n_{max} is differs depending on the selected source node. Figure 1 illustrates the diffusion process for 8 nodes with maximum penetration depth = 4. In the example, Node 1 is selected as the start infected source node at level 0. Then node 1 infects its neighbour, node 2, 7, and 6 at level 1. Then each infected node will infect its neighbours at the next level. The diffusion process stopped at level 3. Node three is considered as an isolated node because it has no relations with the other nodes. Therefore, node 3 is kept as a non-adopter node. If node 3 is selected as a source node, then the penetration depth will equal to 0 because no nodes are connected to it.

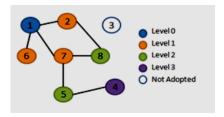


Figure 1. The diffusion process for a network with 8 nodes and maximum number of penetration depth=4.

The second important parameter in the diffusion process is the degree of diffusion α . The degree of diffusion has been calculated in [7] as a ratio between the current adoption rate (Yn) to the predicted future adoption rate (Yn+1) as shown in equation (2). Then, from (2), the future adoption rate can be calculated as (3).

$$\partial = \frac{Y_{n+1}}{(1 - Y_{n+1})} / \frac{Y_n}{(1 - Y_n)}$$
 (2)

$$Y_{n+1} = \frac{\partial Y_n}{1 + \partial (1 - Y_n)} \tag{3}$$

In order to calculate the degree of diffusion in a network, the following steps should be taken as set in [7]:

- Construct a network.
- Select a source node. The selection process could be random.
- Calculate the percentage of adopter nodes. There is an assumption that has been set for this step. The assumption is that any node connected to the source node will become an adopter. Then the diffusion proceeds to the next penetration level. At each penetration level, the adoption rate is calculated.
- Then, the obtained results will be fitted in equation (4) to get the value of alpha.

$$Minimize \mathop{\overset{N-1}{\stackrel{}{\circ}}}_{n=0} (Y_{n+1} - \frac{\partial Y_n}{1 + (\partial - 1)Y_n})^2$$
 (4)

The equation (2), (3), and (4) have been constructed in [7]. Figure 2 shows the algorithm that represents the diffusion process to obtain the degree of diffusion α .

3.3 Reverse Diffusion

Reverse diffusion process is defined in [7] as an opposite process of diffusion process. In reverse diffusion, all the nodes in the network have been assumed to be adopters. Then, a source node will be selected and its status will be changed to non-adopter. After that, in the next level (penetration depth) all the source node neighbours that have outgoing links towards the source node will be infected and become non-adopters. Similar to diffusion process, reverse diffusion will stop if all the nodes become non-adopters or no more nodes can be reached.

```
    Input: Transition matrix T representing a network with size N nodes.

• Output: Y_n, n and \alpha.
• Algorithm:
      Eliminate isolated nodes
              set color[i] = white
      for j \leftarrow 0 to max n
              Y_n = 0
      count = 0, n = 0
      Select source of diffusion S
      If S has no neighbors, then \alpha = 0, END.
              Y_{n} = 1/N
11.
              for j \leftarrow 1 to N
                  if (T(S,j)=1 \& S \neq j \& color[j]=White)
13.
                       color[j] = Gray
14.
15.
                       count = count + 1
16.
              Y_{\cdot \cdot} = count / N
17.
              While there exist nodes with color = Gray
                      n = n + 1
                      \forall Gray node(k)
                             color[k] = Black
20.
21.
                              for j \leftarrow 1 to N
22
                                  if (T(k,j)=1 \& k \neq j \& color[j]=White)
23.
                                       color[j] = Gray
24.
                                       count = count + 1
25.
                      Y_n = count/N
26.
              \alpha = calculate \_alpha(Y_n,n)
27.
      END.
```

Figure 2. Algorithm for calculating α , Yn, and n for a selected source node S[7].

3.4 Diffusion and Reverse Diffusion Results

Diffusion and reverse diffusion processes have been applied in [7] on different types of directed networks: random network, scale-free network, small-world network and virtual friendship social network. In [17], we applied diffusion and reverse diffusion processes on directed real networks to verify the work done in [7]. Then, we generated random, scale-free, and small-world undirected networks and applied diffusion and reverse diffusion processes on them to obtain the average degree of diffusion for undirected networks. Finally, we applied both processes on undirected real network in ordered to categorize them. Several outcomes have been founded. All the results related to the real networks shows that those networks should be classified as small-world or scale-free networks. This is rational since the relations and the behaviour of these types of networks should not be random. This is different than the results obtained using the classical classification techniques of networks. On the other hand, the results of applying the diffusion and reverse diffusion processes on the undirected networks illustrates that the average degree of diffusion is equal since the links were treated as outgoing and ingoing links at the same time. Therefore, both algorithms will function similarly.

4 REAL COMPLEX NETWORKS

The term Degree of Diffusion α defines each network type along with predicting the proportion of future adopters over non-adopters at any given penetration level through the diffusion process while, in reverse diffusion, it is assumed that all nodes are adopted the information and then the process starts by choosing a single node (non-adopter) to be a source of the reverse diffusion process. In the next penetration level, all neighbouring nodes with outgoing links to the source will be infected and become non-adopters.

To experiment the above-explained Diffusion and Reverse Diffusion processes, seven different real networks have been chosen. The networks vary in type (directed & undirected) and model with different parameter as shown in Table 1. The chosen networks are described below.

4.1 Collaboration Network in Science of Networks

The network NetScience contains a coauthor ship network of scientists working on network theory and experiment, as compiled by M. Newman in May 2006. An edge is drawn between a pair of scientists if they co-authored one or more articles during the same time period [25]. The network was compiled from the bibliographies of two review articles on networks, M.E.J. This network is considered as a collaboration network and contains 1589 scientists working together as shown in Figure 3.

The work in [20] elaborates on the degree of distribution in networks and defines it to be the fraction of vertices that have degree k. Equivalently, it is the probability that a vertex chosen uniformly at random has degree k [20]. Networks with power-law degree distributions are referred as scale-free networks [20]. The study in [20] also argues that real-world networks unlike the random graphs do not have Poisson distribution for their degree of distributions. Instead, they are highly right-skewed, meaning that their distribution has a long right tail of values that are far above the mean. The work done in [20] shows that NetScience network has an exponential form of degree of distribution and thus it is considered as a scale-free network.

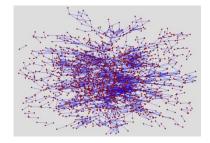


Figure 3. NetScience network.

4.2 Bottlenose Dolphin Social Network

This network is constructed from observations of a community of 62 bottlenose dolphins over a period of seven years from 1994 to 2001 as shown

in Figure 4. Nodes in the network represent the dolphins and ties between nodes represent association between dolphin pairs occurring more often than expected by chance. In [14] the bottlenose dolphin social network is considered a scale-free network with complex power-law distribution for large k.



Figure 4. Bottlenose Dolphin Social network.

4.3 US Power Grid Network

This network is the high-voltage power grid in the Western States of the United States of America. The nodes are transformers, substations, and generators, and the ties are high-voltage transmission lines. This network was originally used in [11]. In [11] this network is classified as a small-world network based on the structural properties of this network. Figure 5 shows the network.

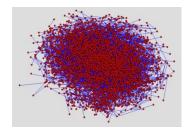


Figure 5. US Power Grid network.

4.4 Open Flights

The dataset contains flights in 8056 different airports around the world including two airports in the US. The data belongs to OpenFlights.org. The work in [21] has categorized it as a small-world network. Figure 6 shows the network.

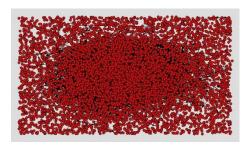


Figure 6. Open Flights network.

4.5 Extraction, Visualization & Analysis of Corporate Inter-relationships (EVA)

The EVA is a prototype system developed by [14] for extracting, visualizing, and analysing corporate ownership information as a social network. The information has been retrieved from sources on online text including corporate annual reports within the United States. The network contains companies as vertices with relationships among those companies as shown in Figure 7. According to [14], the network is highly clustered with over 50% of all companies connected to one another in a single component. An arc (X,Y) from company X to company Y exists in the network if in the company X is an owner of company Y. [14] has analysed the characteristics of the network such as degree, betweenness, cut points and cliques. Analysis reveals power law distributions for two important network metrics, namely component size (number of companies connected together) and company degree (number of ownership relationships in which a company is involved) [14]. This information is for [14] enough to consider the network as a scale-free network.

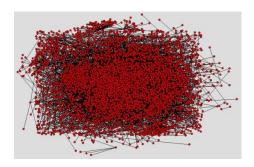


Figure 7. Extraction, Visualization & Analysis of Corporate Inter-relationships (EVA) network.

4.6 Gnutella Peer-2-Peer Network

A sequence of snapshots of the Gnutella peer-to-peer file-sharing network from August 2002. There are total of 9 snapshots of Gnutella network collected in August 2002. Nodes represent hosts in the Gnutella network topology and edges represent connections between the Gnutella hosts. Gnutella peer-to-peer network is a scale-free network [22][23]. Figure 8 represents sample of the network.

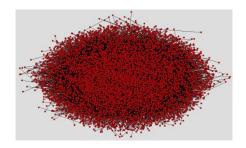


Figure 8. Gnutella Peer-2-Peer network.

4.7 US 500 Busiest Airports

It is a network of the 500 busiest commercial airports in the United States as shown in Figure 9. The data has been collected in year 2002 and a tie between two airports implies a flight scheduled between them. The work in [24] studies the behaviour of basic Reaction-Diffusion (RD) process on networks with heterogeneous topology. According to analysis of phase diagram and critical threshold properties of diffusion process of

particles done by [24], the US 500 network is considered a scale-free network.

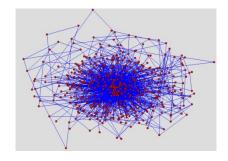


Figure 9. US 500 Busiest Airports network.

5 EXPERIMENTS AND RESULTS

The experiment section is divided into two parts. First, the diffusion and reverse diffusion processes have been applied and analysed on three different undirected networks, Collaboration network in science of networks, US power grid network and US 500 busiest airports, explained in section 5. Second, the same process and analysis have been applied to four different directed networks, dolphins network, US open flights network, extraction – visualization and analysis of corporate inter-relationships network and Gnutella Peer-to-Peer network. Then the average degree of diffusion and average degree of reverse diffusion for each network has been obtained. This section explains in details the two parts of the experiment with their results and a discussion about the results.

5.1 Part 1: Diffusion and Reverse Diffusion Processes on Undirected Real Networks

As stated in section IV part B, the same steps have been taken to calculate the average alpha for each network. Table 2 shows the results of applying both diffusion and reverse diffusion processes on Collaboration network, US power grid and US 500 busiest airports. Figure 10 and Figure 11 show the adoption rate for three randomly selected nodes in US 500 busiest airports and US power grid network respectively. In US 500 airports, they all converge to 100% at the same penetration depth (6). But, in US power grid network the penetration

depth varies based on the selected network from 30 to 37. Obviously, the node with the least penetration depth is the node that can help in propagation of information faster than the others. In general, in any network there are some nodes that are the best candidates for adoption of information.

The degree of diffusion for US 500 airports as shown in Table 2 suggests that this network is a random network. In Figure 10 it is obvious that with a small number of hops (penetration depths), the entire airports network can be covered. Observing the results of $\bar{\alpha}$ for US power grid shows that although [11] has classified it as smallworld network, our results show that it is a scalefree network. As stated in [17], that a large number of networks that are usually classified using regular characterization techniques are in fact networks that are close to small-world or what we named small-work scale free networks. Also, the results for the Collaboration network indicate that it is a small-world network rather than a scalefree network as stated in [25] for the same reasons mentioned above. Since the studied networks are undirected, the average degree of diffusion and the average degree of reverse diffusion are the same. That is because in undirected networks the incoming and outgoing links are no different from each other. Figure 12 illustrates both diffusion and reverse diffusion on the same node in each network.

5.2 Part 2: Diffusion and Reverse Diffusion Processes on Directed Real Networks

In the second part, the networks that were inspected were real directed networks. Table 2 shows the results of applying the diffusion and reverse diffusion processes on Dolphins network, US open flights network, Extraction-Visualization and analysis of corporate inter-relationships (EVA) network, and Gnutella Peer-2-Peer network. Figure 13 shows the adoption rate of three different nodes in Gnutella network and US Open Flights network. They follow an S-shape curve where all reach to the same level adoption at the end. In Gnutella network as shown in Figure 13(a), the curves show that the adoption rate is

around 97% of the whole network which is an indication of a strongly connected Peer-to-Peer network with a small number of isolated nodes. Usually, in Peer-to-Peer networks the clients (peers) are connected to a big number of other clients to share files between each other. The average degree of diffusion for Gnutella network is in the scale-free network range. Figure 13(b) illustrates that the adoption rate in US Open Flights network is around 36% of the whole network since most of the nodes in the network are isolated. The degree of diffusion for this network is in the small-world network range, which agrees with what [21] has earlier categorized it.

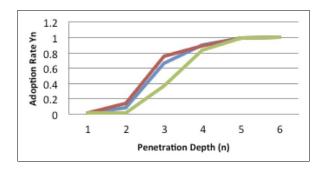


Figure 10. Adoption rate for three different nodes in US 500 busiest airports network.

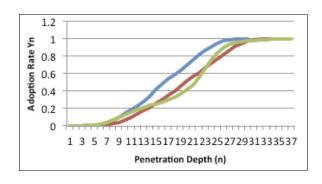
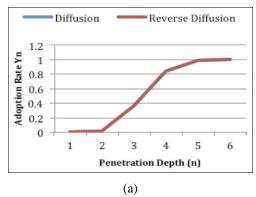
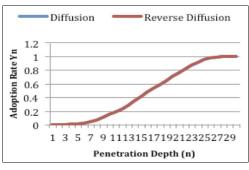


Figure 11. Adoption rate for three different nodes in US power grid network.





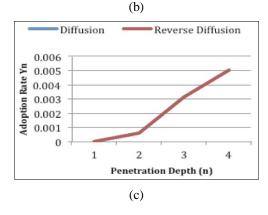
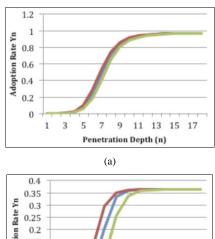


Figure 12. Adoption rate curves for diffusion and reverse diffusion processes for selected nodes for: (a) US 500 airports (b) US Power Grid (c) Network of Sciences.

In Figure 14, a graph for calculated alphas (α) for different nodes have been plotted. Those values have been calculated using the minimization equation (4). In Figure 14(a) the different values of Dolphins network have been plotted. As shown in the plot in Figure 14(b), most of the first 100 nodes of the network have adoption rate of zero indicating that they are isolated nodes. On the other hand, one node is observed to have a very high adoption rate, which indicates that this node is a very good candidate to be used for information adoption.



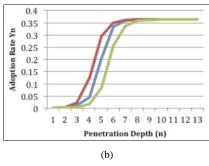
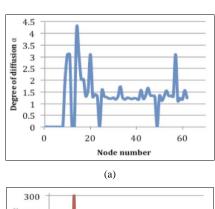


Figure 13. Adoption rate vs. penetration depth n for selected nodes for: (a) Gnutella (b) US Open Flights.



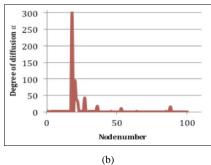
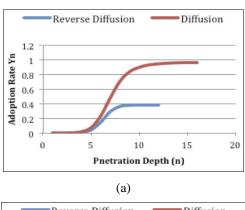


Figure 14. The degree of diffusion values for: (a) Dolphins network (b) EVA first 100 nodes.

For the sake of comparison between diffusion and reverse diffusion, one node has been chosen for each network. Figure 15 shows the result of a sample node chosen to illustrate the adoption rate in diffusion and reverse diffusion processes. In Figure 15(a), it is shown that the reverse diffusion process curve is different than that of diffusion curve which matches the results in [7]. In Figure 15(b), there is a slight small difference between the diffusion and reverse diffusion curves. This is a small-world behavior since every node has 2k neighbors and 2k outgoing links exist at each node [7].



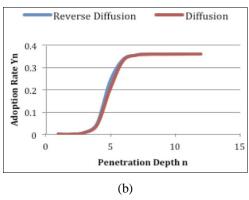


Figure 15. Plot for Diffusion vs. Reverse Diffusion on selected node for: (a) Gnutella (b) US Open Flights.

6 CONCLUSION

Directed and undirected real and model networks were studied. Information diffusion and reverse diffusion in real networks is used to categorize a network as random, scale-free, small-world, or mixed networks depending on a new parameter called the degree of diffusion α . The degree of diffusion α has been defined as the percentage between the adopters and non-adopters in a network during the diffusion process [7].

7 FUTURE WORK

For the future work, we are planning to modify the diffusion and reverse diffusion process used in this paper to be applied on weighted graphs where the weight represents the relations and how strong they are between the nodes in the network.

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 Table 1. Illustrates the real networks characteristics and parameters.

Network Type Network Parameters	NetScience (1589 nodes)	Dolphin (62 nodes)	US Power Grid (4941)	Open Flights	EVA (8348 nodes)	Gnutella P-2-P	US 500 Airports
Watts-Strogatz clustering coefficient	0.87820556	0.15146614	0.10653888	0.56643868	0.03130506	0.00464493	0.72645793
Transitivity	0.69344141	0.15438787	0.10315322	0.25158444	0.00034849	0.00379897	0.35138176
Betweenness	0.02635359	0.01384186	0.28483100	0.00918323	0.00008572	0.01168033	0.21194731
Degree	30.28786840	3.55	16.33751772	232.77166625	550.30119983	40.40988241	133.61445783

Table 2. Experiment results: part I and II – real complex networks.

Network Model	Network Type (Obtained from other works)	Number of nodes	$\bar{\alpha}$	Network Type (From our work)	Range (α_{min} - α_{max}) α isolated nodes=0	Reverse α
Collaboration Network in Science of Networks	Undirected / Scale-free	1589	2.42259	Small-world	1.04716 – 21.2679	2.42259
Dolphins	Directed / Scale-free	62	1.59621	Scale-free	1.10779 - 4.2069	1.55243
US Power Grid	Undirected / Small-world	4941	1.20793	Scale-free	1.02147- 1.4833	1.20793
Open Flights	Directed / Small-world	7976	1.0531	Small-world	1.03055 - 7.00527	1.04866
EVA	Directed / Scale-free	8343	3.24684	Small-world	1.00488-299.156	1.71562
Gnutella P-2-P	Directed / Scale-free	8846	1.23081	Scale-free	1.01225 - 11.0125	1.0665
US 500 Airports	Undirected / random	500	36.343	Random	1.20243 - 157.32	36.343

Table 3. Experiment results: part III – generated complex networks.

Network Type	Network Parameters	Number of nodes	$ar{lpha}$	Reverse α	Entropy (S)
Random		50	1.93	1.93	2.98
		100	13.37	13.37	3.25
	p=0.05	150	19.28	19.28	4.08
		200	37.35	37.35	4.53
		250	29.42	29.42	5.32
		300	23.80	23.80	5.51
Scale-free		90	4.52	4.52	3.06
	$M=5, M_0=5$	100	3.41	3.41	3.21
		110	3.19	3.19	3.08
		130	1.37	1.37	3.27
		150	6.89	6.89	3.53
Small-world	1 4 005	100	3.35	3.35	3.24
	k=4, p=0.05	150	2.11	2.11	4.04